Physics 176
JUL 3 1933

THE PHYSICAL REVIEW

Cphy 2

VOLUME 43 *Second Series* NUMBER 12

JUNE 15, 1933

Published for the

AMERICAN PHYSICAL SOCIETY

by the

AMERICAN INSTITUTE OF PHYSICS
Incorporated

LANCASTER, PA., AND NEW YORK, N. Y.

THE AMERICAN PHYSICAL SOCIETY

OFFICERS OF THE SOCIETY

PAUL D. FOOTE, *President*
The Gulf Companies
Pittsburgh, Pennsylvania

ARTHUR H. COMPTON, *Vice-President*
University of Chicago
Chicago, Illinois

W. L. SEVERINGHAUS, *Secretary*
Columbia University
New York, New York

G. B. PEGRAM, *Treasurer*
Columbia University
New York, New York

JOHN T. TATE, *Editor*
University of Minnesota
Minneapolis, Minnesota

L. B. LOEB, *Local Secretary for the Pacific Coast*
University of California
Berkeley, California

THE PHYSICAL REVIEW

BOARD OF EDITORS

JOHN T. TATE, *Editor* — J. W. BUCHTA, *Assistant Editor*

Associate Editors

A. J. DEMPSTER — R. C. GIBBS — K. K. DARROW
DAVID M. DENNISON — E. O. LAWRENCE — J. C. HUBBARD
J. C. SLATER — J. H. VAN VLECK — A. E. RUARK

THE PHYSICAL REVIEW, which was established in 1893, is a journal of experimental and theoretical physics. It is published for the American Physical Society by the American Institute of Physics Incorporated. There are two volumes annually with twelve issues in each volume.

Manuscripts for publication should be submitted to John T. Tate, University of Minnesota, Minneapolis, Minnesota.

Proof and all correspondence concerning papers in the process of publication, should be addressed to the Editorial Secretary, American Institute of Physics, 11 East 38th Street, New York, New York.

Subscription price $15.00 per year. Single copies $1.50 each for issues prior to July 1, 1929 (Vols. 1–33); $.75 each for issues subsequent thereto. Subscriptions and orders for back numbers should be addressed to Prince and Lemon Streets, Lancaster, Pennsylvania, or to the American Institute of Physics, 11 East 38th Street, New York, New York.

Advertising is not accepted for the *Physical Review* but attention is called to the *Review of Scientific Instruments with Physics News and Views*, a companion publication of the Institute, and a medium through which the field of physics can be reached most effectively. Rates and other information will be furnished on request to the Editorial Secretary.

Changes of address and complaints of failure to receive the *Physical Review*, in the case of members of the American Physical Society, should be addressed to the Treasurer; in the case of other subscribers, to the Editorial Secretary. New copies can be sent free in response to complaints of non-delivery only if notice of non-delivery is received within three months of date of issue.

The Physical Review is published semi-monthly at Prince and Lemon Streets, Lancaster, Pennsylvania.

Entered at the Post Office at Lancaster, Pennsylvania, as second class matter.

Acceptance for mailing at special rate of postage provided for in section 1103, Act of October 3, 1917, authorized September 6, 1926.

THE
PHYSICAL REVIEW
A Journal of Experimental and Theoretical Physics
VOL. 43, No. 12 JUNE 15, 1933 SECOND SERIES

Effect of Temperature on the Intensity of X-Rays Scattered by Powdered Sodium Fluoride

J. J. SHONKA, *University of Chicago, Chicago, Illinois*
(Received April 10, 1933)

The ratio of the integrated intensities of a number of regularly reflected x-ray lines from powdered sodium fluoride crystals at room and liquid air temperatures has been studied by means of a photographic method. The results of the experiments show that the ratio varies with the angle as predicted by the Debye-Waller formula. The characteristic temperature of sodium fluoride was found to be equal to 442°K. This value agrees as well as one could expect with values calculated by the Lindemann melting-point formula and the known characteristic temperatures of sodium chloride or potassium chloride crystals. Using the known structure factor of sodium fluoride atoms at room temperature and assuming zero-point or no zero-point energy the structure factors of the atoms at rest have been calculated. A comparison with the theoretical structure factors of sodium fluoride and neon (sodium fluoride atoms are neon-like) and with the known experimental structure factor of gaseous neon atoms indicates that the assumption of zero-point energy is correct. The square root of the mean square amplitude of thermal vibration of the sodium and fluorine ions at room temperature was found to be equal to 0.182A.

THE first theoretical investigation of the effect of temperature on the intensity of x-rays reflected from simple cubic crystals composed of atoms of one kind was made by Debye.[1] He showed that if f is the structure factor of the atoms at rest, the structure factor F at any temperature T is

$$F=fe^{-M/2}, \tag{1}$$

where

$$M=\frac{6h^2}{mkT_c}\left\{\frac{\varphi(x)}{x}+\frac{1}{4}\right\}\frac{\sin^2\theta}{\lambda^2} \tag{2}$$

or

$$M=\frac{6h^2}{mkT_c}\cdot\frac{\varphi(x)}{x}\,\frac{\sin^2\theta}{\lambda^2},$$

according as one assumes zero-point energy of $\frac{1}{2}$ quantum per degree of freedom or no zero-point energy. Here h is Planck's constant, m is the mass of the atom, k is the gas constant, T_c is the characteristic temperature of the substance, $x=T_c/T$, $\varphi(x)$ is a function of x which function Debye evaluates in his paper, θ is one-half of the angle of deviation and λ is the wave-length of the incident radiation.

The above formula having been found to be in qualitative but not quantitative agreement, Waller[2] extended the theoretical study of this subject and modified Debye's formula to read

$$F=fe^{-M}. \tag{3}$$

This equation can be tested experimentally, except as regards zero-point energy, by measuring the ratio of the integrated intensity of any regularly reflected line at a temperature T_0 and the integrated intensity at various other temperatures T_1, for from Eqs. (2) and (3), noting that the integrated intensity ρ is proportional to F^2,

[1] P. Debye, Verh. deutsch Phys. Ges. **15**, 678 (1913).

[2] I. Waller, Zeits. f. Physik **17**, 398 (1923).

we obtain,

$$\log (F/f)^2 = -2M, \tag{4}$$

whence

$$\frac{\lambda^2}{\sin^2\theta}\log\frac{\rho_1}{\rho_0} = \frac{12h^2}{mkT_c}\left\{\frac{\varphi(x_0)}{x_0} - \frac{\varphi(x_1)}{x_1}\right\}, \tag{5}$$

where ρ_1/ρ_0 is the ratio of the integrated intensities of a regularly reflected line at temperatures T_0 and T_1. It is assumed that the angle θ is the same for a line at all temperatures, which, of course is only approximately true. The left-hand side of Eq. (5) can be evaluated experimentally while the right-hand side can be calculated.

In experiments of this kind performed by James and Firth,[3] James and Brindley[4] and James, Brindley and Wood,[5] with single crystals of sodium chloride, potassium chloride or aluminum, and a temperature range that extended from liquid air to 900°K, agreement was found between theory and experiment up to about 400°K. At higher temperatures the experimental values dropped off more quickly than the calculated ones. They attributed this disagreement at higher temperatures to the fact that the assumptions in the theory lose validity at these elevated temperatures.

Even if the characteristic temperature of the crystal is not known, the Debye-Waller formula can still be experimentally tested. For two given temperatures the right-hand side of Eq. (5) is independent of λ and θ. Whether the left-hand side actually remains constant can be experimentally tested. If a constant is obtained, one can determine the characteristic temperature of the crystal by solving Eq. (5) for T_c. This being done one can, except as far as the assumptions of zero-point energy are concerned, test the formula completely.

In order to find a somewhat approximate value of T_c for the sake of comparison one can make use of Lindemann's melting-point formula[6]

$$(k/h)T_c = \nu = C(T_m/AV^{\frac{2}{3}})^{\frac{1}{2}} \tag{6}$$

where ν is the characteristic frequency, C is a known constant, T_m is the melting point temperature, A is the atomic weight and V is the specific volume of the crystal. From Eq. (5) one obtains,

$$T_c = (A'T_m V'^{\frac{2}{3}}/AT_m' V^{\frac{2}{3}})^{\frac{1}{2}} T_c', \tag{7}$$

where the unprimed quantities refer to the crystal in question and the primed quantities refer to the crystals whose characteristic temperatures are known.

One can therefore calculate the characteristic temperature of a crystal in terms of known characteristic temperatures of other crystals. This equation gives results which are only approximately correct, especially when one uses room temperature volumes, as can easily be seen when one actually calculates the characteristic temperature of one crystal in terms of others.

Waller[7] has shown that the temperature effect can be expressed in terms of the amplitude of vibration of the atoms. He obtains the equation

$$M = (8/3)\pi^2\overline{u_1^2}(\sin^2\theta)/\lambda^2, \tag{8}$$

where $\overline{u_1^2}$ is the mean square displacement of the atoms at a temperature T_1. With the aid of this equation one can calculate the square root of the mean square amplitude of thermal vibration of the atoms at any temperature provided one knows whether or not zero-point energy exists.

To determine experimentally whether or not zero-point energy exists, one must know the magnitude of the temperature effect and the structure factors of the atoms at rest and at some other temperature.

Since the atoms of a monatomic gas are supposed to scatter independently of each other, therefore, as if at rest, f could be determined by placing the substance into the gaseous state. F, and the temperature effect, would, naturally, be determined when the substance were in the crystalline state. Though theoretically possible, experimentally it is very difficult to do this, since one can hardly find a substance which could be satisfactorily worked with in both the gaseous and solid states.

An alternative method is to calculate the value of f from some theory of atomic structure, and compare this calculated value with the experi-

[3] R. W. James and E. Firth, Proc. Roy. Soc. **A117**, 62 (1927).

[4] R. W. James and G. W. Brindley, Proc. Roy. Soc. **A121**, 155 (1928).

[5] R. W. James, G. W. Brindley and R. G. Wood, Proc. Roy. Soc. **A125**, 401 (1929).

[6] W. Nerst und F. A. Lindemann, Zeits. f. Electrochemie **17**, 817 (1911).

[7] I. Waller and R. W. James, Proc. Roy. Soc. **A117**, 214 (1927).

mental value of F as measured from the crystal. This method, which has been applied by James and Brindley[4] and others,[8] is however subject to the criticism that the theoretical structure factors are known to be only approximate. A more direct comparison is obtained if we choose a crystal such as sodium fluoride, which is composed of neon-like atoms, and determine the structure factor f of the corresponding gaseous atoms.

A similar comparison of the reflection from chlorine ions in the sodium chloride crystal with the scattering by argon gas has been made in a preliminary way by Compton,[9] with data from various sources, and more carefully by Wollan with his own data for the scattering by argon[10] and those of James and Firth[3] for the reflection by the sodium chloride crystal.

The experiments about to be described were undertaken primarily to obtain evidence regarding the existence of a zero-point energy. For this purpose measurements were made of the effect of temperature on the integrated reflection from powdered crystals of sodium fluoride. Using existing data[11] for the reflection by this crystal at room temperature enables one to calculate the structure factor of sodium fluoride atoms at rest according as zero-point energy is or is not assumed.

The structure factor thus determined should correspond closely to the experimental structure factor determined by the scattering of x-rays from neon gas,[10] since the average of the sodium and fluorine ions should closely resemble the neon atom. A similar correspondence should exist between the theoretical structure factors of these substances. It will be shown by a comparison of the theoretical with the experimental curves that the same correspondence exists between the experimental as between the theoretical curves only when zero-point energy is assumed.

The comparison of sodium fluoride and neon has the advantage over that between other combinations insofar as, due to its low atomic weight and high characteristic temperature, the zero-point energy plays a more prominent part in the motion of its atoms. The present experiments have the further advantage that, since the measurements have been made with powdered crystals, difficulties due to extinction are avoided.

Because of the work that has already been done by other experimenters on other crystals over a wide range of temperatures, the author of this paper considered it sufficient to make measurements at room and liquid air temperatures only.

Apparatus and Procedure

The source of x-rays was a coolidge, water-cooled, oil-immersed x-ray tube having a molybdenum target. The rest of the apparatus except for slits and a film-holder was contained in a large hard-rubber cylinder C, 5 inches in diameter and 24 inches high, shown partly in section in Fig. 1. Enclosed within this cylinder

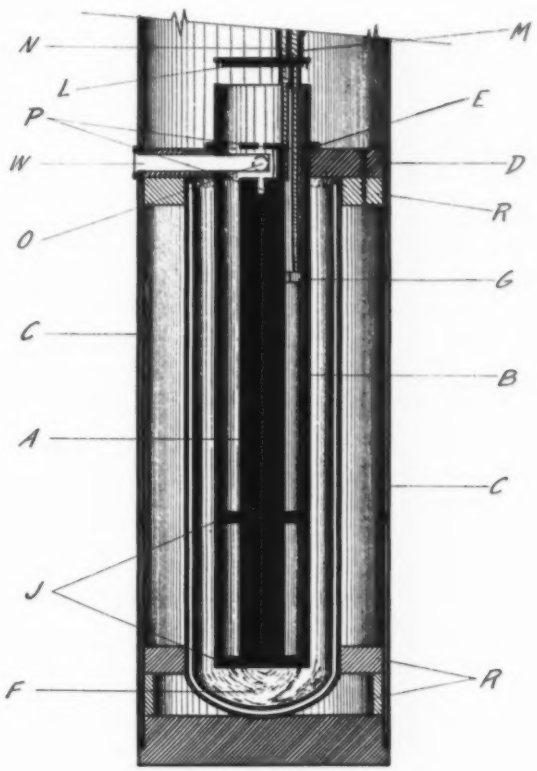


Fig. 1. Part-section of apparatus.

and held firmly in place by wooden rings R rested the liquid-air flask F measuring $2\frac{3}{4}$ inches in diameter and 11 inches in depth. A heavy brass

[8] E. O. Wollan, Rev. Mod. Phys. **4**, 205 (1932). This paper contains an excellent summary of the work on the effect of temperature on the motion of atoms in a crystal

[9] A. H. Compton, Tech. Rev. **33**, No. 1, 19 (1930).

[10] E. O. Wollan, Phys. Rev. **37**, 862 (1931).

[11] R. J. Havighurst, Phys. Rev. **28**, 869 (1926).

cylinder B, 2 inches in diameter and 12 inches long, supported by means of a brass collar E on the wooden ring D which in turn rested on the flask F, extended well down to the bottom of the flask. A solid brass rod A, 1 inch in diameter, ran down the center of this cylinder and was held firmly in place by two perforated brass disks J. The axis of this rod coincided with the axes of the brass and hard-rubber cylinders. A slot was cut into the rod at right angles to its axis so that the powdered crystals, which were pressed into small thin glass tubes sealed at both ends, when placed in position P, came into thermal contact with the brass rod both above and below the x-ray beam. Thus the powdered crystals were surrounded, except for the apertures at O and W, by heavy brass walls which were immersed throughout the greater part of their length in liquid air. Openings were cut through the hard-rubber cylinder, the wooden ring D, and the brass cylinder at O and at W, O being a small hole through which the primary beam passed to the crystal and W a wide angular window at right angles to O through which the scattered rays came forth. Both O and W were sealed with celluloid which was kept dry quite easily during the winter months by means of a stream of air. The wooden rings R and D were made to fit snugly into the hard-rubber cylinder C, the space above, below, and around the flask was filled with felt, and the ends of the outside container C were sealed air-tight so that the whole was quite rigid and compact.

Two small hard-rubber tubes M and N passing through the brass lid L were the only external openings the apparatus had. M carried the buoy, liquid-air level indicator G, and N was used for adding liquid air and for introducing a thermocouple for measuring the temperature. Liquid air was added at intervals of one hour and a half, since the level dropped off at the rate of about 2 inches per hour.

The photographic film was held in a brass frame on the outside of the hard-rubber cylinder opposite the window W so that the film was always at room temperature. The film was wide enough so that both the liquid air and room temperature pictures were taken on it side by side. Exposures varied from a few hours to about thirty-eight hours.

The integrated intensity corresponding to the various lines on the film was determined by a method similar to that used by Froman,[12] except that a new photoelectric photometer constructed and described by Dershem was used.[13]

A line at room temperature was found to have the same shape as at liquid air temperature, as one would expect, so that it was only necessary to take the average of ten readings 0.002 cm apart over the maximum of the lines. The width of the photometer slit was 0.002 cm and its height was sometimes 0.3 and sometimes 0.5 cm.

It has been the experience of persons employing the photographic method in their work and using the above-mentioned photometer to make the measurements, that because of the great reliability of the instrument, the degree of reproducibility of results is high. Errors arising from non-uniformity of the emulsion on the film and from development were small since the lines to be compared were close together. Furthermore, it is probable that the errors caused by motion of the focal spot, thermal variation of the length of the target rod and variation in the line voltage averaged themselves out during the long exposures.

Quantitative measurements of the ratio of the integrated intensities of a line at liquid-air and room temperatures were not made for lines at small angles. No satisfaction could be expected from such measurements since the ratio is so nearly equal to unity at these small angles that a reasonably small error in its measurement would introduce a rather large error into the calculated quantity $(\lambda^2/\sin^2\theta) \log (\rho_2/\rho_1)$.

Measurements at large angles were not made either since at large angles the intensities are very small and very long exposures are necessary. To make measurements at large angles one would rather use the single crystal method. This method had however already been used by other experimenters on other crystals, and therefore it was considered unnecessary to make these measurements on sodium fluoride, especially since one needs to know only the number $(\lambda^2/\sin^2\theta) \log (\rho_2/\rho_1)$ which is independent of the angle, in order to determine the characteristic temperature

[12] D. K. Froman, Phys. Rev. **36**, 1330 (1930).
[13] E. Dershem, Rev. Sci. Inst. **3**, 43 (1932).

of the crystal and to discuss the validity of the assumption of zero-point energy.

Experimental Results

The sodium fluoride crystal was considered as being a simple cubic crystal composed of atoms of one kind of atomic weight 21 and atomic number 10. This assumption is nearly satisfied in sodium fluoride since the Na^+ and F^- ions have 10 electrons each and their atomic weights and numbers are nearly equal.

The results of the measurements on the two final films are given in Table I. The ratio of the integrated intensity at liquid air temperature to the integrated intensity at room temperature is represented by ρ_2/ρ_1. With the powdered crystals first at room temperature (295°K) and later at liquid air temperature (86°K), exposures were made on the one film for 24 hours and 20 hours, respectively, and on the other for 30 hours and 27 hours, respectively.

Table I. *Temperature effect.*

Reflection index	$\frac{\sin\theta}{\lambda}$	$\frac{\rho_2}{\rho_1}$	$\frac{\lambda^2}{\sin^2\theta}\log\frac{\rho_2}{\rho_1}$
420	0.484	1.249	0.947
422	0.531	1.349	1.059
440	0.612	1.434	0.957
600	0.650	1.531	0.993
620	0.684	1.566	0.956
622	0.718	1.635	0.949
		Average	0.976±0.012

These data indicate no variation in the quantity $(\lambda^2/\sin^2\theta)\log(\rho_2/\rho_1)$ greater than the probable error of the experiments.

Upon solving the equation

$$\frac{12h^2}{mkT_c}\left\{\frac{\varphi(x_1)}{x_1}-\frac{\varphi(x_2)}{x_2}\right\}=0.976 \qquad (9)$$

for T_c one obtains

$$T_c=442°\text{K}. \qquad (10)$$

The value of T_c could not be found in physical tables so that it has probably never been determined by other experimental methods. T_c calculated by Eq. (6) in terms of the known characteristic temperature of sodium chloride is equal to 436°K and in terms of the characteristic temperature of potassium chloride is equal to 454°K. These calculations show that a value of 442°K for T_c of sodium fluoride is certainly reasonable.

In Table II, are given the structure factors of the atoms at rest, calculated by Eq. (3) for the cases of both no zero-point and of zero-point energy. The values of the structure factor F at room temperature are those of R. J. Havighurst.[11] The value of M from Eq. (2) is $0.871\sin^2\theta/\lambda^2$ or $0.512\sin^2\theta/\lambda^2$ according as zero-point or no zero-point energy is assumed.

Table II. *Experimental values of structure factors of sodium fluoride.*

Reflection index	$\frac{\sin\theta}{\lambda}$	F room temp.	f no z-p-e	f z-p-e
200	0.217	7.12	7.29	7.42
220	0.306	5.40	5.66	5.86
311	0.360	4.42	4.62	4.98
222	0.375	4.19	4.50	4.73
400	0.433	3.29	3.62	3.87
331	0.471	2.86	3.26	3.47
420	0.4845	2.75	3.10	3.37
422	0.531	2.37	2.73	3.03
511	0.562	2.12	2.49	2.79
440	0.613	1.73	2.10	2.39
531	0.640	1.55	1.91	2.22
600	0.650	1.52	1.89	2.20
620	0.687	1.28	1.63	1.92
622	0.718	1.19	1.55	1.87
642	0.810	0.93	1.30	1.65
731	0.831	0.87	1.26	1.59
820	0.892	0.71	1.06	1.42
822	0.918	0.67	1.00	1.39
840	0.970	0.61	0.99	1.37
842	0.995	0.52	0.84	1.22

Average theoretical structure factor values of sodium fluoride atoms were taken from the individual values for sodium and fluorine ions as calculated by James and Brindley[14] according to Hartree's method of self-consistent fields. These values and also the theoretical structure factor values of neon as similarly calculated[14] are given in Table III. The experimental structure factor values of neon as determined by E. O. Wollan[10] are also given.

All values of Tables II and III are shown graphically in Figs. 2 and 3. A glance at Fig. 2 suggests that the curve which passes through the points representing the values of f when zero-

[14] R. W. James and G. W. Brindley, Phil. Mag. 12, 81 (1931).

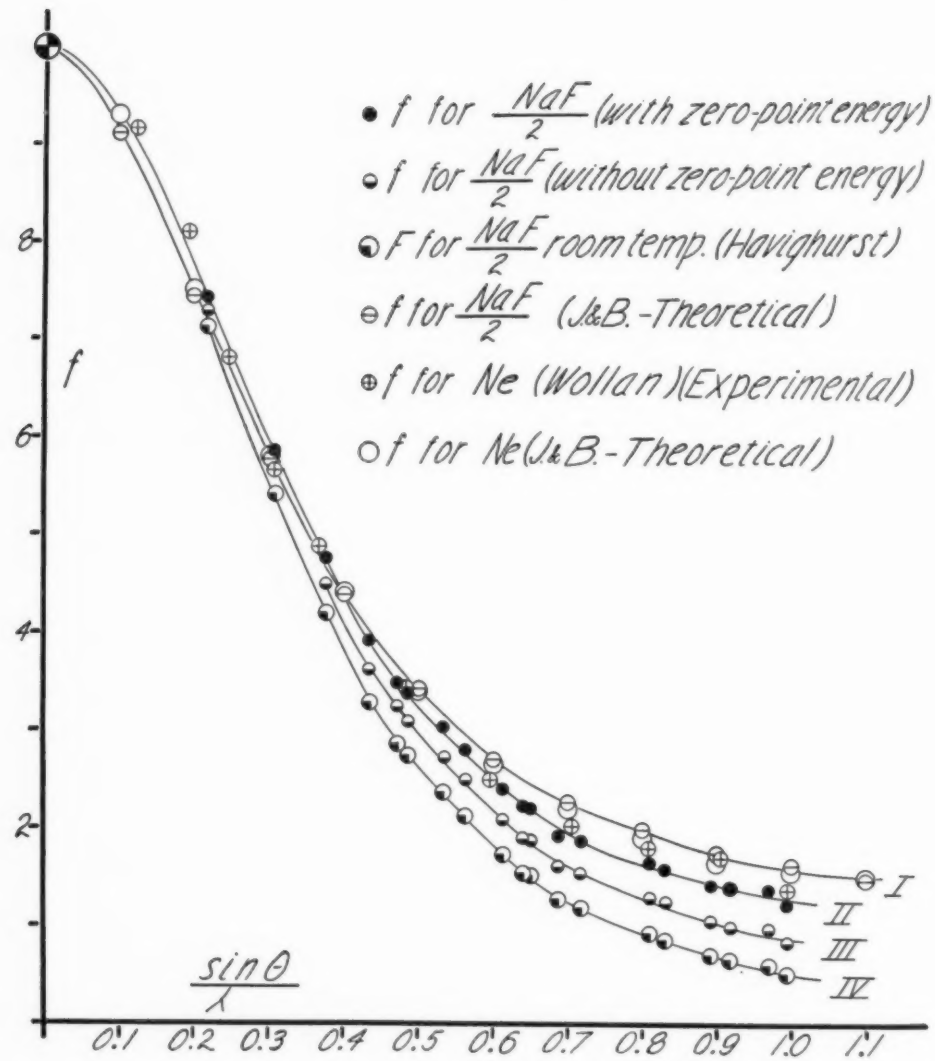


FIG. 2. Curve I, theoretical f(NaF); curve II, experimental f(NaF) with zero-point energy; curve III, f(NaF) without zero-point energy; curve IV, F(NaF) room temperature.

TABLE III. *Theoretical structure factor values of sodium fluoride and neon, and experimental structure factor values of neon.*

$\frac{\sin\theta}{\lambda}$	f(NaF) theory	f(Ne) theory	$\frac{\sin\theta}{\lambda}$	f(Ne) exp.
0.000	10.00	10.00	0.000	——
0.100	9.10	9.30	0.123	9.16
0.200	7.45	7.50	0.190	8.08
0.300	5.75	5.80	0.245	6.80
0.400	4.38	4.40	0.306	5.64
0.500	3.42	3.40	0.365	4.86
0.600	2.70	2.65	0.482	3.43
0.700	2.27	2.20	0.596	2.49
0.800	1.98	1.90	0.704	2.02
0.900	1.75	1.65	0.808	1.80
1.000	1.62	1.55	0.905	1.70
1.100	1.48	1.50	0.995	1.37

point energy is assumed should be given first consideration.

The curves representing the theoretical values of the structure factors of sodium fluoride and neon, the experimental values of the structure factor of neon and the structure factor values of sodium fluoride when zero-point energy is assumed are reproduced in Fig. 3. The experimental values are displaced through one unit in the negative direction of the structure factor axis in order to enable one to study the curves more easily and in greater detail.

An analysis of the theoretical curve shows that the structure factor values of sodium fluoride are

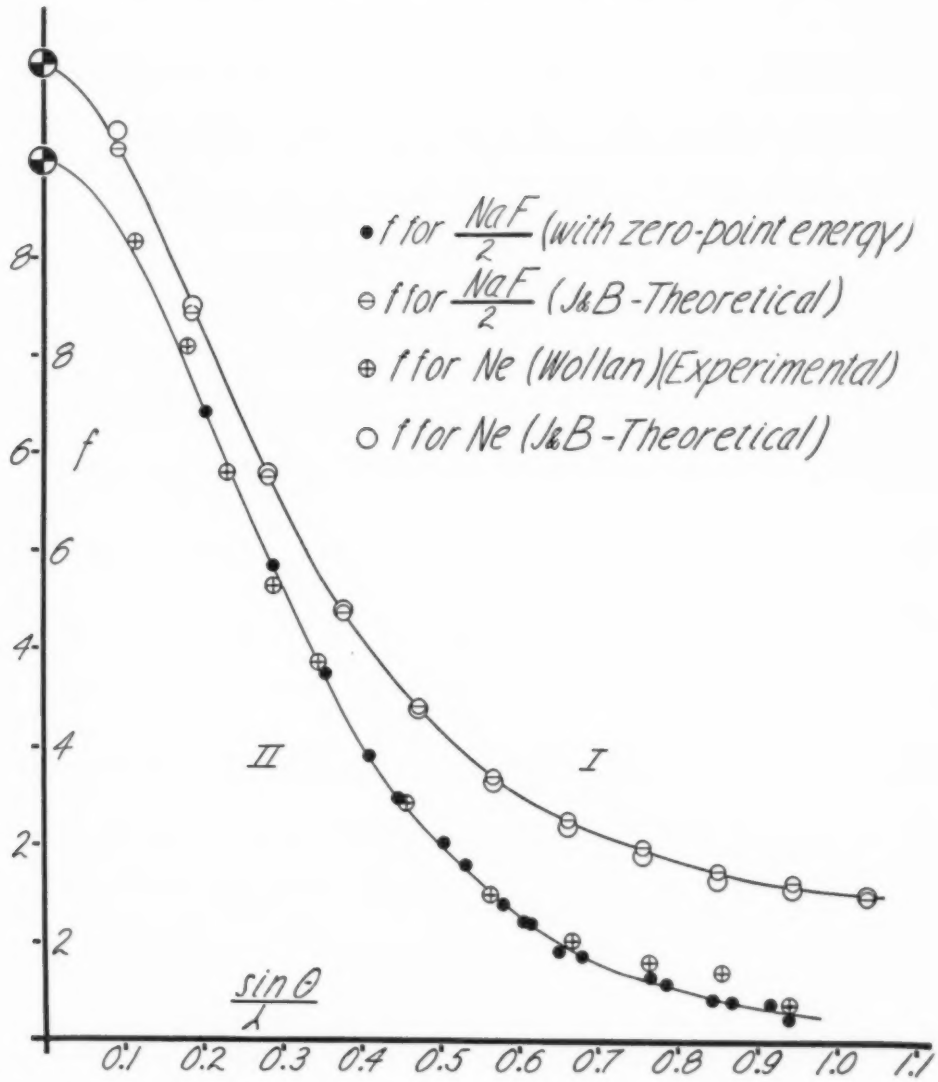

• f for NaF/2 (with zero-point energy)
⊖ f for NaF/2 (J&B-Theoretical)
⊕ f for Ne (Wollan)(Experimental)
○ f for Ne (J&B-Theoretical)
f
sin θ/λ
II
I
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
2 4 6 8

FIG. 3. Curve I, theoretical f(NaF); curve II, experimental f(NaF) with zero-point energy, shifted one unit in the negative direction of the structure factor axis.

slightly smaller than that of neon at small angles, slightly larger at larger angles and about equal at intermediate angles. The experimental curve, when corrected for zero-point energy, discloses similar characteristics, except that at large angles the neon values fall above the sodium fluoride values. This, however, can have nothing to do with the effect of the thermal motions, since it can be accounted for in terms of experimental errors.

A further study of the curves in Figs. 2 and 3 brings out the fact that the difference between the structure factors of neon and sodium fluoride, when no zero-point energy is assumed, is greater than the experimental error. When, however, zero-point energy is assumed, the crystal experiments are consistent with the gas experiments.

The square roots of the mean square amplitude of thermal vibration of the sodium fluoride atoms at any temperature can now be determined by Eq. (8). At room temperature

$$(\overline{u^2})^{\frac{1}{2}} = (3/8)0.871\pi^2 = 0.182\text{A}. \qquad (11)$$

CONCLUSION

The above data give further proof of the correctness of the Debye-Waller formula for the temperature effect and more conclusive evidence

that the assumption of zero-point energy is correct. They also furnish us with the structure factor of sodium fluoride atoms at rest. Incidentally they provide us with a value of T_c of sodium fluoride crystals and the magnitude of the thermal displacements of these atoms.

It is interesting to note that no definite conclusions could have been drawn by studying the relation between the theoretical and experimental structure factor curves of sodium fluoride alone. It might also be mentioned that the theoretical structure factor values of both sodium fluoride and neon are apparently too small at the small angles and too large at the large angles.

In conclusion, the author wishes to express his appreciation to Professor A. H. Compton, under whose direction this work has been done, as also to E. Wollan and E. Dershem for their suggestions and continued assistance during the working of this problem.

Measurements of Intensity of Compton Modified Radiation by Means of Filters

E. O. WOLLAN,* *Zurich, Switzerland*

(Received March 14, 1933)

A new method of measuring the ratio of the intensity of the modified to the unmodified scattering by using a triple balanced filter has been developed. With the K_α radiation of molybdenum three balanced absorption filters containing zirconium, yttrium and strontium, respectively, make it possible to absorb or transmit either the modified or unmodified K_α lines or both if the angle of scattering is sufficiently large so that the shortest wave-length of the modified line is greater than the critical absorption wave-length of yttrium. This method makes possible measurements of the modified radiation not only for the scattering by solids but also for the scattering by gases for which case the theory and experiment can be directly compared. Measurements of the ratio of intensity of the modified to the unmodified scattering have been made at $\varphi = 120°$ for oxygen, nitrogen and argon as well as for several solid elements and compounds. The results for gases are found to be in good accord with the wave-mechanics theory of scattering as developed independently by Wentzel and Waller.

INTRODUCTION

THE use of a double balanced filter for isolating a given wave-length band or spectral line was introduced by Ross[1] and has since been found very useful in experiments on the scattering of x-rays by gases and liquids. Ross has also attempted to study the structure of the modified line by the use of a properly selected double filter.

A double balanced filter for isolating the K_α lines of molybdenum has been previously used and described by Barrett,[2] the author[3] and others.[4] It consists of one filter of ZrO_2 and one of Sr O. The critical absorption limit of Zr lies at $\lambda = 0.6872$A and that of Sr at 0.7693A. The K_α lines of molybdenum ($\lambda = 0.71$A) lying between these limits are readily transmitted by the former and strongly absorbed by the latter. The thickness of these filters can be so chosen that the amount of general radiation transmitted by each is very nearly equal. Hence if readings of the intensity of scattering are made by putting these filters alternately in the primary beam the difference between the readings thus obtained gives the effect due to the band of radiation lying between the critical absorption wave-lengths of the filters which in this case includes the intense K_α lines as well as a small amount of general radiation. The K_β and K_γ lines are almost entirely absorbed by either filter. A more ideal double filter for this case would be obtained if yttrium whose absorption limit lies at $\lambda = 0.7255$A were substituted for the strontium. Such a filter would give a narrower band of general radiation accompanying the K_α lines. The use of strontium has been resorted to because of the difficulty of obtaining a sufficiently pure and suitable compound of yttrium. Fortunately, a relatively pure sample of Yt_2O_3 has been obtained and in the present experiment a triple balanced absorption filter has been constructed from the oxides of the three above-mentioned elements, zirconium, yttrium and strontium. Such a system of filters makes possible not only the isolation of the K_α lines but also the isolation of a wave-length lying between the critical absorption wave-lengths of yttrium and strontium. With the aid of this triple filter, measurements of the ratio of intensity of the modified to the unmodified scattering by gases and solids have been made.

THE TRIPLE BALANCED FILTER

The relative position of the critical absorption wave-lengths of zirconium, yttrium and strontium and the wave-lengths of the K_{α_1} and K_{α_2}

* National Research Fellow at Physikalisches Institut der Eidgenössischen, Technischen Hochskule.

[1] P. A. Ross, Phys. Rev. **28**, 425 (1926).

[2] C. S. Barrett, Phys. Rev. **31**, 159 (1928).

[3] E. O. Wollan, Phys. Rev. **37**, 862 (1931).

[4] W. C. Pierce, Phys. Rev. **38**, 1409 (1931).

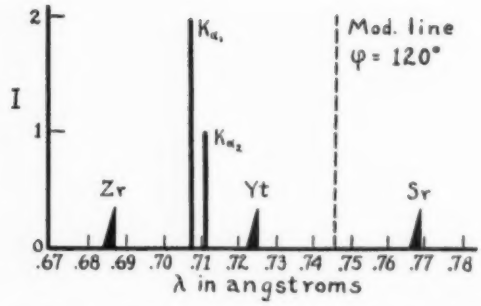


FIG. 1. Relative position of absorption edges and modified and unmodified lines.

lines of molybdenum are shown in Fig. 1. If the K_α lines are scattered from a relatively light element a part of the scattered radiation will consist of the unshifted lines of average wave-length (0.71A) while the remainder consisting of the modified radiation will have a larger wave-length whose value depends on the scattering angle. The wave-length of the modified lines is given by the well-known Compton formula

$$\lambda_{\text{mod.}} = \lambda_{\text{unm.}} + (h/mc)(1-\cos\varphi), \quad (1)$$

where $h/mc = 0.0243$ and φ is the scattering angle. From this relation one can calculate the angle φ at which the wave-length of the modified K_{α_1} and K_{α_2} lines will be greater than the wave-length of the critical absorption limit of Yt. This works out to be 74° 14′ for the K_{α_1} line and 63° 23′ for the K_{α_2} line. In view of the considerable breadth of the modified lines as predicted some time ago by Jauncey[5] and recently shown by DuMond,[6] the whole of the modified radiation will not be displaced past the absorption edge until some angle larger than 75° has been reached. At $\varphi = 120°$ the center of gravity of the modified line should have a position as shown in Fig. 1, at which undoubtedly the whole of the modified line will be included in the region between the Yt and Sr absorption limit.

If the three filters under consideration are properly balanced and are alternately placed in the scattered beam for which the scattering angle is sufficiently large, say 120°, one has in effect two double filters, one for the unmodified line and one for the modified line. The use of these filters should then make it possible to make separate measurements of the intensity of the unshifted and shifted lines.

Construction and Calibration of the Filters

The zirconium filter is one of the type obtained from the General Electric Company. It is made from ZrO_2 from which the impurity hafnium has been removed.

The strontium filter was made by finely powdering SrO and suspending the powder in paraffin. The filter was made homogeneous by kneading the paraffin and SrO mixture with the fingers, the warmth of which keeps it malleable. It was then pressed into a flat plate.

Yt_2O_3 is difficult to obtain in a pure state since it contains a considerable amount of erbium, the removal of which is very difficult. Fortunately the author was able to obtain some relatively pure Yt_2O_3 and from this a filter was constructed in the same manner as for SrO.

The filters of Yt and Sr were balanced against the Zr filter by successively scraping away small amounts from the surface. It was also found necessary, however, to pad the filters slightly with aluminum, this being apparently due to a small amount of impurity which the Yt_2O_3 still contained.

The calibration curve of the filters is shown in Fig. 2. The curve represents the intensity of the

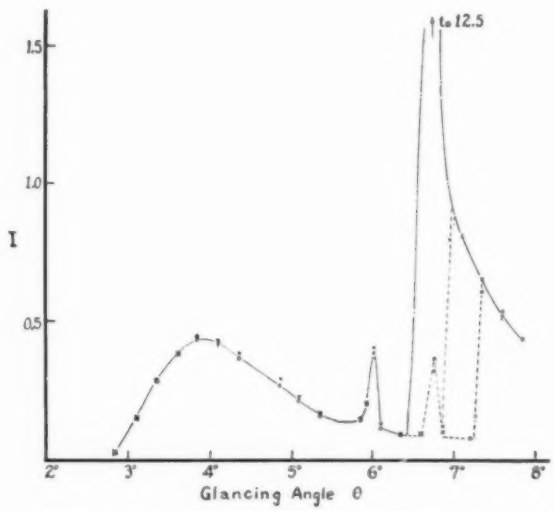


FIG. 2. Calibration curve of absorption filters.

[5] G. E. M. Jauncey, Phil. Mag. **49**, 427 (1925).
[6] J. DuMond, Phys. Rev. **37**, 136 (1931).

various wave-lengths in the spectrum of the x-ray tube as reflected from a calcite crystal and absorbed by the various filters in turn. The transmission by the Zr filter is represented by the solid points, the transmission of the Yt by the crosses and the transmission of the Sr by the circles. The balance achieved is seen to be quite satisfactory.

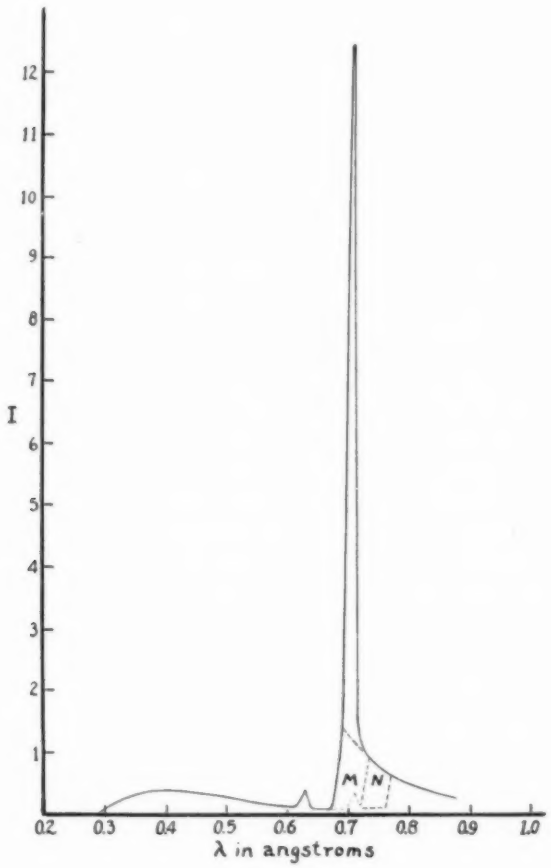


FIG. 3.

Fig. 3 shows the curve as a whole and one sees the approximate relation between the intensity of the K_α lines to the general radiation both outside and between the absorption limits.

DETERMINATION OF GENERAL RADIATION BETWEEN ABSORPTION LIMITS OF FILTERS

The presence of general radiation between the absorption limits of the filters presents a difficulty in regard to the measurement of the ratio of intensity of the modified and unmodified radiation. Because of the presence of this general radiation Fig. 2 does not represent a complete calibration of the filters.

For convenience the general radiation between the Zr and Yt absorption limits is referred to as M and that between the limits of the Yt and Sr filters is referred to as N. When the filters are used in the scattered beam a certain amount of the radiation in the region to the left of M will be moved into the region M, a part of the radiation M will be displaced into N while a part of N goes into the region to the right. There will always be a small amount of intensity due to the general radiation between the absorption limits of the filters which must be subtracted from the total readings to obtain the intensity of the characteristic line radiation. That part of the K_α lines which is modified, however, will be displaced bodily into the region N.

The value of M can be obtained if one scatters from a substance, for which all the radiation is modified. When scattering from such a substance, that part of the intensity which still remains between the absorption limits of the Zr and Yt filters will be the value of M desired. Scattering by lithium should very nearly satisfy this condition.

To determine the value of N one might scatter from a body for which all the radiation remains unmodified or at a scattering angle sufficiently small so that none of the modified radiation is displaced into the region N. It should also be possible to obtain N from the difference of intensity between the Yt and Sr filters when these filters are used in the primary beam. Insofar as it is difficult to obtain a suitable scatterer to satisfy the conditions mentioned in the first of these three methods the second two methods have been here used. Fig. 4 shows the scattering by lithium between 35° and 120°. At all angles the total intensity has been taken equal to 1000. Curve I represents the percent of the total radiation which lies between the Yt and Sr filters, while curve II represents the radiation lying between the Zr and Yt filters. Curve I is seen to begin with a constant but relatively small amount of intensity at small angles and then gradually rises until an approximately constant value is reached at large angles. At 35° both the modified and the unmodified K_α radiation lies between the Zr and Yt

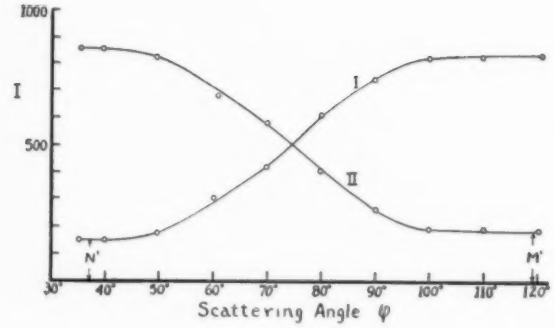


FIG. 4. Scattering curves for lithium as a function of scattering angle. Curve I represents the difference of intensity between Yt and Sr filters and curve II represents the difference between the Zr and Yt filters.

filters and hence the height of curve I at this point is a measure of N, the general radiation between the Yt and Sr limits. The fact that M and N are primed in the figure indicates that certain corrections have not yet been made. At 120° the greater percent of the radiation lies between the Yt and Sr filters. If we assume that all the radiation from lithium at large angles is modified, the intensity corresponding to curve II at 120° would then be a measure of M. If, however, we assume that the scattering by lithium at such large values of $(\sin\varphi/2)/\lambda$ is the same as the scattering by the element in the gaseous state, that is, that the interference between neighboring atoms is negligible, one can obtain the theoretical value of I_m/I_u for lithium. Referring to Table II we see that the calculations for lithium give $I_m/I_u=64$. On this basis a correction corresponding to 1.56 percent of the modified K_α radiation must be subtracted from the value of M obtained on the assumption that $I_m/I_u=\infty$. The average of a number of measurements of the scattering by lithium at $\varphi=120°$ gives $M'=187$. After applying the above-mentioned correction together with the corrections for differential absorption as discussed below we obtain the value $M=172$.

A value of $N'=148$ was obtained by scattering from lithium at 35°. As mentioned above N should also be obtained by measurements of the intensity of scattering at 120° when the filters are used in the primary beam. In this way a value of $N'=144$ was obtained. This value was found to be approximately the same regardless of the atomic number of the scatterer. The average of these two values when corrected for the differential absorption gives $N=151$* and this is the value which is used in the calculations of I_m/I_u. In view of the fact that in most of the following measurements the value of I_m/I_u is greater than unity the accuracy of the determination of the value of M is of greater importance than that of N.

Experimental Arrangement

A molybdenum target x-ray tube of the Siemens type was used. The filament current was supplied by lead storage cells and the current was controlled by a fine adjustment rheostat, which could be controlled from the workers position while the tube was in operation. In this way the current and voltage supplied to the tube were kept quite constant.

A Compton electrometer was used for measuring the ionization current and this was operated at a sensitivity of about 8000 mm per volt at a distance of a meter. The ionization chamber was 30 cm long and contained argon at 1 atmosphere.

Measurements of the Scattering by Solids

The materials to be used as scatterers were finely powdered, except in the cases of paraffin and lithium, and were pressed into a briquet of convenient size. In the case of lithium the metal was cleaned and crushed under oil and pressed into a brass ring. The oil was wiped away and the sample again scraped clean under hot paraffin. On removing, a very thin coat of paraffin protected its surface from oxidizing. The layer of paraffin was so thin that its contribution to the scattering was negligible.

These samples were placed on the center of the spectrometer table such that the face exposed to the radiation made an angle of $\varphi/2$ with the primary beam. A comparatively wide slit[7] was used in the ionization chamber, the angular divergence being about 5°.

The filters were carried on a slide in front of the ionization chamber slit such that each filter could be conveniently moved into a position in front of the slit. Readings were then taken alternately for each of the three filters. Calling the total intensity

* These values of M and N are reduced to correspond to a total intensity of 1000.

[7] A. W. Coven, Phys. Rev. **41**, 422 (1932).

(i.e., the difference between the readings for the Zr and Sr filters) C, the intensity corresponding to the difference between the Zr and Yt filters A' and that corresponding to the differences between the Yt and Sr filters B', the results of these measurements are given in the first two columns of Table Ia. For convenience the total intensity,

TABLE Ia.

Element	A'	B'	A	B	$I_{mod.}/I_{unm.}$ Wollan	Woo
KCl	534	466	505	495	1.03	
Sulphur	499	501	466	534	1.30	
Aluminum	403	597	376	624	2.31	1.45
NaF	362	638	335	665	3.15	
MgO	347	653	322	678	3.51	
Carbon	286	714	276	724	5.56	2.26
Boron	257	743	246	754	8.15	
Paraffin	222	778	216	784	14.30	5.49

C, has been taken equal to 1000 in all cases. By applying the corrections for differential absorption (as discussed below) to the values of A' and B' the resulting values are given in the third and fourth column as A and B. These values have also been reduced so that their sum is again equal to 1000.

It can be readily seen that the ratio of the modified to the unmodified scattering is given by the following formula

$$I_m/I_u=(B-N)/(A-M). \qquad (2)$$

The final values of I_m/I_u for solids are given in the fifth column of Table Ia. In column six of this table are given the values of I_m/I_u as measured by Woo.[8] It is seen that his values are all considerably lower than those obtained here. It would seem that an explanation of this can be found in connection with the measurements of DuMond on the width of the modified lines in which it was found that the modified lines are considerably broader than the unmodified. Since Woo used a spectroscopic method of measuring the intensity of the modified and unmodified lines the evaluation of the ratio of the integrated intensities would be considerably affected, it would seem, if the width of one line was much greater than that of the other. The intensity of the narrower unmodified lines would be overestimated and hence he would obtain a too small value of I_m/I_u.

One would expect that the value of I_m/I_u for solids would vary according to the physical condition of the scattering body, i.e., depending on the amount of crystal reflection one took in. In using powdered crystals as scatterers the result should, however, be a constant for a definite angle and angular divergence.

SCATTERING FROM GASES

In order to increase the intensity of the scattering by gases a special scattering chamber was designed to admit of the use of higher pressures than was possible in the previous work of the writer on scattering by gases. The general scheme is shown in Fig. 5. A rectangular brass block 10

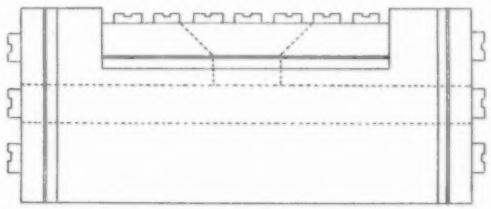


FIG. 5. Gas scattering chamber.

cm long was milled out with a rectangular passage 1 cm wide by 2 cm high. A circular opening was made in the center of one side 1.5 cm in diameter. The ends and side opening were fitted with windows of celluloid 0.7 mm thick. Rubber gaskets were used between the celluloid and the chamber and the windows were held in place by screws and brass plates. It was found possible to use this chamber at a pressure of 30 atmospheres with practically no leak. The x-rays traversing the gas were scattered through the window on the side. This type of construction does not admit of measurements at angles much greater than 40° on each side of the normal to the x-ray beam.

By using this chamber, measurements of the ratio of modified to unmodified scattering have been made for nitrogen, oxygen and argon at $\varphi=120°$. The values of A' and B' thus obtained as well as the corrected values A and B are given in Table Ib. The values of I_m/I_u are calculated as for solids according to Eq. (1), the value of the correction factor being slightly different for this

[8] J. H. Woo, Phys. Rev. **27**, 119 (1926).

TABLE Ib.

Element	A'	B'	A	B	$I_{mod.}/I_{unm.}$ Exp.	Wave mechanics	Classical
Argon	484	516	470	530	1.27	1.44	1.58
Oxygen	308	692	295	705	4.49	4.33	4.71
Nitrogen	294	706	283	717	5.10	5.08	5.41

note $C=A+B=1000$

case as is pointed out below. The final values of I_m/I_u are given in the fifth column of Table Ib.

CORRECTIONS

On account of the difference in wave-length of the modified and unmodified lines the measured intensity ratio will have to be corrected for the different absorption which these lines experience when traversing any matter interposed in their path. In consideration of this, the following corrections will have to be made.

(1) The differential absorption within the filters themselves

The zirconium filter was found to transmit 36 percent of the unmodified K_α lines. Since the yttrium and zirconium filters transmit equally at the wave-length corresponding to the modified line the transmission of the modified line can be calculated from its wave-length. By taking the absorption coefficients as proportional to λ^3 it is found that the ratio of absorption of the modified line to that of the unmodified line is equal to 1.18.

(2) The differential absorption in the ionization chamber

Since the ionization chamber contained argon at one atmosphere pressure not all the radiation was absorbed. Taking the mass absorption coefficient of argon as $\mu_\rho=14.4$, for a chamber 30 cm long the ratio of the absorption of the modified to the unmodified line comes out as 1.12. Here the greater absorption of the modified line, however, tends to increase its apparent intensity while in the above case (1) the modified line was absorbed away and hence weakened in intensity. Combining corrections 1 and 2 gives a factor 1.04 by which the values of B' must be multiplied.

(3) A calculation of the differential absorption within the scattering body

This gives the relation

$$\frac{I_m}{I_u}=\frac{I_m'}{I_u'}\left(\frac{\mu+\mu_1}{2\mu}\,\frac{1-e^{-2\mu b/\sin\varphi/2}}{1-e^{-(\mu+\mu_1)b/\sin\varphi/2}}\right)=\beta\frac{I_m'}{I_u'},* \qquad (3)$$

where the primes represent the uncorrected value, μ and μ_1 represent the linear absorption coefficients of the unmodified and modified lines, respectively, and b is the thickness of the scattering body.[9] Eq. (3) represents the case where the face of the scatterer makes an angle $\varphi/2$ with the primary beam. Taking the absorption coefficients as proportional to λ^3 we find $\mu_1=1.16\mu$ for this case. This simplifies β to

$$\beta=1.08(1-e^{-\mu b})/(1-e^{-1.08\mu b}). \qquad (4)$$

It can also be seen that when the thickness b of the sample is sufficient to absorb all the radiation the latter term reduces to unity and $\beta=1.08$. For the scattering bodies MgO and heavier, this latter condition is fulfilled. Correction (3) is negligible for the case of scattering by gases at the pressures here used.

(4) The differential absorption in the windows of the ionization chamber and of the scattering chamber

The former of these corrections is negligible and the latter is about 2 percent and of course only applies to the scattering by gases.

THEORY OF SCATTERING BY INDIVIDUAL ATOMS

The theory of scattering of x-rays by a free atom has been worked out independently by Wentzel[10] and Waller[11] on the basis of wave mechanics and independently by Raman[12] and Compton[13] on the basis of classical theory.

To a first approximation the theories as worked out by Wentzel and by Waller are identical, giving for the intensity of the modified and unmodified scattering the relations

---

* I_m' and I_u' correspond here to the total measured intensities before M and N have been subtracted and hence the various corrections are applied to B' and the resulting $B+A$ reduced to 1000.

[9] Jauncey and De Foe, Phil. Mag. **1**, 711 (1926).

[10] G. Wentzel, Zeits. f. Physik **1**, 43 and 779 (1927).

[11] I. Waller, Phil. Mag. **4**, 1228 (1927).

[12] C. V. Raman, Ind. J. Phys. **3**, 357 (1928).

[13] A. H. Compton, Phys. Rev. **35**, 925 (1930).

$$I_{\text{unm.}} = I_e\left(\sum_1^Z f_n\right)^2$$

$$I_{\text{mod.}} = I_e\left[Z - \sum_1^Z (f_n)^2\right], \tag{5}$$

where I_e is the intensity of scattering by a free electron as obtained by the Thomson classical theory, f_n is the structure factor for the nth electron which is defined by the relation

$$f_n = \int_0^\infty u_n(r) \frac{\sin(xr)}{xr} dr, \tag{6}$$

where $x = 4\pi/\lambda \sin \varphi/2$, and $u_n(r)$ is the charge for the nth electron lying between the spherical shells of radius r and $r+dr$ and expressed in electron units.

Eq. (5) has been worked out on the assumption that the incident wave-length is sufficiently long so that relativity effects are negligible. For wavelengths shorter than 1A the relativity effect becomes quite appreciable and it is necessary to apply a correction to the incoherent part of Eq. (5). For the case of the molybdenum radiation ($\lambda = 0.71$A) which was used in the above described experiments, the relativity effects should be accurately accounted for if the modified part of Eq. (5) is divided by the Breit[14]-Dirac[15] factor $(1+\gamma \text{ vers } \varphi)^3$ where $\gamma = h/mc\lambda$. Waller's derivation of Eq. (5) includes a small additional factor which arises from the consideration of the Pauli exclusion principle. This term has the form $-\sum(f_{nm})^2$ where the sum is taken over all pairs of L electrons of the same spin and $n \neq m$. Including this term the modified intensity given by Eq. (5) will now have the form

$$I_{\text{mod.}} = I_e[Z - \sum(f_n)^2 - \sum(f_{nm})^2]. \tag{7}$$

The inclusion of this term has no effect on the intensity as calculated for nitrogen and oxygen at this large value of $(\sin \varphi/2)/\lambda$ used, but it does have a small effect on the intensity of the modified radiation from argon.

In order to calculate the values of I_m and I_u from Eq. (5) or (7) it is necessary to know the values of the structure factor, f_n, corresponding to all the electrons in the atoms in question. Hartree[16] has made calculations of the Schrödinger charge distribution for a few atoms by his method of self-consistent fields and from this one can calculate f_n by means of Eq. (6). From the few cases for which the charge distribution has been completely worked out James and Brindley[17] have shown that it is possible by a method of interpolation to obtain reliable values of f_n for all elements of atomic number less than 18 and they have given a table from which the values of f_n for the various electrons can be directly read. From these tables the values of f_n at $(\sin \varphi/2)/\lambda = 1.22$ ($\varphi = 120°$) have been taken and are given in the first three columns of Table II. From these data the values of I_m/I_u have been calculated for a number of elements by means of Eqs. (5) and (7) and with correction for the Breit-Dirac factor. These values are given in the next to the last column of Table II and are plotted to give the curve in Fig. 6.

In the classical theory of scattering as developed by Raman and by Compton the atom has been assumed to be of the form of a degenerate electron gas. On this assumption the classical theory gives the following expression for the total scattering.

$$I_{\text{tot.}} = I_e[f^2 + Z - f^2/Z], \tag{8}$$

where f represents the structure factor of the atom as a whole and hence $f = \sum f_n$.

In view of the fact that the classical theory does not distinguish a modified or unmodified scattering, Compton compared Eq. (8) with Eq. (5) and hence concluded that the term f^2 represents the unmodified scattering and the term $[Z - f^2/Z]$ represents the modified part of the scattered radiation. If we write this term in the form $[Z - (\sum f_n)^2/Z]$, a comparison with the same term of Eq. (5) shows that the two expressions are identical only if the values of the structure factors f_n are identical for each electron.

Taking the values of f_n given in Table II which were used to calculate I_m/I_u by means of Eq. (5) one can now calculate this quantity on the basis of Eq. (8) where, as mentioned above, $f = \sum f_n$. The values so calculated are given in the

[14] G. Breit, Phys. Rev. **27**, 242 (1926).
[15] P. A. M. Dirac, Proc. Roy. Soc. **A111**, 405 (1926).
[16] D. R. Hartree, Proc. Camb. Phil. Soc. **24**, 89 (1928).
[17] James and Brindley, Phil. Mag. **12**, 89 (1931).

TABLE II.

At. No.	Element	$f_{(1s)}$	$f_{(2s)}$	$f_{(2p)}$	$\left[\sum_{1}^{Z} f_n\right]^2$	$\frac{Z-\Sigma(f_n)^2-\Sigma(f_{nm})^2}{(1+\gamma \text{ vers } \varphi)^3}$	$\Sigma(f_{nm})^2$	$I_{\text{mod.}}/I_{\text{unm.}}$ Wave mechanics	Classical
3	Lithium	0.10			0.04	2.56		64.00	
4	Beryllium	.225			.203	3.34		16.50	
5	Boron	.320			.409	4.11		10.10	
6	Carbon	.440			.774	4.82		6.24	
7	Nitrogen	.523			1.093	5.56		5.08	5.41
8	Oxygen	.600			1.44	6.25		4.34	4.71
9	Fluorine	.658			1.73	6.99		4.04	
10	Neon	.698			1.95	7.70		3.95	
11	Sodium	.750			2.25	8.48		3.77	
12	Magnesium	.790			2.49	9.23		3.70	
13	Aluminum	.815	0.01	0.04	2.82	10.00		3.54	
16	Sulphur	.876	.03	.12	5.25	12.36	0.042	2.35	
17	Chlorine	.888	.062	.16	7.30	12.97	.155	1.78	
18	Argon	.900	.097	.18	9.45	13.65	.298	1.44	1.63
19	Potassium	.920	.134	.23	12.20	14.12	.503	1.16	

last column of Table II for the gases under consideration. It is seen, as has been previously pointed out by Herzog,[18] that the assumption that the atom scatters as a degenerate gas leads to considerably larger values of I_m/I_u than one obtains from a Schrödinger atom as calculated by Hartree. The classically calculated values are shown in Fig. 6 as short lines above the solid curve.

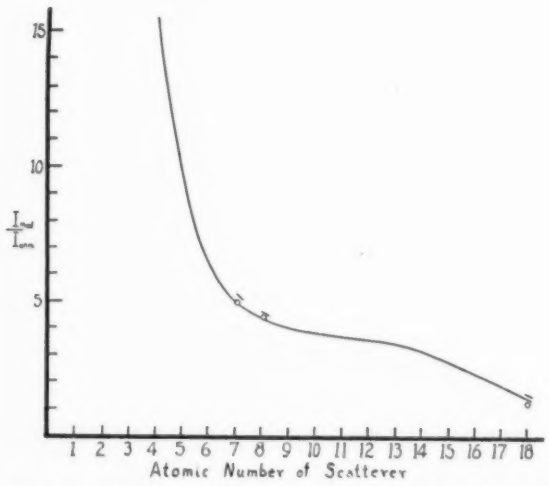


FIG. 6. Ratio of modified to unmodified scattering as a function of atomic number. Solid curve calculated according to Waller, short lines calculated according to Compton and circles represent measured values. Scattering angle, 120°; wave-length, 0.71A.

The measured values of I_m/I_u seem to be in better agreement with the calculations based on wave mechanics.

[18] G. Herzog, Zeits. f. Physik **69**, 207 (1931).

CONCLUSIONS

The present measurements of the ratio of the intensity of the modified to the unmodified scattering by gases, together with previous measurements of the total scattering of x-rays by gases as made by the writer for neon and argon and by Herzog for argon, are in good accord with the theory of scattering as developed by Wentzel and Waller and the calculations of the Schrödinger charge distribution by Hartree's method of self-consistent fields. Taken as a whole the measurements are in better accord with the wave-mechanics theory than with the classical derivation on the basis of an atom of the form of a degenerate electron gas.

In view of this fact measurements of the total intensity of scattering by a gas are not, in general, sufficient to determine the structure factor of the atoms of which the gas is composed. This can be seen to be the case from Eq. (5) since the total intensity corresponding to the sum of the modified and unmodified scattering is a function of $\sum(f_n)^2$ and hence cannot be solved for the atomic structure factor $f=\sum f_n$ in terms of the measured total intensity. This is the case not only for the scattering by gases but for the intensity of the diffuse scattering of x-rays by crystals as measured by Jauncey[19] and his collaborators.

There are, however, special cases in which the theories represented by Eqs. (5) and (8) are identical. In the case of scattering by helium, the

[19] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **38**, 1071 and 1925 (1931); **42**, 453 (1932).

example discussed by Compton in his initial paper, the theories give identical results because the values of f_n for the two electrons are identical and hence f can be directly obtained. For the extreme case of scattering by a very heavy gas or vapor the modified radiation is negligible and hence the atomic structure factors according to either Eq. (5) or (8) are related to the total intensity by

$$I_{\text{tot.}} = I_e f^2. \tag{9}$$

The measurements of the scattering of x-rays by mercury vapor as made by Scherrer and Staeger[20] are an example of the case where the modified radiation is negligible and hence f can be directly calculated according to Eq. (9).

For cases in which the atomic number lies between these limits, measurements of I_m/I_u, together with the measurements of the total scattered intensity, make it possible to solve for the unmodified radiation and hence for the atomic structure factor. Since $I_{\text{tot.}} = I_{\text{mod.}} + I_{\text{unm.}}$ multiplying by I_m/I_u and substituting the value for $I_{\text{unm.}}$ from Eq. (5) we obtain

$$f^2 = [1/(R+1)] I_{\text{tot.}}/I_e, \tag{10}$$

where R represents I_m/I_u and I_e is the Thomson scattering by a single electron and f is the atomic structure factor equal to $\sum f_n$.

Since the present method of measuring I_m/I_u cannot be applied to all values of $(\sin \varphi/2)/\lambda$ one is not able in this way to obtain a complete structure factor curve. The above facts have been pointed out, however, to show the difficulties involved in the determination of the atomic structure factor from measurements of x-ray scattering involving the modified radiation.

The present experiment seems to furnish additional evidence of the correctness of the wave-mechanics theory of the scattering of x-rays by gases as well as giving another method of obtaining the ratio of the modified to the unmodified scattering at certain values of $(\sin \varphi/2)/\lambda$.

In conclusion I wish to acknowledge the assistance of Professor P. Scherrer who has kindly permitted me to work in his laboratory and has put the necessary equipment at my disposal.

I am also indebted to Professor B. S. Hopkins of Illinois from whom the sample of pure Yt_2O, was obtained.

[20] P. Scherrer and A. Staeger, Helv. Phys. Acta **1**, 518 (1928).

A Spectroscopic Search for H^3 in Concentrated H^2

GILBERT N. LEWIS AND FRANK H. SPEDDING, *Department of Chemistry, University of California*

(Received April 17, 1933)

With hydrogen of which about two-thirds consists of the heavy isotope, H^2, a search has been made for the isotope H^3. This appears not to be present in ordinary hydrogen to the extent of 1 part in 6,000,000. Photographs are reproduced showing the relative fine structure resolution of H^1 and H^2.

HAVING available[1] a sample of water in which about two-thirds of the hydrogen consists of the heavy isotope H^2, we have looked for a third possible isotope, H^3, which should have been concentrated in at least the same degree as H^2.

By means of a large Hilger spectrograph (E. 185 Littrow mounting), supplemented with extra glass prisms, we were able to separate the $H^1\beta$ from the $H^2\beta$ by about 1.2 mm and $H^1\gamma$ from the $H^2\gamma$ by about 2.0 mm on the plates. Since the separation of the H^3 lines from the H^2 lines should be one-third of the separation of the H^2 from the H^1 lines, the lines of H^3 should be easily detectable if this third isotope is present in appreciable amount.

Using a discharge tube with aluminum electrodes of the type designed by R. W. Wood, we found that the H^2 lines, originally strong, rapidly faded out, doubtlessly because of a sort of "clean up," accompanied by the evolution of ordinary hydrogen from the electrodes, and from the water occluded in the glass walls. This effect may account for the great disparity between the various estimates that have been made by spectroscopic methods of the normal ratio of H^2 to H^1. We found that this disappearance could be largely avoided by repeatedly filling the tube with the concentrated isotope, subjecting it to the discharge for an hour or two, and then exhausting. At best, however, the loss of H^2 continued, so that although we have on several occasions obtained visually an H^2 line more intense than the H^1, after long exposures the H^1 line prevails.

[1] Lewis, J. Am. Chem. Soc. **55**, 1297 (1933); Lewis and Macdonald, J. Chem. Phys. **1**, 341 (1933).

In order to obtain the atomic spectrum as free as possible from molecular lines, it is known that a certain amount of water vapor is necessary, and we spent some time in attempting to introduce gas containing the right amount of water vapor of the same isotopic composition, until we hit upon the happy expedient of introducing nothing but water vapor, which we find to give a very pure atomic spectrum. A side tube containing a drop of the heavy water was connected to the discharge tube through a stopcock. By opening this cock small amounts of water vapor could be introduced into the discharge tube as often as desired.

With a discharge of such intensity as to show the H^2 line on the photographic plate after one or two minutes, exposures were made for forty hours on both the $H\beta$ and $H\gamma$ lines. No trace of the H^3 line was obtained in either case.

Although the non-appearance on the photographic plate of an extremely weak line is sometimes deceptive, it seems reasonably safe to say that the ratio of the amount of the H^3 to the amount of H^2 is not greater than one to a thousand.

Since it has been shown by Lewis and Macdonald that the ratio of H^2 to H^1 in ordinary hydrogen is about 1 to 6500, it seems that ordinary hydrogen can not contain more than one part in six million of H^3.

Having been engaged with Professor C. D. Shane in a search for H^2 in the solar spectrum, we have availed ourselves of the higher resolving power in the 3rd order of the 10 meter plain grating of the Astronomy Department to obtain photographs from our discharge tube. Fig. 1 shows the two $H\beta$ lines, the one on the right

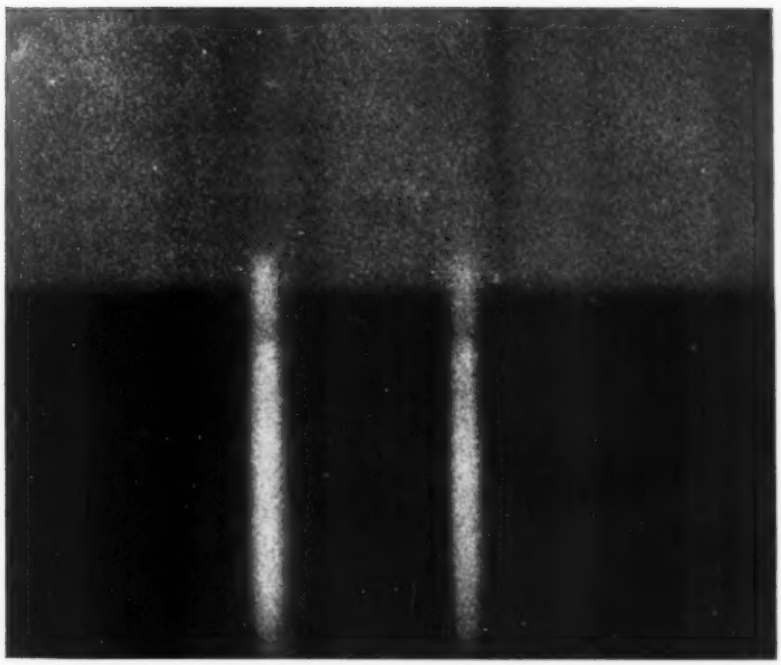


FIG. 1. Hβ lines of hydrogen in 3rd order 10 meter plane grating.

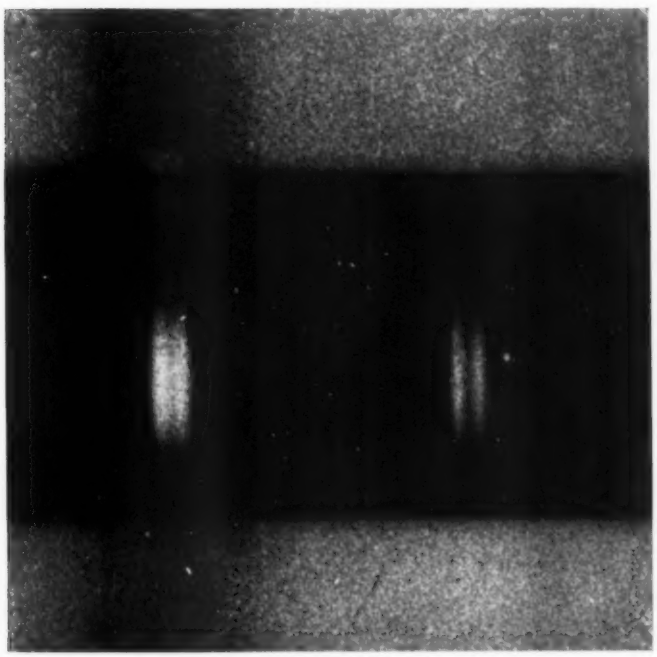


FIG. 2. Hα lines of hydrogen in 3rd order 10 meter plane grating.

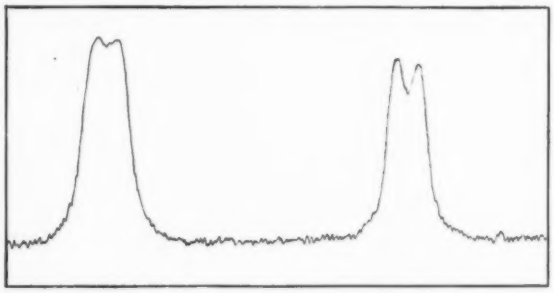


FIG. 3. Microphotometer curve of Hα lines.

belonging to the heavy isotope. On careful inspection the fine structure doublets can be seen. This is more evident in H^2 than in H^1, which was to have been expected, for the breadth of the lines is chiefly due to the Doppler effect and the H^1 atoms are on the average moving 1.4 times as fast as the H^2 atoms, at the same temperature.

This difference is brought out more strikingly in Figs. 2 and 3, which reproduce the photograph of the Hα line and the corresponding photometric curve. Fig. 4 is a microphotometer curve of $H^2\alpha$ under high magnification and shows very

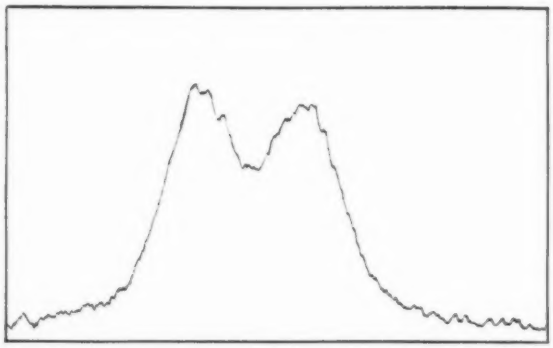


FIG. 4. Microphotometer curve of $H^2\alpha$ showing fine structure.

clearly the beginning of the further fine structure resolution. The curve is almost identical with the curve of $H^1\alpha$ obtained by Kent, Taylor and Pearson[2] with an interferometer, their discharge tube being cooled with liquid air. By cooling our discharge tube it is probable that a far greater resolution of the fine structure of $H^2\alpha$ may be obtained.

---

[2] Kent, Taylor and Pearson, Phys. Rev. **30**, 266 (1927)

The Infrared Absorption of Benzene in the Liquid, Solid and Vapor States

C. E. LEBERKNIGHT, *Rowland Hall, Johns Hopkins University*

(Received March 16, 1933)

The 2.19μ, 2.46μ, and 3.25μ infrared absorption regions of benzene were investigated in the liquid, solid and vapor states with a grating spectrograph whose theoretical resolving power was almost 15,000. For all the bands measured the solid absorption was approximately the same as that of the liquid but with the bands shifted slightly towards longer wave-lengths; the 3.25μ and 2.46μ bands showed a shift of about 13A, and the bands lying in the 2.19μ region showed a shift of about 20A. In the vapor the absorption was quite different from that of the liquid. The 3.25μ region had two strong bands in the vapor, each showing evidences of three components, while the liquid had three bands; interpreting the three components of the vapor bands as the envelopes of the vibration-rotation bands the value, 14×10^{-39} g cm^2, is obtained for the moment of inertia of the benzene molecule. The 2.46μ region showed a doublet in the vapor while the liquid had a single strong band. The 2.19μ region showed two bands incompletely resolved in the vapor, but the liquid and solid showed a group of five bands with indications of a sixth.

INTRODUCTION

IN investigating the problem of the structure of molecules, it is customary to proceed from the dynamical standpoint, that is, by the determination of the modes of vibration of the atoms or radicals within the molecule. These vibrations, being of comparatively low frequency, give rise to absorption bands in the infrared portion of the spectrum and can be determined from an analysis of the absorption spectrum of the molecule. Much work has been done with powerful instruments on the absorption of substances in the gaseous state, but little has been done on the absorption bands of a substance in the liquid, solid and vapor states with instruments of high resolving power.

In the case of benzene, few studies have been conducted with particular regard to its infrared absorption in the three states. Also, the region between 2μ and 4μ has been only meagerly explored with instruments of high resolving power. Barnes[1] studied the 3.25μ band of liquid benzene and Meyer,[2] the same band in the vapor state. Dreisch[3] measured the absorption of liquid and gaseous benzene up to 2.7μ. He reports bands at 2.188μ and 2.488μ, and a change in the magnitude of the absorption accompanied by a slight shift of the bands towards shorter wave-lengths in the vapor. This work was done with a prism spectrograph whose resolving power above 2μ was low. Using a grating spectrograph of high resolving power, Barnes found the 3.25μ band of liquid benzene to consist of three components of approximately the same intensity lying close together. Meyer, with a grating spectrograph, found a broad doublet in the vapor in the same region. Since the shift in the absorption bands as found by Dreisch between the liquid and vapor states is small and since there is no experimental information concerning the infrared absorption of solid benzene, this work was undertaken to measure the 2.188μ, 2.488μ, and 3.25μ bands of benzene in the three states with a grating spectrograph of high resolving power which would enable a careful study to be made of the changes in the absorption involved in the change of state.

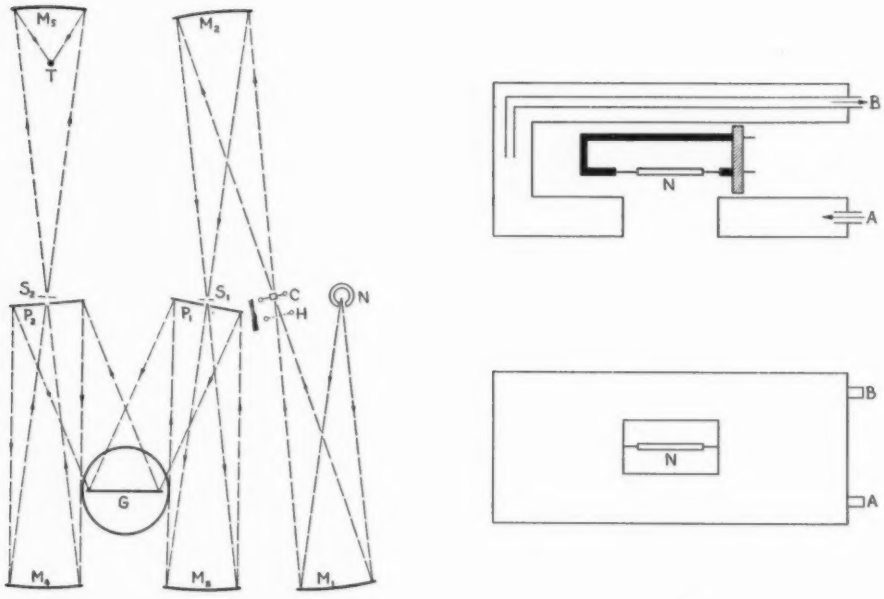
APPARATUS

A diagram of the apparatus used in this investigation is shown in Fig. 1a. G is an echelette grating; two gratings were used, both being 3600 line gratings with a ruled surface of four inches thus giving a theoretical resolving power of almost 15,000. One of these gratings concentrated the energy at 4μ while the other had its region of concentration at 2.7μ. The optical system has been described by Barnes.[1] The path of light

[1] Barnes, Phys. Rev. **35**, 1528 (1930); Phys. Rev. **36**, 296 (1930).

[2] Meyer, J. O. S. A. **15**, 257 (1927).

[3] Dreisch, Zeits. f. Physik **30**, 200 (1924).



a b

FIG. 1a. Diagram of the optical system; b. Diagram of the water cooler for the Nernst glower.

through the system is made clear in the diagram. By employing plane mirrors in the manner shown it is possible to use the concave mirrors along their axes and thus minimize the aberrations; in fact, with the width of the first slit equal to 0.1 mm the image of the visible mercury lines is formed straight and well defined.

A vacuum thermocouple was used in series with a high sensitivity Kipp and Zonen galvanometer to measure the intensity of the radiation. Although the thermocouple was not a compensating one, the drift of the galvanometer current was so very small that continuous readings could be taken without determining zero readings at each setting of the grating. The galvanometer deflections were read to 0.5 mm on a scale at a distance of five meters.

The source of radiation was a Nernst glower which operated at 220 volts and 0.6 amperes from a large storage battery. The glower was shielded with a water cooler to minimize convection currents. (Fig. 1b.)

CALIBRATION

By observing the grating settings which directed the various orders of the visible mercury spectrum on the second slit, a calibration more accurate than 5A was obtained. The grating, mounted on a spectrometer table, could be rotated by the observer from a distance of three meters at which distance the rotation was measured by the motion of the image of an incandescent filament formed on a scale. The light from an incandescent lamp was reflected twice from a mirror fixed to the grating and then focussed on the scale so that the image of one of the filaments could be used as an indicator. A motion of the image on the scale of 1 mm corresponded to a change of wave-length falling on the slit S_2 of about 9A. In order to avoid the use of the long scale required to read from 2μ to 3.5μ a single scale 1 meter in length was used, but instead of a single mirror attached to the grating, two mirrors were used inclined at a small angle to each other in such a manner that when the image from the one mirror left the scale the image from the other came on. The instrument was calibrated before and after each set of readings by means of the higher orders of the visible mercury lines; in addition, the calibration was checked by taking readings on the third order of the 1.0140μ emission line of mercury.

PROCEDURE

In taking measurements in any part of the spectrum precautions had to be taken to elimi-

nate the effect of superimposed orders on the thermocouple which results from the fact that the radiation falling on S_1 contains a wide range of wave-lengths. This effect was practically eliminated by the use of filters whose preparation has been described by Pfund.[4] A film of zinc oxide deposited on a quartz plate was found to be quite satisfactory for the region between 2μ and 3.5μ. In Fig. 2 is shown the transmission curve

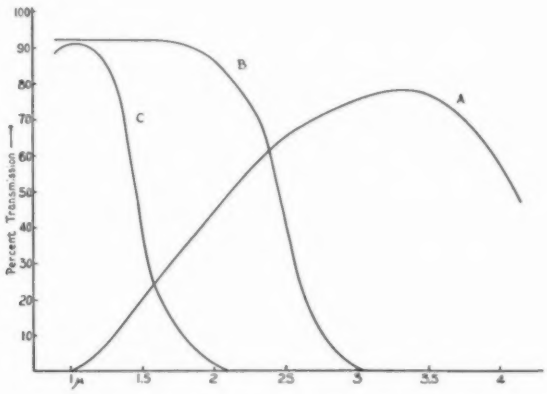


FIG. 2. Transmission curves of the filters; *A*, zinc oxide deposited on a quartz plate; *B*, corex "A" glass, 5.37 mm thick; *C*, pollopas, 4.45 mm thick.

of this filter whose efficiency is such that the contamination of lower wave-length energy is less than 2 percent at 2μ; about 7 percent at 2.6μ; and about 35 percent at 3.3μ, using the Nernst glower as the source. To be quite certain that the bands measured were not higher orders of lower wave-lengths, two other filters were used whose transmission curves are also shown on Fig. 2. As is seen from the curve, the transmission of the Corex glass filter is zero above 3μ and high at 1.5μ; hence the bands at 3.25μ should disappear when this filter is used, and such was the case. The same procedure was followed with the Pollopas filter to test the bands at 2.19μ and 2.46μ. Pollopas is a urea-formaldehyde condensation product.

In taking readings, settings were made at 1 mm intervals on the scale which corresponded to a change of wave-length of about 9A; tenths on the scale could be estimated easily. With the exception of the measurements on the solid and those on the vapor at 2.19μ, at each setting of the grating a zero reading was taken, and then readings were taken with the cell in the light path and with the radiation passing through a dummy cell. In the case of the measurements on the solid and those with the long cell required for the vapor at 2.19μ this method was not feasible. In these cases continuous readings were taken; this procedure being justified because of the fact that there was little drift in the thermocouple current. To be certain of the small shifts occurring between the bands in the liquid and solid states measurements were made with the liquid, then solid, then liquid, etc.

The benzene used was obtained from Merck and was dried by distilling it with phosphorus pentoxide. For the liquid measurements the benzene was contained in small cells with microscope cover glass windows; these cells have been described by Barnes.[1] In the case of the frozen benzene these same cells were placed in a brass tube which was packed in carbon dioxide snow; at first there was trouble with moisture condensing on the windows but this was eliminated by blowing a stream of dry air on the windows. For the measurements in the vapor state the benzene was contained in brass tubes with quartz windows; the length of the cell being so chosen with respect to the thickness of the liquid cell that there were approximately the same number of molecules in the light path in the two cases.

RESULTS

The absorption curves of benzene in the three states for the bands lying in the 2.19μ, 2.46μ and 3.25μ regions are shown in Figs. 3–5; and the wave-lengths of the absorption bands are listed in Table I. In this table column 1 gives the wave-lengths of the absorption bands for the liquid, column 2 gives the shifts of the bands in passing from the liquid to the solid at the temperature of carbon dioxide snow, and column 3 gives the wave-lengths of the bands in the vapor state. For all the bands measured it is evident that there is no difference in the type of absorption in passing from the liquid to the solid; in each of the regions of absorption the relative intensity of the individual bands is approximately the same in the two states, but there is a small shift in the position of the bands towards longer wave-

[4] Pfund, Phys. Rev. **36**, 71 (1930).

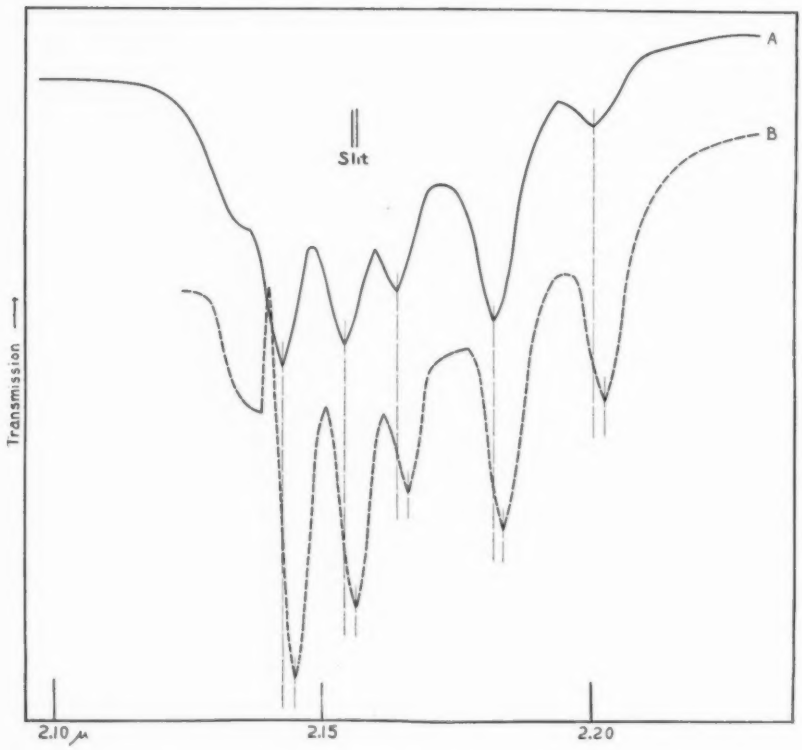


FIG. 3. Absorption curves for the 2.18μ band in the liquid and solid states; *A*, liquid cell = 0.15 mm; *B*, solid cell = 0.15 mm; slit = 12A.

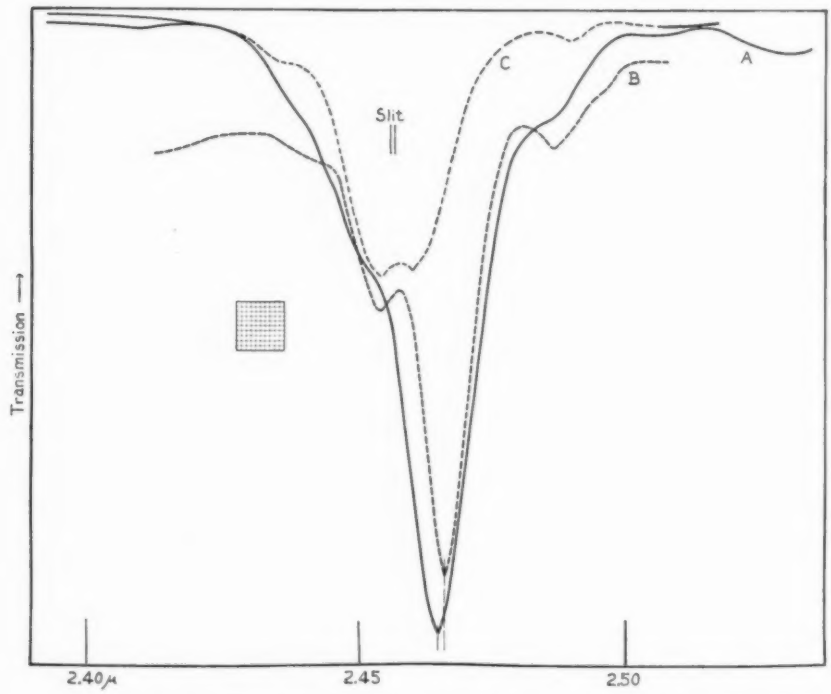


FIG. 4. Absorption curves for the 2.46μ band in the liquid, solid, and vapor states; *A*, liquid cell = 0.06 mm; *B*, solid cell = 0.06 mm; *C*, vapor cell = 10.3 cm; slit = 11A.

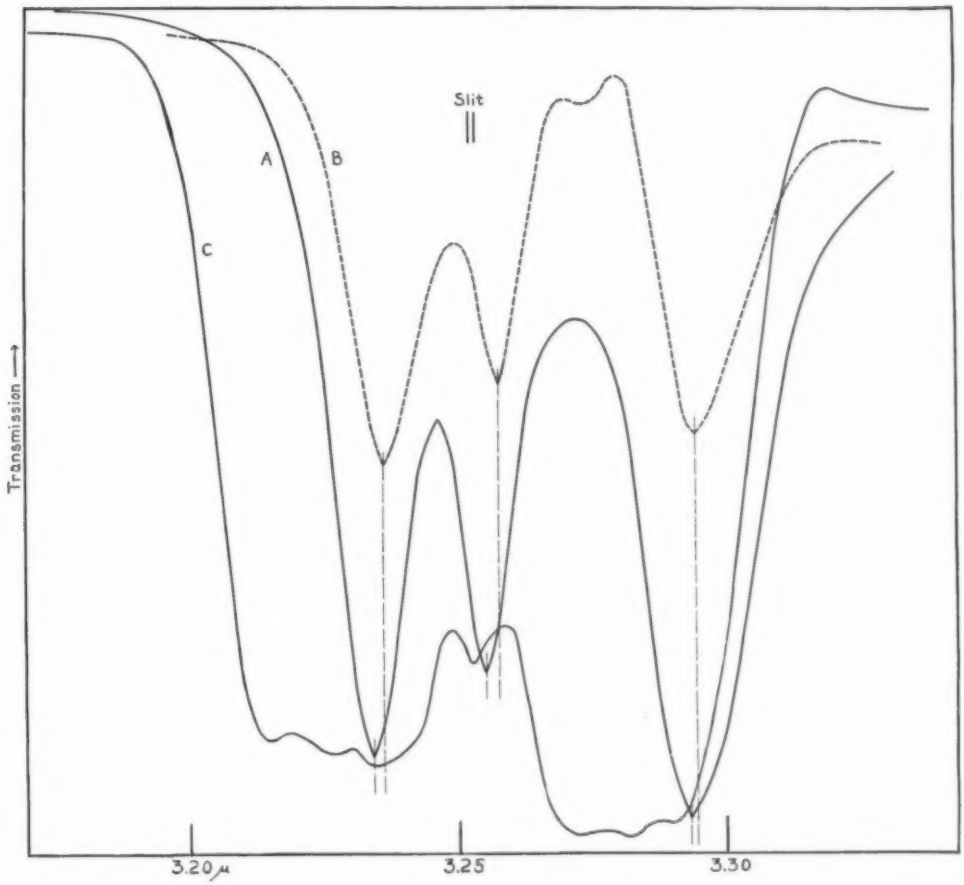


FIG. 5. Absorption curves for the 3.25μ band in the liquid, solid, and vapor states; *A*, liquid cell =0.02 mm; *B*, solid cell=0.02 mm; *C*, vapor cell=4 cm; slit=13A.

TABLE I.

Liquid	Liquid to Solid	Vapor
3.235μ	+0.0013μ	3.217μ 3.226 3.236
3.256	+0.0024	
3.294	+0.0012	3.273 3.283 3.292
	2.454μ	
2.464	+0.0013 2.486μ	2.452 2.462
2.144	+0.0020	
2.155	+0.0020	2.148
2.164	+0.0020	
2.183	+0.0019	2.175
2.202	+0.0020	

lengths in the solid absorption. The curves show very clearly, however, that there is a difference in absorption in passing from the liquid to the vapor state, and also that in the vapor the envelope of the vibration-rotation bands appears.

3.25μ region

The absorption curves for the bands lying in this region are shown in Fig. 3. The measurements on the liquid and solid were made with cells of 0.02 mm in thickness while those on the vapor were made with a cell of 4 cm in length. In the liquid there are three absorption bands of approximately the same intensity as has been reported by Barnes. The solid shows the same type of absorption as the liquid, having three bands of approximately the same intensity but there is a small average shift of 13A towards longer wave-lengths. In the vapor the absorption

is quite different from that of the liquid; there are two strong absorption bands and these in turn show evidences of finer structure, each band, in fact, showing evidences of three components. If these components are interpreted as the envelopes of the vibration-rotation bands of the benzene molecule they show a doublet separation of 0.019μ. Considering the benzene molecule to be plane and the vibrational frequency to be due to the vibration of the C–H atoms then these bands will be of the perpendicular type and the results of the calculations of Gerhard and Dennison[5] can be applied to the calculation of the moment of inertia about an axis lying in the plane of the molecule; this calculation gives the value, 14×10^{-39} g cm^2, which is in agreement with the value obtained by Kettering and Sleator[6] from measurements on the 9.5μ band of benzene.

2.46μ region

Fig. 4 shows the absorption curves for the band lying in this region. The thickness of the absorption cell was 0.06 mm for the liquid and solid measurements and 10.3 cm for those on the vapor. In the liquid there is a strong absorption band at 2.464μ with indications of two weaker bands, one on each side of the strong band. In the solid the strong band shows a shift of 13A towards longer wave-lengths and two weaker bands appear, caused, no doubt, by the narrowing of the lines by the low temperature. The vapor shows towards shorter wave-lengths a band which is double.

2.19μ region

In this region the curves (Fig. 5) show that instead of a single band, as reported by Dreisch, there is a group of five bands with indications of a sixth. In the solid state all these bands are shifted towards longer wave-lengths by approximately 20A. These measurements were made with cells of 0.15 mm in thickness. The sharp upward break in the solid absorption curve seemed to be due to some emission line in the particular Nernst glower used in the experiment since it did not occur when this part of the curve was repeated with an incandescent lamp as the source. The vapor appeared to have two main absorption bands incompletely resolved, lying at 2.148μ and 2.175μ.

In conclusion the writer wishes to express his appreciation to Professor A. H. Pfund for his assistance during the course of the investigation of the problem.

[5] Gerhard and Dennison, Phys. Rev. **43**, 197 (1933).
[6] Kettering and Sleator, Physics **4**, 39 (1933).

Zeeman Effect in the Arc Spectrum of Nickel

H. H. MARVIN AND A. E. BARAGAR, *University of Nebraska*

(Received April 7, 1933)

Measurements of the Zeeman patterns for 113 lines between λ5500 and λ3000 are presented, from which g-factors for 61 terms are calculated. The g-sum rule holds for configuration d^9s and for $d^8s(^2F)\cdot 4p$, but not for $d^9\cdot 4p$ or $d^8s(^4F)\cdot 5s$. It does hold for $d^9\cdot 4p$ and the quintet terms of $d^8s(^4F)\cdot 4p$ taken together, indicating that these groups interact. It is surmised that $d^8s(^4F)\cdot 5s$ is perturbed by $d^9\cdot 4d$. The theory of two-electron systems fits d^9s very well. It works fairly well with d^8s^2, but breaks down when applied to $d^9\cdot 4p$, probably because of increased importance of the magnetic interactions which are neglected in the theory.

THE arc spectrum of nickel (Ni I) provides an excellent opportunity to study the Zeeman effect in electron coupling intermediate between the (LS) and (jj) types. Henry Norris Russell[1] has extended the analysis of the spectrum to include nearly all of the lines, and has assigned the terms to electron configurations. The energy level separations in the triplet and quintet multiplets deviate markedly from the simple interval rules. It is evident that electron coupling in all configurations is intermediate, and fairly remote from (LS) coupling.

Several configurations, such as d^9s, d^8s^2, $d^9\cdot 4p$ and $d^9\cdot 4d$, are of the two-electron type. The d^9 group, which lacks one electron of completion, plays the rôle of a single electron except that it causes multiplet terms to be inverted. The d^8 group is related likewise to a d^2 group. The theory of two-electron systems in intermediate coupling has been developed sufficiently so that it accounts quite well for the arrangement of terms in configurations such as d^9s; but it fails when applied to such configurations as $d^9\cdot 4p$, probably because of neglected magnetic interactions of the electrons. Knowledge of the Zeeman effect in such configurations will be useful as a guide in the extension of the theory.

The energy levels of the intermediate and the high configurations are intermingled sufficiently so that fairly strong perturbations may be expected. Zeeman effect data will be useful in the study of these interactions.

Not much experimental work has been done on the Zeeman effect in the nickel spectrum. Several observers[2] have made measurements on a few lines, which do not permit fixing values of g-factors. The patterns for λ3597 and λ3722 observed by Beals[3] are consistent with the classification of the lines. The only precise measurements are those of Bakker,[4] who has observed the patterns of twenty-five lines, by using a vacuum arc as the source, and has calculated the g-factors for twenty terms.

This communication is a record of the investigation of the Zeeman effect for 113 lines in the range λ5500 to λ3000. The g-factors for 61 terms are calculated, and the results are tested by the g-sum rule. The experimental values are compared with the theoretical g-factors in intermediate coupling in the two-electron configurations d^9s and d^8s^2.

EXPERIMENTAL PART

The spectrum was photographed in the second and third orders of an Anderson 21-foot concave grating on a Paschen mounting. A quartz lens and a calcite plate were used to form separate images on the slit of the components of vibration parallel and perpendicular to the magnetic field. Several spectrograms were made of each com-

[1] Henry Norris Russell, Phys. Rev. **34**, 821 (1929).

[2] Reese, Astrophys. J. **12**, 120 (1900); Kent, Astrophys. J. **13**, 289 (1900); Kent, Johns Hopkins Univ. Circular **20**, 82 (1901); Peterke, Halle Inaug. Diss. (1909); Graftdijk, Thesis, Amsterdam (1911); Lüttig, Ann. d. Physik **38**, 43 (1912); Takahashi, J. Coll. Sci. Tokyo **41**, Art. 8 (1921); Yamada, J. Coll. Sci. Tokyo **41**, Art. 870 (1921).

[3] Beals, Proc. Roy. Soc. **A109**, 369 (1925).

[4] Bakker, Proc. Akad. Amsterdam **35**, 82 (1932).

ponent, and the best three of each were measured. Realizing that most of the Zeeman patterns would be unresolved, we aimed at the best quality for these patterns rather than at the highest resolution. The pole-pieces of the magnet were about 12 mm in diameter and 6 mm apart. The field strength, about 26,000 gauss, was computed from the Zeeman patterns of the Ca II lines λ3968.47 and λ3933.67, which appeared on the spectrograms because of calcium present in the carbon electrode of the arc.

The arc electrodes were strips of carbon and pure nickel which crossed each other at right angles between the magnet poles. The nickel electrode was fixed in position, and insulated from the pole by a fused quartz disk. The carbon electrode vibrated to produce an intermittent arc. Current from a 220 volt d.c. source was adjusted by a variable resistance to keep the nickel as cool as possible—in general, at a cherry-red heat. Operation of the arc in vacuum was unbearably tedious because magnetic debris had to be removed frequently from the pole-pieces. Fortunately, it could be operated in the open air without serious detrimental effect upon the quality of the spectrograms. The time of exposure was 4 to 8 hours.

The spectrograms were measured with a Gaertner comparator, the eyepiece of which was provided with several pairs of index lines ruled on glass. The comparator, in a darkened room, was illuminated by a 75 watt lamp through a green gelatin filter. Additional screens of thin white paper were used at the discretion of the operator. Zeeman patterns of weak lines, which were all but invisible by daylight illumination, were brought out clearly. The magnification was adjusted so that a pair of index lines spanned a component of the pattern, with the lines at the positions of sharp contrast on opposite sides of the center of intensity.

The unresolved Zeeman patterns of lines due to transitions between terms with nearly equal g-factors were easily measured. The observer had only to guard against errors of judgment in case of unequal shading off on opposite sides of the center of intensity. The patterns of lines due to transitions between terms with widely differing g-factors, which were nearly resolved, were troublesome because their centers of intensity were ill-defined. Reliance was placed, in these instances, upon the average of a large number of measurements made at different times, in order to eliminate personal errors; nevertheless, the measurements of these patterns are relatively inaccurate.

Many lines due to transitions from intermediate to deep configurations were reversed in the no-field exposures, and tended to be reversed in the field exposures. In some instances, notably λ3619, λ3524, λ3492, λ3458 and λ3414, reversal was clean-cut and complete. The separation of the parts of a reversed Zeeman component was nearly equal to the split of the reversed no-field line. We believe that this tendency toward reversal has given the arc in air an evil reputation which it does not deserve as a source for the study of the Zeeman effect. We were able to measure reversed and partially reversed unresolved patterns about as precisely as those which were not reversed.

The patterns of nearly all lines between λ5500 and λ3380 were measured on two or three spectrograms. Below λ3380 only one spectrogram was secured, for the perpendicular component. The number of measurements ranged from about 20 on patterns easy to measure consistently to more than 70 on those which were most difficult. The self-consistency of the g-factor values calculated from the measurements indicates that this procedure was effective in reducing error due to width and fuzziness of the components.

The g-factors were calculated from resolved patterns by the method of Landé, and from unresolved patterns by formulae given by Shenstone and Blair.[5] The combinations of intermediate terms with the deep terms a^3D, a^1D, a^1S and a^3F were considered first. In most instances, two to five combinations appeared on the spectrograms. The computed g-factor values were weighted in accord with our judgment of the relative precision of the measurements of the patterns, and weighted mean values were computed. The g-factors of the high terms were calculated in the same manner, from their combinations with intermediate terms. Finally, the g-factors of a few intermediate terms, which had no combinations with deep terms on the spectro-

[5] Shenstone and Blair, Phil. Mag. **8**, 765 (1929).

TABLE I. *Zeeman patterns and g-factors for lines of the Ni I spectrum.*

λ	Combination $x-y$	Zeeman Effect Patterns Observed		Calculated	g-factors g_z	g_v
5476.91	$a^1S_0-z^1P_1^0$	(0), 0.997	(a)	(0), 0.997	0/0	0.997
5081.12	$z^1F_3^0-e^1G_4$	(0), 1.025	(d)	(0), 1.025	1.019	1.021*
5080.53	$z^3F_4^0-e^3G_5$	(0), 1.066		(0), 1.021	1.282	1.195
5035.36	$z^3F_3^0-e^3G_4$	(0), 1.014		(0), 1.014	1.078	1.055
5017.61	$z^5F_3^0-e^5F_3$	(0), 1.398		(0), 1.395	1.395	1.395
4984.12	$z^3F_2^0-e^3G_3$	(0), 0.821		(0), 0.821	0.718	0.770
4980.17	$z^5F_4^0-e^3G_5$	(0), 1.009		(0), 1.019	1.283	1.195
4918.37	$z^3G_4^0-f^3F_3$	(0), 0.996		(0), 0.996	1.046	1.079
4904.40	$z^3P_2^0-e^3S_1$	(0), (0.514), 1.253	(b)	(0), (0.463), 1.228	1.459	1.922
4873.45	$z^5F_3^0-e^5F_2$	(0), 1.500		(0), 1.497	1.225	0.953
4866.28	$z^5F_2^0-e^5F_4$	(0), 1.510	(e)	(0), 1.527	1.395	1.329
4855.42	$z^3P_2^0-e^3P_2$	(0), 1.445		(0.050), 1.445	1.459	1.431
4831.19	$z^5F_4^0-e^5F_3$	(0), 1.426	(e)	(0), 1.370	1.283	1.225
4829.04	$z^3P_2^0-f^3D_3$	(−), 1.172		(−), 1.172	1.459	1.316
4806.99	$z^3D_3^0-f^3F_4$	(−), 1.250		(−), 1.205	1.310	1.268
4786.54	$z^5G_5^0-e^5F_5$	(0.537), 1.329		(0.524), 1.331	1.267	1.395
4763.95	$z^3F_4^0-f^3F_4$	(0), 1.266		(0.053), 1.275	1.282	1.268
4756.52	$z^5G_4^0-e^5F_4$	(0.635), 1.241		(0.573), 1.243	1.157	1.329
4715.76	$z^5G_3^0-e^5F_3$	(0.807), 1.083		(0.774), 1.080	0.935	1.225
4714.42	$z^5G_6^0-e^5F_5$	(0), 1.147		(0), 1.147	1.324	1.395
4686.21	$z^5G_2^0-e^5F_2$	(0.622), (1.203), −	(b)	(0.618), (1.236), −	0.335	0.953
4648.66	$z^5G_5^0-e^5F_4$	(0), 1.167		(0), 1.143	1.267	1.329
4604.99	$z^5G_4^0-e^5F_3$	(0), 1.058		(0), 1.055	1.157	1.225
4600.36	$z^5G_2^0-e^5F_1$	(0), (0.277), 0.453	(b)	(0), (0.265), 0.467	0.335	0.070
4592.53	$z^5G_3^0-e^5F_2$	(0), 0.914		(0), 0.917	0.935	0.953
4470.49	$z^5D_2^0-e^5F_3$	(0), 0.852		(0), 0.837	1.613	1.225
4462.46	$z^5D_1^0-e^5F_2$	(0), (0.575), 0.666	(b)	(0), (0.574), 0.666	1.527	0.953
4459.05	$z^5D_3^0-e^5F_4$	(0), 1.057		(0), 1.047	1.517	1.329
4401.55	$z^5D_4^0-e^5F_5$	(0), 1.162		(0), 1.162	1.512	1.395
4359.59	$z^5D_2^0-e^5F_2$	(−), 1.297		(−), 1.283	1.613	0.953
4331.64	$b^1D_2-y^1D_2^0$	(−), 0.997		(−), 0.997	1.141	0.853
4325.61	$z^5D_3^0-e^5F_3$	(−), 1.378		(−), 1.371	1.517	1.225
4288.01	$z^3G_5^0-g^3F_4$	(0), 1.078		(0), 1.078	1.205	1.268
3973.55	$a^1D_2-z^3P_2^0$	(0.838), 1.213		(0.799), 1.237	1.015	1.459
3944.10	$z^3F_3^0-f^3G_4$	(−), 0.950		(−), 0.950	1.078	1.027
3858.28	$a^1D_2-z^3F_3^0$	(0), 1.152		(0), 1.141	1.015	1.078
3807.14	$a^1D_2-z^3D_3^0$	(0), (0.302), (−), 1.647	(b)	(0), (0.295), (−), 1.605	1.015	1.310
3783.52	$a^1D_2-z^5F_3^0$	(0), 1.455		(0), 1.435	1.015	1.225
3775.56	$a^1D_2-z^3D_2^0$	(0), 1.026		(0.040), 1.026	1.015	1.037
3749.04	$a^3F_3-z^5D_3^0$	(−), 1.301		(−), 1.300	1.083	1.517
3739.23	$a^3F_3-z^5G_4^0$	(0), 1.332		(0), 1.268	1.083	1.157
3736.81	$a^1D_2-z^5F_2^0$	(0), 0.996		(0.075), 0.994	1.015	0.972
3722.48	$a^3D_1-z^3P_2^0$	(0), (0.929), 0.474, 1.445, 2.403	(a)	(0), (0.959), 0.500, 1.459, 2.418	0.500	1.459
3688.41	$a^3F_2-z^3F_3^0$	(0), 1.562		(0), 1.488	0.668	1.078
3674.11	$a^1D_2-z^3F_2^0$	(0.586), 0.826	(e)	(0.555), 0.866	1.015	0.718
3670.42	$a^3F_3-z^3P_2^0$	(0), 0.661		(0), 0.707	1.083	1.459
3669.23	$a^3F_3-z^5G_3^0$	(0.451), 1.008		(0.381), 1.009	1.083	.0935
3664.09	$a^3F_2-z^3P_1^0$	(0), (.811), 0, −	(a)	(0), (0.783), 0, 0.668	0.668	1.451
3624.73	$a^3F_4-z^5G_5^0$	(0), 1.314	(e)	(0), 1.301	1.250	1.267
3619.39	$a^1D_2-z^1F_3^0$	(0), 1.039		(0), 1.023	1.015	1.019
3612.73	$a^3F_2-z^3D_2^0$	(0.686), 0.840		(0.664), 0.852	0.668	1.037
3610.45	$a^3D_2-z^3P_2^0$	(0.588), 1.295		(0.553), 1.305	1.152	1.459
3609.31	$a^3D_2-z^5G_3^0$	(0), 0.729		(0), 0.718	1.152	0.935
3602.28	$a^3F_3-z^5F_4^0$	(0), 1.613		(0), 1.583	1.083	1.283
3597.70	$a^3D_1-z^3P_1^0$	(0.956), 0.511, 1.425	(a)	(0.951), 0.500, 1.451	0.500	1.451
3587.93	$a^3D_3-z^5G_4^0$	(0), 0.890		(0), 0.893	1.333	1.157
3571.87	$a^3F_3-z^3F_3^0$	(0), 1.077		(0), 1.081	1.083	1.078
3566.37	$a^1D_2-z^1D_2^0$	(0), 1.020		(0), 1.017	1.015	1.018
3561.75	$a^3F_4-z^5G_4^0$	(−), 1.192		(−), 1.203	1.250	1.157

TABLE I—*Continued*

λ	Combination $x-y$	Zeeman Effect Patterns Observed	Calculated	g-factors g_x	g_y
3548.19	$a^3D_1-z^3D_2^0$	(0), −, 1.066, 1.586 (c) (e)	(0), 0.500, 1.037, 1.574	0.500	1.037
3527.99	$a^3F_3-z^3D_3^0$	(−), 1.201	(−), 1.197	1.083	1.310
3524.54	$a^3D_3-z^3P_2^0$	(0), 1.204	(0), 1.207	1.333	1.459
3523.45	$a^3D_3-z^5G_3^0$	(1.009), 1.134 (e)	(1.023), 1.134	1.333	0.935
3519.78	$a^3F_2-z^3F_2^0$	(0), 0.707	(0), 0.693	0.668	0.718
3515.06	$a^3D_2-z^3F_2^0$	(0), 0.998	(0), 1.004	1.152	1.078
3513.95	$a^3D_1-z^5F_2^0$	(0), (−), −, 0.950, 1.477 (a)	(0), (0.472) 0.500, 0.972, 1.444	0.500	0.972
3510.34	$a^3D_1-z^3P_0^0$	(0), 0.498 (a)	(0), 0.500	0.500	0/0
3500.85	$a^3F_3-z^3D_2^0$	(0), 1.122	(0), 1.129	1.083	1.037
3492.97	$a^3D_2-z^3P_1^0$	(0), 1.005	(0), 1.003	1.152	1.451
3483.78	$a^3F_2-z^3D_1^0$	(0), 0.710	(0), 0.732	0.668	0.540
3472.55	$a^3D_2-z^3D_3^0$	(0), 1.418	(0), 1.468	1.152	1.310
3469.48	$a^3F_2-z^1F_3^0$	(0), 1.355	(0), 1.370	0.668	1.019
3467.51	$a^3F_3-z^5F_2^0$	(0), 1.206	(0), 1.194	1.083	0.972
3461.66	$a^3D_3-z^5F_4^0$	(0), 1.167	(0), 1.208	1.333	1.283
3458.47	$a^3D_1-z^3F_2^0$	(0), 0.815	(0), 0.827	0.500	0.718
3452.89	$a^3D_2-z^5F_3^0$	(0), 1.275	(0), 1.298	1.152	1.225
3446.26	$a^3D_2-z^3D_2^0$	(0), 1.081	(0.207), 1.094	1.152	1.037
3437.28	$a^3F_4-z^5F_4^0$	(0), 1.270	(0.110), 1.262	1.250	1.283
3433.57	$a^3D_3-z^3F_3^0$	(0.670), 1.180 (e)	(0.656), 1.205	1.333	1.078
3423.71	$a^3D_1-z^3D_1^0$	(0), 0.509	(0.020), 0.520	0.500	0.540
3414.77	$a^3D_3-z^3F_4^0$	(0), 1.183	(0), 1.205	1.333	1.282
3413.94	$a^3D_2-z^5F_2^0$	(−), 1.072 (d)	(−), 1.062	1.152	0.972
3413.48	$a^3F_3-z^3F_2^0$	(−), 1.499 (d)	(−), 1.448	1.083	0.718
3392.99	$a^3D_3-z^3D_3^0$	(0), 1.321	(0.059), 1.322	1.333	1.310
3391.05	$a^3F_4-z^3F_4^0$	(−), 1.278	(−), 1.266	1.250	1.282
3380.89	$a^3F_2-z^3G_3^0$	(0), 0.814 (d)	(0), 0.814	0.668	0.741*
3380.58	$a^1D_2-z^1P_1^0$	(0), 1.017 (d)	(0), 1.024	1.015	0.997
3374.64	$z^5G_4^0-e^5H_7^0$	(−), 1.101 (d)	(−), 1.101	1.324	1.264*
3374.23	$a^3D_3-z^5F_3^0$	(−), 1.197 (d)	(−), 1.279	1.333	1.225
3372.00	$a^3F_3-z^3G_4^0$	(−), 0.981	(−), 0.990	1.083	1.046
3369.58	$a^3F_4-z^3D_3^0$	(−), 1.158	(−), 1.160	1.250	1.310
3366.17	$a^3F_3-z^1F_3^0$	(−), 1.067 (d)	(−), 1.051	1.083	1.019
3365.77	$a^1D_2-y^3F_3^0$	(−), 1.317 (d)	(−), 1.285	1.015	1.150
3361.56	$a^3D_2-z^3F_2^0$	(−), 0.938	(−), 0.935	1.152	0.718
3322.32	$a^1D_2-y^3D_3^0$	(−), 1.442	(−), 1.442	1.015	1.229
3320.26	$a^3F_3-z^1D_2^0$	(−), 1.170	(−), 1.148	1.083	1.018
3315.67	$a^3D_2-z^1F_3^0$	(−), 0.923	(−), 0.888	1.152	1.019
3250.75	$a^1D_2-y^3D_2^0$	(−), 1.086	(−), 1.100	1.015	1.185
3248.44	$a^3D_3-z^3G_4^0$	(−), 0.664	(−), 0.616	1.333	1.046
3243.06	$a^3D_3-z^1F_3^0$	(−), 1.173	(−), 1.176	1.333	1.019
3234.66	$a^3D_2-z^3G_3^0$	(−), 0	(−), 0**	1.152	0.741*
3232.95	$a^3F_4-z^3G_5^0$	(−), 1.115	(−), 1.115	1.250	1.205
3225.03	$a^1D_2-y^3D_1^0$	(−), 1.276	(−), 1.262	1.015	0.522
3221.66	$a^3F_4-z^1F_3^0$	(−), 1.617	(−), 1.597	1.250	1.019
3134.11	$a^3D_1-y^3F_2^0$	(−), 0.948	(−), 0.934	0.500	0.789
3101.88	$a^1D_2-y^1F_3^0$	(−), 1.021	(−), 1.021	1.015	1.018
3101.56	$a^3D_2-y^3F_3^0$	(−), 1.149	(−), 1.148	1.152	1.150
3080.76	$a^3D_1-y^3D_2^0$	(−), 1.570	(−), 1.528	0.500	1.185
3057.65	$a^3D_1-y^3D_1^0$	(−), 0.540	(−), 0.511	0.500	0.522
3054.32	$a^3D_2-y^3F_2^0$	(−), 0.966	(−), 0.971	1.152	0.789
3037.94	$a^3D_3-y^3F_3^0$	(−), 1.241	(−), 1.242	1.333	1.150
3012.01	$a^1D_2-y^1D_2^0$	(−), 0.934	(−), 0.934	1.015	0.853
2994.46	$a^3D_3-z^1G_4^0$	(−), 0 (e)	(−), −	1.333	−

(a) Resolved pattern.
(b) Parallel component pattern resolved.
(c) Perpendicular component pattern resolved.
(d) Perpendicular components of adjacent lines overlap slightly.
(e) Decides classification of the line.
* g_y not less than value given.
** Strongest perpendicular components in the Landé pattern are at −0.08.

grams, were determined from their high term combinations; and the g-factor for b^1D_2 was calculated from its single combination with an intermediate term. A critical survey of the whole array of g-factor values indicated that the self-consistency of the results could not be improved by altering arbitrarily the g-factors of the deep terms which were first considered.

The wave-lengths and classifications of the spectrum lines, the observed Zeeman patterns, the weighted g-factors and the Zeeman patterns calculated from these for comparison with the observed patterns are given in Table I. Parallel components are lacking for lines below λ3380, and for a few other weak lines. This is of little moment, for the perpendicular components are more useful in determining g-factors. The perpendicular components of adjacent lines overlapped slightly, in a few instances, so that it was impossible to resolve them cleanly. Zero weight was given to these measurements when the g-factors could be calculated from other lines. In three instances in which the g-factor is calculated from one of these questionable patterns alone, the value is indicated as a lower limit. Several instances are noted in which the observed Zeeman pattern decides the classification of the line, where Russell gives an alternative.

The g-factors are compared with their theoretical values for (LS) coupling in Table II. Bakker's values are given for comparison. The agreement of the two sets of experimental values is excellent. In three instances only is the discrepancy greater than we would expect from the estimates of ex-

TABLE II. *Comparison of observed g-factors with their theoretical values for (LS) coupling.*

Configuration	Term	g-factors (LS) coupling	g-factors Observed	g-factors Bakker
d^{10}	a^1S_0	0/0	0/0	0/0
d^9s	a^3D_3	1.333	1.333	1.33
	a^3D_2	1.167	1.152	1.15
	a^3D_1	0.500	0.500	0.50
	a^1D_2	1.000	1.015	1.01
d^8s^2	a^3F_4	1.250	1.250	1.25
	a^3F_3	1.083	1.083	1.08
	a^3F_2	0.667	0.668	0.67
	b^1D_2	1.000	1.14	—
$d^9\cdot 4p$	$z^3F_4^0$	1.250	1.28	1.25
	$z^3F_3^0$	1.083	1.08	1.08
	$z^3F_2^0$	0.667	0.72	0.74
	$z^3D_3^0$	1.333	1.31	1.29
	$z^3D_2^0$	1.167	1.04	1.03
	$z^3D_1^0$	0.500	0.54	0.55
	$z^3P_2^0$	1.500	1.46	1.49
	$z^3P_1^0$	1.500	1.45	1.43
	$z^3P_0^0$	0/0	0/0	0/0
	$z^1F_3^0$	1.000	1.02	1.04
	$z^1D_2^0$	1.000	1.02	1.06
	$z^1P_1^0$	1.000	1.00	1.02
$d^8s\cdot 4p$ (^4F)	$z^5G_6^0$	1.333	1.32	
	$z^5G_5^0$	1.267	1.27	
	$z^5G_4^0$	1.150	1.16	
	$z^5G_3^0$	0.917	0.93	
	$z^5G_2^0$	0.333	0.33	
	$z^5F_5^0$	1.400	1.40	
	$z^5F_4^0$	1.350	1.28	
	$z^5F_3^0$	1.250	1.23	
	$z^5F_2^0$	1.000	0.97	
	$z^5D_4^0$	1.500	1.51	
	$z^5D_3^0$	1.500	1.52	
	$z^5D_2^0$	1.500	1.61	
	$z^5D_1^0$	1.500	1.53	
$d^8s\cdot 4p$ (^2F)	$z^3G_5^0$	1.200	1.20	
	$z^3G_4^0$	1.050	1.05	
	$z^3G_3^0$	0.750	0.74*	
	$y^3F_3^0$	1.083	1.15	
	$y^3F_2^0$	0.667	0.79	
	$y^3D_3^0$	1.333	1.23	
	$y^3D_2^0$	1.167	1.19	
	$y^3D_1^0$	0.500	0.52	
	$y^1F_3^0$	1.000	1.02	
	$y^1D_2^0$	1.000	0.85	
$d^9\cdot 4d$	e^3G_5	1.200	1.20	
	e^3G_4	1.050	1.05	
	e^3G_3	0.750	0.77	
	f^3D_3	1.333	1.32	
	e^3P_2	1.500	1.43	
	e^3S_1	2.000	1.92	
	e^1G_4	1.000	1.02*	
$d^9\cdot 5d$	f^3G_4	1.050	1.03	
$d^8s\cdot 5s$ (^4F)	e^5F_5	1.400	1.40	
	e^5F_4	1.350	1.33	
	e^5F_3	1.250	1.23	
	e^5F_2	1.000	0.95	
	e^5F_1	0.000	0.07	
	f^5F_4	1.250	1.27	
	f^5F_3	1.083	1.08	
$d^8s\cdot 4d$ (^4F)	e^5H_7	1.286	1.26*	
$d^8s\cdot 5s$ (^2F)	g^3F_4	1.250	1.27	

* Value not less than that given.

perimental error. Bakker states that his values are reliable to about 0.01. We estimate that our values for the deep multiplets a^3D, a^1D and a^3F are reliable to about 0.003 and those for the configurations $d^9 \cdot 4p$, $d^8s(^4F) \cdot 4p$ and $d^8s(^4F) \cdot 5s$ to about 0.01, while others may be uncertain by as much as 0.02 in instances where the term appears in but one line on our spectrograms.

Theoretical Part

The g-sum rule holds for a configuration unless it is perturbed. Perturbations are usually small unless the energy levels of the configurations concerned are actually interspersed. There is considerable opportunity in the nickel spectrum for perturbations, especially among the intermediate and high configurations. The data presented in the preceding section enable conclusions to be drawn concerning interactions among a few configurations, by applying the g-sum rule and the theory of two-electron systems.

The d^9s configuration is not perturbed. Laporte and Inglis[6] have shown that Houston's theory[7] of the two-electron system with one electron in an s state fits this configuration. Table III shows

Table III. d^9s *configuration.*

Term	Level Exp.	Level Theor.	g-factor Exp.	g-factor Theor.
a^3D_3	205	205	1.333	1.333
a^3D_2	880	872	1.152	1.152
a^3D_1	1713	1713	0.500	0.500
a^1D_2	3411	3402	1.015	1.015

how accurately Houston's formulae give the energy levels and the g-factors. The constants which appear in the formulae have the values $X = -7.809$ and $\gamma = -301.6hc$. The experimental g-factors obey the g-sum rule.

The g-sum rule cannot be applied to the d^8s^2 configuration, since only four g-factors are known. Johnson's formulae[8] for the d^2 configuration were adjusted to fit as well as possible, and theoretical values for the g-factors were calculated by the formulae of Inglis and Johnson.[9]

[6] Laporte and Inglis, Phys. Rev. **35**, 1340 (1930).
[7] Houston, Phys. Rev. **33**, 297 (1929).
[8] Johnson, Phys. Rev. **38**, 1628 (1931).
[9] Inglis and Johnson, Phys. Rev. **38**, 1642 (1931).

The empirical and theoretical energy levels, and the experimental and theoretical g-factors are shown in Table IV. The constants in Johnson's

Table IV. d^8s^2 *configuration.*

Term	Level Exp.	Level Theor.	g-factor Exp.	g-factor Theor.
a^3F_4	0	0	1.250	1.250
a^3F_3	1332	1332	1.083	1.083
a^3F_2	2216	2260	0.668	0.668
a^3P_2	15610	16685	—	1.388
a^3P_1	15734	16832	—	1.500
a^3P_0	16017	17091	—	0/0
a^1G_4	22102	22102	—	1.000
b^1D_2	13521	14417	1.14	1.112
$^1S_0$	—	54179	—	0/0

formulae were adjusted to bring the levels of a^1G and a^3F into good agreement with their empirical values, and to leave the multiplet a^3P undistorted. The values assigned to the constants are $\alpha = 53106hc$, $\beta = 15500hc$, $\gamma = 13845hc$, $\delta = 21078hc$ and $a = -655.8hc$. The calculated values for a^3P and b^1D are about 1080 and 800, respectively, above their empirical values. The displacement is probably a consequence of the insufficiency of the theory of the two-electron system which, at present, takes account of the interactions of the electrons only to the zeroth order approximation. It is probable that the electrostatic interaction needs to be worked out to a higher order, in this instance. The fact that the calculated values for a^3P and b^1D are pushed apart may be due to perturbation by the configuration d^{10}, since its a^1S level lies between a^3P and b^1D and near to both of them. The magnitude of the perturbation, if any, cannot be determined until the theory of the two-electron system is extended to higher order approximations.

The term ^1S, which has not been found, is far above a^1G. It should be sought between 50,000 and 56,000.

The theoretical g-factors are very near to the values for (LS) coupling, except for those terms for which $J = 2$. The values for these terms are very sensitive to changes in the values assigned to the constants in the formulae. Since the theory of the two-electron system fits this configuration imperfectly, the g-factors calculated for the terms a^3P_2 and b^1D_2 are relatively uncertain. The experimental g-factors for the multiplet a^3F agree very well with the theoretical

TABLE V. *Application of g-sum rule to quintet terms.*

Configuration	Multiplet	$J=6$	$J=5$	$J=4$	$J=3$	$J=2$	$J=1$	$J=0$
$d^9\cdot 4p$	z^3F^0			1.28	1.08	0.72		
	z^3D^0				1.31	1.04	0.54	
	z^3P^0					1.46	1.45	0/0
	z^1F^0				1.02			
	z^1D^0					1.02		
	z^1P^0						1.00	
g-sum, observed				1.28	3.41	4.24	2.99	0/0
g-sum, (LS)				1.250	3.417	4.333	3.000	0/0
$d^8s\cdot 4p$ (^4F)	z^5G^0	1.32	1.27	1.16	0.93	0.33		
quintets only.	z^5F^0		1.40	1.28	1.23	0.97	—*	
	z^5D^0			1.51	1.52	1.61	1.53	—
g-sum, observed		1.32	2.67	3.95	3.68	2.91	1.53	—
g-sum, (LS)		1.333	2.667	4.000	3.667	2.833	1.500	0/0
Sum of g-sums, observed		1.32	2.67	5.23	7.09	7.15	4.52	—
Same, (LS)		1.333	2.667	5.250	7.083	7.167	4.500	0/0

* g-factor for $z^5F_1^0$ is not known. Its value is 0.000 for (LS) coupling.

values. The agreement for the term b^1D_2 is satisfactory, considering that both values are relatively uncertain.

The configuration $d^9\cdot 4p$ overlaps the quintet levels of $d^8s(^4F)\cdot 4p$, while the triplet levels of the latter are far higher. It seems possible that these quintet terms may be practically independent of all other terms of $d^8s\cdot 4p$, on account of their isolation from them. It seems probable that they interact with $d^9\cdot 4p$. This point is tested by applying the g-sum rule, in Table V. The rule does not hold for either group of terms alone. The deviations of the sums in the column $J=2$, particularly, from the sums for (LS) coupling are too large to be accounted for as due to errors of measurement. The rule does hold, however, for both groups taken together. This indicates fairly strong interaction. It is unfortunate that no combination of the term $z^5F_1^0$ appeared on our spectrograms. The g-sum in the column $J=1$ is uncertain, for this reason; but it appears that a g-factor for $z^5F_1^0$ close to 0.000, the (LS) value, would bring this sum into line.

Bakker concluded that the g-sum rule holds for $d^9\cdot 4p$. If all of his Zeeman patterns are given equal weights in the calculation of g-factors, however, a deviation from the g-sum rule is found in the same direction as in Table V, though not so large.

TABLE VI. *Application of g-sum rule to $d^8s(^2F)\cdot 4p$.*

Configuration	Multiplet	$J=5$	$J=4$	$J=3$	$J=2$	$J=1$
$d^8s\cdot 4p$ (^2F)	z^3G^0	1.20	1.05	0.74*		
	y^3F^0		—	1.15	0.79	
	y^3D^0			1.23	1.19	0.52
	z^1G^0		—			
	y^1F^0			1.02		
	y^1D^0				0.85	
g-sum, obs.		1.20	—	4.14	2.83	0.52
g-sum, (LS)		1.200	2.883	4.167	2.833	0.500

* Value not less than that given.

TABLE VII. *Failure of g-sum rule for $d^8s(^4F)\cdot 5s$.*

Configuration	Multiplet	$J=5$	$J=4$	$J=3$	$J=2$	$J=1$
$d^8s\cdot 5s$ (^4F)	e^5F	1.40	1.33	1.23	0.95	0.07
	f^3F		1.27	1.08	—	
g-sum, obs.		1.40	2.60	2.31	—	0.07
g-sum, (LS)		1.400	2.600	2.333	1.667	0.000

Inglis and Ginsburg[10] have pointed out that the theory of two-electron systems, in its present incomplete form, does not yield exact results when the outer electron is in a p state because of neglected magnetic interactions. This is the case with the $d^9\cdot 4p$ configuration. Johnson's formulae for the $d\cdot p$ system distort the multiplets seriously, and displace them considerably. The g-factors computed by the method of Inglis and Johnson exhibit deviations from the observed values about as great as those found by Inglis and Ginsburg in the $2p^5\cdot 3p$ configuration of neon.

The g-sum rule is applied to $d^8s(^2F)\cdot 4p$ in Table VI. The triplets of this configuration are so close to the singlets of $d^9\cdot 4p$ that interaction is possible. It is evident, from the table, that the interaction is very small if it exists at all.

Table VII indicates that the g-sum rule does not hold for $d^8s(^4F)\cdot 5s$. The levels of this configuration are so intermingled with those of $d^9\cdot 4d$ that interaction may be expected. Indeed, the evidences of interaction found are surprisingly small. The strong perturbation of the g-factor of the term e^5F_1 may be caused by f^3D_1, which is very close beside it.

[10] Inglis and Ginsburg, Phys. Rev. **43**, 194 (1933).

Further Studies in the Spectrum of the OH Molecule: A New (2,2) Band; Satellite Series in λ3122; Λ-Type Doubling and Electronic Spin Doubling

DAVID H. DAWSON[1] AND HERRICK L. JOHNSTON, *Department of Chemistry, Ohio State University*

(Received April 15, 1933)

The investigations begun in the neighborhood of the (0′,1″) band at λ3428 (cf. reference 4) have been extended to include the bands of the (0,0) and (1′,0″) sequences. Approximately 400 lines were observed in this region of the spectrum, of which 120 are reported for the first time. All but 18 of these have been successfully assigned. The assignments include the identification of a new (2,2) band with its principal head at λ3185; the identification of $^{RR}R_{21}$ and of $^{PP}P_{12}$ satellite branches of the (1,1) band; complete assignments of the regular R-branches of the (1,1) and (2′,1″) bands; and extensions of nearly all the remaining branches of the bands which appear in this region. The new data are also utilized to obtain Λ-doublets and spin doublets to $K=35$ for the 0″ level and to somewhat smaller values of K for the 1″ and 2″ levels. The experimental results are compared with equations in the literature. Molecular constants derived from the new (2,2) band are in good agreement with those derived from the (1′,2″) band.

INTRODUCTION

THE spectrum of the neutral OH molecule is of considerable interest both from a theoretical standpoint, because of its more than usual complexity[2] for a diatomic molecule, and from a practical standpoint because of its importance in the interpretation of numerous photochemical reactions and in the determination of certain chemical equilibria. Yet, in spite of its importance and of its early investigation[3] only a few bands are known and the assignments in them are incomplete.

In an earlier paper[4] we described the results of our investigations on the "water vapor" bands in the region between λ3429 and λ3570, which is the neighborhood of the (0′,1″) band. Those results included, among other things, the identification of the (1′,2″) band, with its principal head at λ3484, and permitted the formulation of the vibrational energy equation for the normal state of OH. The present paper describes the results of a similar investigation in the neighborhood of the (0,0) and (1,1) bands with principal heads at λ3064 and λ3122, and in certain other regions of the spectrum.

EXPERIMENTAL PROCEDURE

The method of excitation was similar to that described by J, D and W, and the same spectrograph was employed, a Hilger E 185 with quartz prisms. Exposure times were eight hours (λ3064 region) and twelve hours (λ2811 region), with a slit width of 0.001 mm. We also made some exposures with a new type of source, by substituting commercial 3 percent hydrogen peroxide solution for the distilled water. Oxygen was bubbled through this solution, as with the former procedure. Although the initial 3 percent solution yielded results no different from those with ordinary water, as the hydrogen peroxide became more concentrated by the process of distillation[5] during a series of exposures, the OH spectrum was noticeably intensified without a corresponding intensification of the residual background.

PLATE MEASUREMENTS

Tables I–VI contain *all new* lines which appeared on our plates in the regions under

[1] Dawson, Du Pont Fellow in Chemistry, 1932–33.

[2] Some of the factors responsible for this are: a large magnetic coupling energy leading to an inverted multiplet; an unusually large Λ-type doubling; the presence of numerous satellite branches; a nearly complete transformation from Hund's case (a) to Hund's case (b); etc.

[3] For a fairly complete bibliography of the earlier work on this spectrum, cf. reference 4.

[4] Johnston, Dawson and Walker, Phys. Rev. **43**, 473 (1933). We will henceforth refer to this paper as J, D and W.

[5] Water is the more volatile constituent. By continued distillation a residue with about 70 percent H_2O_2 remains.

investigation, with the possible exception of approximately 15 very faint markings which appeared adjacent to the strongest lines of λ3064 and which we attributed to some characteristic of the optical system.[6] These very faint markings did not appear in the correct positions for either isotope lines or for satellite lines. Furthermore, they resemble in appearance similar markings which were observed adjacent to over exposed iron lines.

The lines were measured with a Gaertner comparator and were calibrated against iron lines which were photographed before and after each OH exposure. Two independent readings were made on each line. Our measurements of 200 lines in λλ3064 and 3122 agreed with the measurements of Grebe and Holtz[7] to within an average 0.01A and our measurements of 135 lines in λλ2811 and 2875 checked the measurements of Watson[8] to within an average 0.001A. However,

TABLE I. *The (2,2) band, λ3185.*

K	λ air	Intensity	$\nu_{vac.}$	λ air	Intensity	$\nu_{vac.}$
	R_1				R_2	
1	3189.64	0	31342.5	3202.76	7*t*	31214.1
2	3188.04	7*t*	31358.2	3199.50	7*q*	31245.9
3	3186.95	2*t*	31368.9	3196.18	2	31278.3
4	3186.00	7*hq*	31378.3	3193.85	2	31301.2
5	3185.27	2*t*	31385.4	3192.37	7*t*	31315.7
6	3184.75	2*t*	31390.6	3191.28	0*t*	31326.4
7	3184.75	2*t*	31390.6	3190.15	0*t*	31337.5
8	3185.27	2*t*	31385.4	3190.15	0*t*	31337.5
9	3185.75	2	31380.7	3190.79	7*t*	31331.2
10	3186.95	2*t*	31368.9	3191.28	0*t*	31326.4
11				3192.76	7*t*	31312.2
	Q_1				Q_2	
1	3195.88	4	31281.3	3208.71	4	31155.8
2	3197.56	7	31264.9	3208.48	4*dq*	31158.5
3	3199.50	7*t*	31245.9	3208.48	4*dq*	31158.5
4	3201.53	9	31226.1	3209.41	7*q*	31149.4
5	3203.94	9*hq*	31202.6	3210.80	7*t*	31136.0
6	3206.57	2	31177.0	3212.65	4*t*	31118.0
7	3209.41	7*q*	31149.4	3215.04	4	31094.9
8	3212.65	4*t*	31118.0	3217.94	4	31066.9
9	3216.57	4	31080.1	3221.25	2	31034.9
10	3221.04	2	31036.9	3224.94	2	30999.4
11	3225.21	7*dq*	30996.8	3229.13	2	30959.2
12	3229.94	7*ht*	30951.4	3233.68	7*dq*	30915.6
13	3235.22	7*dq*	30900.9	3238.24	2	30872.1
14	3241.10	2	30844.9			
	P_1				P_2	
1	3199.09	7*ht*	31249.9			
2	3203.94	9*hq*	31202.6	3214.81	4	31097.1
3	3208.48	4*dq*	31158.5	3217.82	4	31068.1
4	3214.04	4	31104.6	3222.05	2	31027.2
5	3219.48	7	31051.9	3226.45	7*t*	30984.9
6	3225.21	4*dq*	30996.8	3231.29	2	30938.5
7	3231.53	2*t*	30936.2	3236.96	4	30884.3
8	3237.60	4	30878.2	3242.84	2	30828.3
9	3244.34	7*q*	30814.1	3249.18	4	30767.2
10	3251.25	4	30748.6	3256.04	2	30703.4
11	3258.83	2	30677.1	3263.12	2	30636.7
12	3266.68	0	30603.3	3270.74	2	30565.3
13	3274.25	4*t*	30532.6	3277.88	0	30498.8
					$^PQ_{12}$	
1				3211.99	2	31124.4

[6] Cf. Shenstone, Phys. Rev. **34**, 726 (1929).

[7] Grebe and Holtz, Ann. d. Physik **39**, 1243 (1912).

[8] Watson, Astrophys. J. **60**, 145 (1924).

TABLE II. *R-branches and satellite branches in the (1,1) band, λ3122.*

K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$	K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$
		R_1					R_2		
1	3128.23	4*hq*	31957.8	31957.6	1	3140.44	2*t*	31833.4	
2	3126.64	4*dt*	31974.5	31973.6	2	3136.85	7*dt*	31869.9	31867.9
3	3125.28	4	31987.9	31987.7	3	3133.96	2	31899.2	31898.9
4	3123.90	9*t*	32002.0	32001.3	4	3131.44	4*t*	31924.9	31924.5
5	3122.96	4*t*	32011.7*	32011.3	5	3129.49	7*t*	31944.9*	31944.6
6	3122.24	4	32019.1*	32020.0	6	3128.04	2	31959.7*	31959.8
7	3121.62	4*hq*	32025.4*	32024.4	7	3127.02	4	31970.1*	31969.8
8	3121.62	4*hq*	32025.4*	32025.7	8	3126.48	4	31976.0*	31975.8
9	3121.62	4*hq*	32025.4*	32023.0	9	3126.34	4	31977.6*	31977.0
10	3122.49	9*dq*	32016.5*	32017.0	10	3126.64	4	31974.5*	31974.4
11	3123.43	4	32006.9*	32007.4	11	3127.32	4	31967.1*	31966.8
12	3124.63	2	31994.6	31993.2	12	3128.55	2*ht*	31954.5	31954.8
13	3126.64	4*dt*	31974.5	31974.0	13	3130.23	7*t*	31937.2	31938.4
14	3128.77	2	31952.3	31952.7	14	3132.16	2	31917.6	31918.0
15	3131.44	4	31924.9	31925.5	15	3134.61	7*ht*	31892.6	31892.7
16	3134.33	7*ht*	31895.5	31894.2	16	3137.69	7*dq*	31861.3	31861.3
					17	3141.16	4	31826.2	31826.2
					18	3145.12	0	31786.1	31786.0
		$^{P}Q_{12}$					$^{PP}P_{12}$		
1	3150.33	2	31733.5		2	3156.14	7*t*	31675.1	
					3	3162.16	4	31610.4	31608.9
		$^{RR}R_{21}$			4	3169.64	7*t*	31540.3	31540.0
					5	3177.30	4	31464.2	31464.6
1	3118.88	0	32053.5	32054.3	6	3185.27	2*t*	31385.4	31385.1
2	3114.25	0	32101.2	32101.1	7	3193.85	2*t*	31301.2	31301.8
3	3108.81	0	32157.3	32157.7	8	3202.76	7*t*	31214.1.	31215.0
4	3104.79	0	32199.0	32198.7	9	3211.99	2*t*	31124.4	31124.6
5	3101.01	0	32238.2	32238.2					
6	3096.82	9*t*	32281.8	32281.5					

TABLE III. *Extensions in the (0,0) band, λ3064.*

K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$	K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$
		R_1					R_2		
22	3096.08	9*t*	32289.5		22	3098.56	11*t*	32263.7	
23	3101.67	2	32231.3		23	3104.34	4	32203.6	
24	3107.86	0	32167.2		24	3110.55	2	32139.4	
25	3114.70	7*dt*	32096.5		25	3117.11	7*ht*	32070.9	
26	3121.62	4*hq*	32025.4						
		$^{RR}R_{21}$					$^{PP}P_{12}$		
19	3024.25	2	33056.4	33056.9	15	3216.12	2	31084.5	31084.7
20	3026.16	2	33035.6	33034.8	16	3229.13	2	30974.5	30975.5
21	3028.78	0	33007.0	33007.4	17	3239.19	2	30863.1	30863.5
22	3031.88	0	32973.3	32972.9	18	3251.25	4*t*	30748.6	30749.2
23	3035.53	0	32933.6	32932.9	19	3263.58	0	30632.4	30632.4
24	3039.90	0	32886.3	32885.4	20	3276.26	7*t*	30513.9	30513.6
25	3044.81	0	32833.3		21	3289.56	0	30392.2	30392.1
26	3050.24	0	32774.8		22	3302.74	0*t*	30268.7	30268.8
					23	3316.68	0*t*	30142.0	30143.4
					24	3330.91	0	30013.2	30013.2
					25	3345.33	0	29883.9	29883.9

TABLE IV. *Extensions in the (1,0) band, λ2811.*

K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$	K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$
		P_1					P_2		
22	2979.93	2	33548.0		22	2982.39	2	33520.4	
23	2992.31	1	33409.3		23	2994.80	1	33381.5	
24	3005.99	0	33265.5		24	3007.74	0	33237.9	
		R_1					R_2		
19	2850.02	8*t*	35077.1	35075.3	19	2852.60	10*t*	35045.4	35046.4
20	2856.55	2	34997.0	34996.7	20	2858.91	4	34968.1	34967.9
21	2863.70	8*t*	34909.6	34911.6	21	2865.66	7*ht*	34885.8	34882.7
22	2871.08	0	34819.9	34820.2	22	2873.38	2	34792.0	34790.5
		Q_1					Q_2		
22	2926.87	4*t*	34156.3	34156.7	23	2939.16	2	34013.4	34013.5
23	2937.20	1	34036.1	34037.4	24	2950.01	1	33888.3	33889.1
24	2948.05	0*dt*	33910.8	33912.1					

TABLE V. *Extensions in the (2,1) band, λ2875.*

K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$	K	λ air	Intensity	$\nu_{vac.}$	Calculated $\nu_{vac.}$
		P_1					P_2		
11	2941.23	2	33989.5		12	2952.32	2	33861.8	
12	2949.11	1	33898.6		13	2960.46	1	33768.7	
13	2957.33	1	33804.5		14	2969.09	0	33670.6	
14	2966.05	0	33705.1		15	2978.18	0	33567.8	
		R_1					R_2		
1	2877.22	2	34745.6	34744.5	1	2887.99	7*t*	34616.1	34617.5
2	2876.34	4*t*	34756.3	34756.0	2	2885.10	4	34650.7	34650.7
3	2875.67	5*ht*	34764.4*	34763.1	3	2883.06	4	34675.3	34674.2
4	2875.51	6	34766.3	34765.8	4	2881.61	6*t*	34692.7	34690.1
5	2875.27	6	34769.2	34768.9	5	2880.81	6*t*	34702.4	34702.1
6	2875.67	5*ht*	34764.4*	34764.5	6	2880.53	5	34705.7	34705.4
7	2876.34	4*t*	34756.3	34756.3	7	2880.81	6*t*	34702.4	34702.5
8	2877.49	2	34742.4	34742.4	8	2881.61	6*t*	34692.7	34693.4
9	2878.88	7*t*	34725.6	34723.6	9	2882.82	6*t*	34678.2	34678.4
10	2881.14	2	34698.4	34698.4	10	2884.83	4*ht*	34654.0	34656.1
11	2883.65	6*t*	34668.2	34667.8	11	2886.98	2	34628.2	34628.4
12	2886.69	2	34631.7	34630.9	12	2889.78	0	34594.5	34594.1
		Q_1					Q_2		
11	2913.77	2	34309.8	34309.5	11	2916.92	2*t*	34272.7	34273.1
12	2919.21	4*t*	34245.9	34244.7	12	2922.14	2	34211.5	34211.6
13	2925.06	1	34177.4	34176.5	13	2927.89	1	34144.4	34144.2
14	2931.39	1	34103.5	34103.3	14	2934.24	0	34070.4	34071.2
15	2938.09	5*t*	34025.8		15	2940.90	0	33993.3	33993.5
					16	2948.05	0*dt*	33910.8	

TABLE VI. *Unassigned lines in the region between λ2780 and λ3429.*

λ air	Intensity	$\nu_{vac.}$	λ air	Intensity	$\nu_{vac.}$
2820.04	1	35449.8	3131.80	4	31921.2
2906.01	1	34401.4	3153.89	2	31696.7
2917.21	2	34269.4	3172.02	2	31516.5
2926.40	1	34161.8	3172.23	2	31514.4
2934.10	0	34072.0	3176.70	2	31470.1
2937.73	0	34029.9	3204.62	0	31196.0
2942.68	0	33972.7	3229.63	0	30954.4
2967.30	1	33690.9	3232.61	0	30925.9
3077.64	2	32483.0	3240.34	2	30852.1
3105.12	0	32195.5	3242.09	0	30835.5
3111.14	0*d*	32133.3	3265.94	2	30611.2
3116.30	2	32080.1	3274.98	0	30525.8
3125.64	4	31984.2	3341.39	0	29919.1

the wave-lengths of individual lines were not ordinarily reproducible to much better than 0.05A (=0.4 cm^{-1}). The symbols in the "intensity" columns and the asterisks have the significance assigned them in the paper by J, D and W, and are on the same scale of intensities (i.e., lines of apparently equal intensity in λ3064 and λ3484, for instance, bear the same intensity rating).

THE (2,2) BAND, λ3185

A Fortrat diagram of the region of the spectrum encompassed by the intense (0,0) band (λ3064) is shown in Fig. 1, where the principal *P*, *Q* and *R*-branches are represented by full lines and the ${}^{PP}P_{12}$ and ${}^{RR}R_{21}$-branches by broken lines. The circles represent new lines, or in some instances reassignments, which are recorded in Tables I, II and III. Portions of the branches from which circles are omitted were experimentally observed but are recorded, and correctly assigned, in earlier literature.[3, 15]

The most interesting feature of this diagram is the new (2,2) band,[9] with its principal head at λ3185, for which we observed the six principal branches. Our assignments are confirmed by the application of the combination principle as

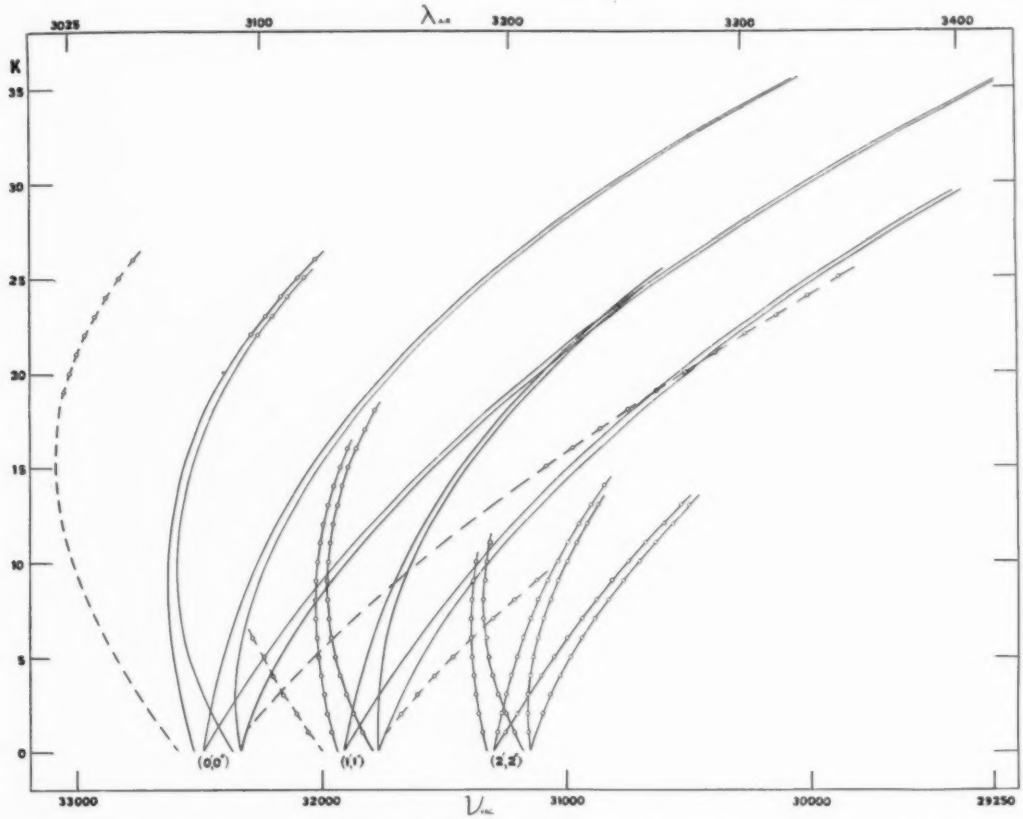


FIG. 1. Fortrat diagrams for λλλ3064, 3122 and 3185. Open circles are lines assigned in this research; full lines, *P*, *Q* and *R*-branches; broken lines, ${}^{RR}R_{21}$ and ${}^{PP}P_{12}$-branches.

[9] Johnston and Dawson, Phys. Rev. **43**, 580 (1933).

TABLE VII.[10] *Application of the combination principle to the upper state of the (2,2) band.*

K	$R_{12}(K)-P_{12}(K)=\Delta_2F_{12}'(K)$			$R_1(K)-Q_1(K)\cong\Delta_1F_1'(K+\frac{1}{2})$			$R_2(K)-Q_2(K)\cong\Delta_1F_2'(K+\frac{1}{2})$		
	(2,0)	(2,1)	(2,2)	(2,0)	(2,1)	(2,2)	(2,0)	(2,1)	(2,2)
1	90.7	90.6	91.2	61.3	61.3	61.2	60.8	58.7	58.3
2	152.3	152.5	152.2	92.2	91.8	93.3	90.9	91.0	87.4
3	212.6	213.8	210.3	121.5	123.5	123.0	121.3	121.4	119.8
4	271.3	272.8	273.9	150.2	150.2	152.2	151.6	152.1	151.8
5	333.6	333.9	332.2	183.0	183.5	182.8	181.5	181.9	179.7
6	392.6	392.7	390.9	212.9	212.4	213.6	211.4	211.2	208.4
7	(451.8)	451.8	453.8	242.8	242.9	241.2	240.8	240.8	242.6
8	509.8	509.5	508.2	272.1	272.5	267.4	270.4	270.5	270.6
9	567.4	568.3	565.3	301.6	303.4	300.6	299.2	300.3	296.3
10	623.1	622.6	621.7	330.1	326.5	332.0	327.5	325.7	327.0

TABLE VIII. *Application of the combination principle to the lower state of the (2,2) band.*

	$R_{12}(K-1)-P_{12}(K+1)$				$R_{12}(K-1)-Q_{12}(K)$			
K	Δ_2F_{12}''		B_2''		Δ_1F_{12}''		B_2''	
	(1,2)	(2,2)	(1,2)	(2,2)	(1,2)	(2,2)	(1,2)	(2,2)
2	167.8	170.0	17.21	17.43	68.4	66.5	17.63	17.15
3	235.4	236.3	17.16	17.22	100.0	99.8	17.03	17.00
4	305.0	304.1	17.25	17.20	135.9	135.7	17.30	17.28
5	374.0	372.6	17.30	17.24	170.6	169.9	17.35	17.28
6	440.6	440.7	17.27	17.27	203.0	202.0	17.22	17.13
7	506.3	507.7	17.22	17.27	236.7	234.9	17.24	17.11
8	573.4	574.0	17.25	17.28	268.4	269.6	17.14	17.21
9					301.9	301.3	17.19	17.16
10					335.0	334.6	17.23	17.21
11					366.7	365.1	17.22	17.14

shown in Table VII, for the upper level, and in Table VIII for the lower level. The treatment of the data in making the calculations of ΔF's and of B_2'' in these tables is identical with that employed for the (1′, 2″) band by J, D and W. The weighted average of B_2'', computed from the data of λ3185 is 17.217 which compares very favorably with the value 17.216 computed from the rotational structure of λ3484.

Null lines calculated from Eqs. (12a) and (12b) of J, D and W are

$$\nu_{01}=31{,}273.5\ \text{cm}^{-1};\quad \nu_{02}=31{,}135.1\ \text{cm}^{-1}.$$

The expected positions of the null lines, calculated from the vibrational constants in Table XII of J, D and W are 31,272.8 cm^{-1} and 31,133.6 cm^{-1}, respectively. The agreement is within the limits of error.

[10] For the (2′,0″) band we employed the data of Jack (Proc. Roy. Soc. **A115**, 373 (1927)) and for the (2′,1″) band, the data of Watson[8] for the P and Q-branches and the data of Table IV for the R-branches.

THE (1,1) BAND, λ3122

The stronger lines which appear near the heads of the R_1 and R_2-branches of λ3122 were measured by Grebe and Holtz[7] and assigned by Heurlinger[11] and by Fortrat.[12] However, many of the assignments appear to be incorrect and since we have also considerably extended the data of these branches we publish them completely in Table II, with the older assignments corrected where necessary. The calculated frequencies, which guided us in our assignments, were computed from the relationships

$$R_1(K)=P_1(K)-\Delta_2F_1'(K) \qquad (1a)$$

$$R_2(K)=P_2(K)-\Delta_2F_2'(K). \qquad (1b)$$

The (Δ_2F')'s were calculated from the data of the (1′,0″) band by the use of Eqs. (2a, b) of J, D and W. Inasmuch as our assignments differ considerably from those of Heurlinger and of Fortrat

[11] Heurlinger, *Dissertation*, Lund (1918).

[12] Fortrat, J. Phys. Radium **5**, 20 (1924).

we have further confirmed our own set of assignments by other combination relationships. Comparison of $(\Delta_1 F')$'s with those of the $(1',0'')$ band, by $(R-Q)$ combinations is given in Table IX. The agreement leaves little doubt as to the correctness of our assignments.

TABLE IX. $\Delta_1 F'$ *Combination differences in the (1,1) band.*

	$R_1(K)-Q_1(K)$		$R_2(K)-Q_2(K)$	
K	(1,1)	(1,0)	(1,1)	(1,0)
1	64.5	64.6		64.7
2	97.7	96.6	98.5	96.0
3	128.4	128.7	127.8	128.2
4	161.7	161.1	160.4	160.0
5	192.9	192.8	192.0	191.9
6	224.3	225.3	223.2	223.5
7	257.6	256.4	254.6	254.5
8	287.3	287.5	285.4	285.1
9	318.2	318.5	316.2	315.8
10	348.3	348.8	346.2	346.3
11	378.6	379.2	375.9	375.9
12	409.4	408.2	404.4	405.6
13	436.2	436.6	432.1	433.8
14	456.2	465.5	461.6	461.9
15	492.0	492.8	489.6	489.4
16	521.0	520.0	515.5	515.9
17			541.7	542.0
18			567.2	567.3

Table II also includes 14 assignments in the weak ${}^{RR}R_{21}$ and ${}^{PP}P_{12}$ satellite branches of λ3122. Although this is the first evidence of the existence of these branches in λ3122, they have been observed previously[13] for λλλλ3064, 2811, 2608 and 3428. The calculated values were obtained from the relationships[14]

$$ {}^{RR}R_{21}(K) = Q_1(K) + R_1(K+1) - P_1(K+1) - 0.216(K+2), \quad (2a) $$

$$ {}^{PP}P_{12}(K) = Q_2(K) - R_2(K-1) + P_2(K-1) + 0.216(K-2). \quad (2b) $$

THE (0,0) BAND, λ3064

Shaw[15] recently reported 63 new assignments in the P and Q-branches of λ3064. With the exception of one or two of the weakest lines reported by Shaw we have confirmed these by our own measurements. In addition we have been able to extend the R-branches and the satellite ${}^{RR}R_{21}$ and ${}^{PP}P_{12}$ to the extent of some 28 assignments, almost entirely with new lines. These are listed in Table III. The calculated lines in the satellite series were computed by Eqs. (2a, b).

THE (1',0'') AND (2',1'') BANDS, λλ2811 AND 2875

Several new lines, which proved to be extensions of the regular P, Q and R-branches of λ2811 and of λ2875, were observed and are tabulated in Tables IV and V. In particular, the R-branches of λ2875 were extended in both directions from the heads and, in some instances, we found it necessary to change assignments of the few lines of these branches which are now in the literature. Accordingly we publish the data for these branches, complete. The calculated positions of regular R-branch lines were computed by Eqs. (1a,b), with $(\Delta_2 F')$'s obtained from the data of the $(2',0'')$ band[10] while the calculated Q-branch lines were obtained from the relationships

$$ Q_i(K) = P_i(K) + [Q_i(K) - P_i(K)]_{v'}; \quad i = 1, 2. \quad (3) $$

The bracketed expression represents the $(Q-P)$ combination for another band with the *same* upper vibrational quantum number. For the $(1',0'')$ band we employed the data of the (1,1) band to evaluate this term and for the $(2',1'')$ band we employed the data of the $(2',0'')$ band. Eq. (3) involves the assumption that the Λ-type doublet intervals are independent of v''. By comparison among the stronger bands we find that this assumption is in error to the extent of only 0.1 or 0.2 cm^{-1} for the range of rotational quantum numbers encompassed in Tables IV and V.

We observed no new lines in the ${}^{RR}R_{21}$ and ${}^{PP}P_{12}$ satellite branches of λ2811 and were unable to detect these branches in λ2875 although we made a special effort to bring them out.

UNASSIGNED LINES

Lines which we identified but were unable to assign are recorded in Table VI. Including the 4 unassigned lines in the neighborhood of the $(0',1'')$ sequence which were reported by J, D and W, there now exist a total of 30 unassigned lines between the head of the $(1',0'')$ band at λ2811 and the tail of the $(1',2'')$ band out to λ3570. The total number of lines which we ob-

[13] Watson, reference 8; Jack, reference 10; Almy, Phys. Rev. **35**, 1495 (1930), and Almy and Rahrer, Phys. Rev. **38**, 1816 (1931).

[14] Cf. Mulliken, Phys. Rev. **32**, 388 (1928).

[15] R. M. Shaw, Astrophys. J. **76**, 202 (1932).

served within this region of the spectrum was 780. Considering the complexity and overlapping of the various branches and various bands we regard this as very satisfactory. Substantially all of the unassigned lines are very weak and some of them may be optical effects. It is probable that only a few, if any, of them belong to the "water vapor" bands.

Band Intensities

The relative intensities of the bands of the OH system are shown in Table X. The intensity ratings are based on an empirical scale as the bands appear to the eye. The numbers in parentheses are the numbers of lines observed in the Q-branches of the respective bands, and parallel the visual ratings. It is apparent that the locus of maximum intensities runs close to the diagonal of the table. Probably a more significant comparison would be in the lengths of exposure necessary to bring out a definite number of the stronger lines. Put on this basis and compared as to the time necessary to bring out about the fifty strongest lines of the respective bands, the (0,0), (1′,0″) and (0′,1″) bands rate in the proportions 50 : 4 : 1.

Table X. *Relative intensities of the bands of OH.*

	$v''=0$	1	2
$v'=0$	10 (35)	6 (28)	
1	6 (25)	6 (25)	2 (19)
2	2 (17)	2 (15)	2 (14)

An effort was made to bring out the (0′,2″), (3′,2″) and (3′,3″) bands but, so far, we have been unsuccessful in accomplishing this.

Λ-Doubling in the OH Bands

The question of Λ-doubling has been discussed, from a theoretical standpoint, by Van Vleck[16] who concludes that for a $^2\Pi$ term the magnitudes of the doublets should be given by the expressions

$$\delta_1=\delta_2=(C_1-C_2)K(K+1) \qquad (4a)$$

for a case (b) molecule, and

$$\delta_1=a(j+\tfrac{1}{2}); \quad \delta_2=0 \qquad (4b)$$

for a case (a) molecule, where C_1, C_2 and a are constants which can be calculated from other molecular constants.[17] Mulliken,[14, 18] has given empirical equations for the Λ-doublets in OH, based on the data then available on the (0,0) and (1′,0″) bands. These are

$$\delta_1=0.04j^2=0.04(K+\tfrac{1}{2})^2, \qquad (5a)$$

$$\delta_2=0.03j^2=0.03(K-\tfrac{1}{2})^2. \qquad (5b)$$

In form, Mulliken's equations resemble that of Van Vleck for a case (b) molecule although the constants differ from the theoretical value by a factor of ten.

The new data make it possible to test these relationships for much higher values of the rotational quantum number. It is necessary to compute the values of the doublets indirectly since a direct evaluation of the individual doublets requires the use of very weak satellite branches[19] for which the data are unreliable. Accordingly we have employed the following relationship, which is that used by Mulliken,

$$\begin{aligned}&[R_i(K)-Q_i(K+1)]-[Q_i(K)-P_i(K+1)]\\&=[F_{iA}''(K)+F_{iA}''(K+1)]\\&\quad-[F_{iB}''(K)+F_{iB}''(K+1)]\\&=\delta_i(K)+\delta_i(K+1)\cong 2\delta_i(K+\tfrac{1}{2}); \quad i=1,\,2. \qquad (6)\end{aligned}$$

Values of δ_1 and δ_2 calculated by this method are given in Table XI for the $v''=0,1$ and 2 levels, respectively, with all the data now available.[20] The values given by Mulliken's formulae (5a,b) are also included in Table XI. It is apparent that,

[16] Van Vleck, Phys. Rev. **33**, 467 (1929).

[17] $(C_1-C_2)=4B^2/\nu$ where ν is the frequency of the electronic transition. For OH, which behaves as a case (b) molecule except at low quantum numbers, this gives $(C_1-C_2)=3.4\times10^{-3}$.

[18] Mulliken, Phys. Rev. **33**, 507 (1929).

[19] $\delta_1=Q_{1A1B}-Q_1$; $\delta_2=Q_{2A2B}-Q_2$. The nomenclature is that of Mulliken.[14]

[20] The following data were used:

$v''=0$: (0,0) band—data of Fortrat and this research.
(1′,0″) band—data of Watson, and this research.
(2′,0″) band—data of Jack.

$v''=1$: (0′,1″) band—data of J, D and W.
(1,1) band—data of Fortrat and this research.
(2′,1″) band—data of Watson and this research.

$v''=2$: (1′,2″) band—data of J, D and W.
(2,2) band—data of this research.

TABLE XI.[21] A-*Doublets in the OH bands.*

K	$\delta_1(K+\frac{1}{2})$											Eq. (5a)
	$v''=0$				$v''=1$				$v''=2$			
	(0,0)	(1,0)	(2,0)	Aver.	(0,1)	(1,1)	(2,1)	Aver.	(1,2)	(2,2)	Aver.	
1	0.07	0.18	0.43	0.33	0.2	0.3	0.3	0.3	0.1	(−0.6)	0.1	0.16
2	0.37	0.25	0.58	0.40	−0.1	0.5	0.7	0.4	(1.5)	(3.0)		0.36
3	0.54	0.58	0.64	0.59	0.4	0.3	1.0	0.6	1.0	0.8	0.9	0.64
4	1.00	0.98	(−0.21)	0.99	0.6	1.2	(−0.1)	0.9	(3.0)	0.8	0.8	1.00
5	1.47	1.22	1.51	1.40	1.5	1.3	1.7	1.5	1.6	1.3	1.5	1.44
6	2.10	2.13	1.95	2.07	1.8	1.8	1.8	1.8	2.2	0.2	1.2	1.96
7	2.66	2.72	2.45	2.61	2.3	3.3	2.8	2.5	2.1	(0.7)	2.1	2.56
8	3.43	3.47	3.12	3.34	3.6	3.2	3.3	3.4	2.9	(0.7)	2.9	3.24
9	4.34	4.22	4.30	4.29	3.5	5.8	3.4	4.2	4.2	6.2	5.2	4.00
10	5.12	5.08	5.18	5.13	4.6	4.8	(3.1)	4.7		6.2	6.2	4.84
11	6.21	6.08	6.26	6.18	7.4	5.6	5.6	6.3				5.76
12	6.93	7.13		7.03	6.5	7.3	6.5	6.8				6.76
13	7.89	7.58		7.74	6.4	7.4		6.9				7.84
14	8.94	8.94		8.94	7.7	8.7		8.2				9.00
15	10.04	10.01		10.03	9.0	9.5		9.3				10.24
16	11.07	11.42		11.25	9.5	11.4		10.5				11.56
17	12.06	12.48		12.27								12.96
18	13.02	13.70		13.36								14.44
19	14.52	15.46		14.99								16.00
20	15.93	16.66		16.30								17.64
21	17.0	14.07		15.54								19.36
22	17.8	17.55		17.68								21.16
23	19.5			19.5								23.04
24	20.8			20.8								25.00
25	21.6			21.6								27.04
26	25.1			25.1								29.16

K	$\delta_2(K+\frac{1}{2})$											Eq. (5b)
	$v''=0$				$v''=1$				$v''=2$			
	(0,0)	(1,0)	(2,0)	Aver.	(0,1)	(1,1)	(2,1)	Aver.	(1,2)	(2,2)	Aver.	
1	−0.21	−0.01	−0.06	−0.11	−0.6	+0.7	−1.3	−0.4	+1.2	−1.6	−0.2	
2	−0.32	−0.19	−0.24	−0.25	−0.3	−0.4	−0.7	−0.5	−0.3	−1.5	−0.9	
3	−0.20	−0.18	+0.14	−0.08	+0.1	+0.1	−0.2	0.0	−0.2	−1.2	−0.7	
4	−0.02	−0.11	+0.20	+0.03	+0.1	+0.2	0.0	+0.1	−0.4	+0.4	0.0	0.48
5	+0.20	+0.22	+0.24	+0.22	+0.9	+0.5	+0.1	+0.5	+0.3	+0.1	+0.2	0.75
6	0.39	0.58	0.47	0.45	−0.2	+1.4	+0.2	+0.5	(0.0)	(−1.1)		1.08
7	0.94	0.63		0.79	+0.7	+1.3	+1.1	+1.0	+1.2	+2.0	+1.6	1.47
8	1.39	1.17	1.60	1.39	1.6	1.9	1.8	+1.8	(−0.4)	+1.5	+1.5	1.92
9	1.42	2.28	1.84	1.85	1.7	2.4	2.5	2.2		(+0.2)		2.43
10	1.92	2.58	2.28	2.26	1.7	3.1	1.6	2.1		+1.6	+1.6	3.00
11	2.46	3.53	2.64	2.88	2.9	3.5	2.9	3.1		+1.4	+1.4	3.63
12	3.26	4.30		3.78	4.1	3.9	3.7	3.9				4.32
13	3.92	4.64		4.28	5.7	5.3		5.5				5.07
14	4.68	5.35		5.02	6.8	7.0		6.9				5.88
15	5.65	6.63		6.14	6.5	7.1		6.8				6.75
16	6.42	7.40		6.91	9.3	8.3		8.8				7.68
17	7.21	8.54		7.88		8.9		8.9				8.67
18	8.13	9.62		8.88								9.72
19	9.26	10.49		9.88								10.83
20	11.49	11.28		11.39								12.00
21	12.92	13.63		13.28								13.23
22	14.3	13.62		13.96								14.52
23	14.6			14.61								15.87
24	15.7			15.7								17.28
25	17.3			17.3								18.75

[21] Figures in parentheses have been omitted in taking averages.

although Mulliken's formulae hold nearly to $(K=20)$, which was about the limit of the data available at the time of Mulliken's work, it fails rapidly at the higher quantum numbers.

In order to make use of the data to still higher quantum numbers $(K=35)$ we may employ a second indirect method, which solves for the *differences* between δ_1 and δ_2 without the need of R-branch lines. Thus, if we express the doublet separations in the excited $^2\Sigma$ state by the equation of Mulliken with the constant of Almy[13]

$$F_1'(K)-F_2'(K)=0.216K \qquad (7)$$

we may write

$$\{[P_1(K)-P_2(K)]-0.216(K-1)\} - \{[Q_1(K)-Q_2(K)]-0.216(K)\} = \delta_1(K)-\delta_2(K). \qquad (8)$$

The results of this calculation are shown graphically in Fig. 2, for the $v''=0$, 1 and 2 levels. In obtaining these values we have averaged the calculations for the several bands as given in reference 20. The broken line was constructed by the use of Mulliken's formulae (5a,b). From Fig. 2 it appears that the Λ-doublets, or at least their differences vary directly with K and not with $(K+1/2)^2$. This is inconsistent with the results to be expected from Van Vleck's theoretical treatment.

The Spin Doublets in the OH Bands

The electronic spin doublets in the OH bands have been considered by Kemble[22] and by Hill and Van Vleck.[23] The latter authors, whose

[22] Kemble, Phys. Rev. **30**, 387 (1927).
[23] Hill and Van Vleck, Phys. Rev. **32**, 250 (1928).

TABLE XII. *Electronic doublets in the OH bands.*

K	$v''=0$												
	R Doublets			P Doublets			Q Doublets			Average		Δ	
	(0,0)	(1,0)	(2,0)	(0,0)	(1,0)	(2,0)	(0,0)	(1,0)	(2,0)	R and P	Q	Obs.	Calc.
1	126.18	126.38	126.14	126.66	126.73	124.94	126.22	126.62	125.93	126.17	126.26	126.22	
2	104.08	103.82	103.91	103.97	104.12	103.67	103.38	103.51	102.00	103.93	102.96	103.45	
3	87.30	87.01	86.18	87.09	87.02	86.82	86.69	86.47	86.14	86.90	86.43	86.72	
4	74.42	74.36	71.90	74.46	74.36	75.95	73.62	73.39	73.45	74.94	73.47	74.21	
5	65.80	64.03	63.77	64.70	64.54	64.24	63.58	63.34	62.50	64.51	63.14	63.83	63.60
6	57.22	57.23	56.95	57.07	57.06	56.90	55.74	55.74	55.64	57.07	55.71	56.39	
7	51.17	51.44	51.27	51.50	51.09	51.16	49.56	49.47	49.51	51.27	49.52	50.40	
8	46.48	46.55	46.87	46.37	41.42	45.88	44.54	44.21	44.40	45.26	44.38	44.82	
9	42.65	42.76	42.80	42.65	42.52	42.19	40.50	40.26	40.59	42.59	40.45	41.52	
10	39.36	39.34	39.29	39.68	39.40	39.64	37.00	37.02	36.94	39.45	36.99	38.22	36.75
11	37.09	37.19	36.80	36.88	36.77	36.69	34.14	34.10	33.43	36.90	33.89	35.40	
12	34.61	34.02	34.00	34.61	33.67	34.65	31.66	31.59	30.79	34.26	31.35	32.81	
13	32.76	32.11		32.77	32.66	32.34	29.61	29.44	28.58	32.55	29.21	30.88	
14	30.97	30.91		30.99	30.84	30.92	26.73	27.62	28.36	30.93	27.57	29.25	
15	29.64	29.20		29.58	29.93	29.27	26.25	26.03	25.88	29.52	26.05	27.79	25.50
16	28.51	28.43		28.50	28.14	27.73	24.66	24.54	24.37	28.26	24.52	26.39	
17	27.28	26.93		27.28	27.23		23.42	23.08	21.09	27.18	23.53	25.36	
18	25.64	25.91		26.28	26.17		22.28	22.15		26.00	22.22	24.11	
19	25.45	27.46		25.53	25.53		21.38	21.13		25.74	21.26	23.50	
20	24.60	24.35		24.76	24.61		20.46	19.98		24.58	20.17	22.38	19.46
21	23.99	22.11		24.04	23.68		19.65	22.08		23.46	20.87	22.17	
22		22.93		23.50	23.10		18.68	18.83		23.18	18.76	20.97	
23				22.89	23.07		18.03	17.76		22.98	17.89	20.44	
24				22.46	22.55		17.26	17.29		22.51	17.28	19.90	
25				22.38			16.71			22.38	16.71	19.55	15.71
26				21.39			16.04			21.39	16.04	18.72	
27				21.23			15.44			21.23	15.44	18.34	
28				20.96			14.25			20.96	14.25	17.61	
29				20.47			14.47			20.47	14.47	17.47	
30				19.92			14.39			19.92	14.39	17.16	13.17
31				19.65			16.12			19.65	16.12	17.89	
32				19.33			13.25			19.33	13.25	16.29	
33				19.15			12.76			19.15	12.76	15.96	
34				18.85			12.43			18.85	12.43	15.64	
35				19.45			12.88			19.45	12.88	16.17	11.33

TABLE XII. (*Continued.*)

K	$v''=1$												
	R Doublets			P Doublets			Q Doublets			Average		Δ	
	(0,1)	(1,1)	(2,1)	(0,1)	(1,1)	(2,1)	(0,1)	(1,1)	(2,1)	R and P	Q	Obs.	Calc.
1	126.8	124.0	127.2			127.0	126.7		126.6	126.3	126.7	126.5	
2	104.2	103.9	105.7	105.4	105.4	105.0	104.1	105.0	104.4	104.9	104.5	104.7	
3	88.7	87.8	88.2	87.8	88.0	88.5	87.6	87.5	86.4	86.8	87.2	87.0	
4	75.4	76.0	72.6	74.8	75.7	75.2	74.8	74.9	74.6	75.0	74.8	74.9	
5	65.9	65.5	65.5	65.9	65.9	66.0	65.6	64.8	64.1	65.8	64.8	65.3	65.14
6	59.2	57.9	57.2	57.9	58.6	58.0	57.1	57.0	56.2	58.1	56.8	57.5	
7	51.8	53.6	52.2	52.5	53.6	52.5	50.6	50.8	50.3	52.7	50.6	51.7	
8	49.5	47.5	47.8	48.0	47.9	47.5	46.0	45.8	46.0	48.0	45.9	47.0	
9	44.0	35.6	45.2	43.9	43.5	43.4	43.4	41.4	42.3	44.3	42.4	43.4	
10	39.9	39.8	42.0	40.5	40.3	40.3	37.6	37.7	41.4	40.5	38.9	39.7	37.85
11	37.6	37.2	37.4	37.9	37.6	37.2	35.3	34.8	34.7	37.5	34.9	36.2	
12	35.3	37.3	34.4	37.5	35.2	34.4	32.9	32.5	31.8	35.7	32.4	34.1	
13	31.0	34.3		33.2	33.1	33.2	30.7	30.4	30.2	33.0	30.4	31.7	
14	29.8	30.0		30.0	31.4	31.6	28.7	28.1	30.1	30.9	29.0	30.0	
15	29.5	29.0		29.0	30.0		28.1	26.6	29.3	29.3	28.0	28.7	26.31
16	26.3	30.8		30.8	28.9		27.2	25.2		29.2	26.2	27.7	
17	(28.0)	25.9		25.9	27.5		24.6	24.1		26.7	24.4	25.6	
18	26.0				26.8		23.1	23.2		26.4	23.2	24.8	
19	25.5				26.7		21.7	21.6		26.1	21.7	23.9	
20	24.7				25.4		21.1	20.8		25.1	21.0	23.1	20.08
21	25.2				24.3		19.8	18.5		24.7	19.2	22.0	
22	23.4				23.8		19.0	16.8		23.6	17.9	20.8	
23	19.3				22.8		18.2	18.7		21.1	18.5	19.8	
24	22.4				22.7		17.1	18.8		22.6	18.0	20.3	
25	21.8				21.1		16.9			21.4	16.9	19.2	16.23
26	21.4				21.1		16.9			21.2	16.9	19.1	
27	23.1				21.1		13.2			22.1	13.2	17.7	
28					20.9					20.9			
29					20.3					20.3			

K	$v''=2$									
	(1,2)	(2,2)	(1,2)	(2,2)	(1,2)	(2,2)	R and P	Q	Obs.	Calc.
1	126.1	128.0		125.5	126.5	125.3	126.5	125.9	126.2	
2	109.9	101.6	106.4	105.3	107.2	106.0	105.8	106.6	106.2	
3	90.0	89.7	90.7	91.2	88.8	86.7	90.4	87.8	89.1	
4	78.6	76.0	76.1	76.7	74.9	75.8	76.9	75.4	76.2	
5	66.9	68.4	68.3	66.1	65.3	65.5	67.8	65.4	66.6	66.58
6	60.5	62.2	58.9	57.2	57.9	57.7	59.7	57.8	58.8	
7	52.8	51.4	53.7	50.6	52.2	53.0	52.1	52.6	52.4	
8	47.2	46.0	47.5	48.4	46.4	48.2	47.3	47.3	47.3	
9	44.9	47.3	48.2	45.2	42.3	43.3	46.4	42.8	44.6	
10	44.2	41.4		43.3	39.2	35.3	42.2	37.4	39.8	38.91
11	40.7			38.2	36.2	35.2	39.5	35.7	37.6	
12	35.4			35.6	34.0	33.2	35.5	33.6	34.6	
13	35.2			31.2	30.7		33.2	30.7	32.0	
14	32.7				30.7		32.7	30.7	31.7	
15	30.9				28.7		30.9	28.7	29.8	27.09
16	29.9				27.1		29.9	27.1	28.5	
17					25.6					
18					23.8					
19					23.0					

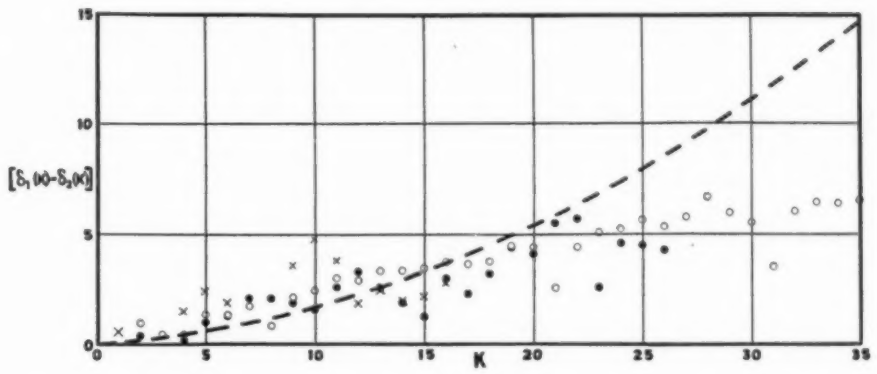


FIG. 2. Λ-doublet differences in F_1 and F_2 states of OH. Open circles, $v''=0$ level; closed circles, $v''=1$ level; crosses, $v''=2$ level. The broken line is Mulliken's formulae (see Eqs. (5a, b)).

treatment is somewhat more exact than that of Kemble, give an equation for the doublets which may be put in the form

$$\Delta = F''_{2AB}(K) - F''_{1AB}(K) = B[\{4(K+\tfrac{1}{2})^2 + (A/B)(A/B-4)\Lambda^2\}^{\frac{1}{2}} - (2K+1)], \quad (9)$$

where the (F''_{AB})'s represent the midpoints of the Λ-doublets and the other symbols have the significance given by J, D and W. The doublets may also be evaluated from the experimental data by the relationships

$$[F''2_A(K) - F''_{1A}(K)] = [Q_1(K) - Q_2(K)] - 0.216K, \quad (10a)$$

$$[F''_{2B}(K) - F''_{1B}(K)] = [P_1(K) - P_2(K)] - 0.216(K-1), \quad (10b)$$

$$= [R_1(K) - R_2(K)] - 0.216(K+1), \quad (10c)$$

$$F''_{2AB}(K) - F''_{1AB}(K) = \tfrac{1}{2}[F''_{2A}(K) - F''_{1A}(K)] + \tfrac{1}{2}[F''_{2B}(K) - F''_{1B}(K)]. \quad (10d)$$

These relationships have been calculated for all of the bands and are recorded in Table XII which also shows values of Δ calculated by Eq. (9) for every fifth value of K, by using the values of the constants given in J, D and W. It is evident that Eq. (9) fails to reproduce the data within the limits of experimental accuracy beyond about $K=10$.

The Magnetic Susceptibilities of Some Common Gases

GLENN G. HAVENS, *University of Wisconsin*

(Received April 20, 1933)

The Faraday test body method, which had been developed and used by the author for finding the magnetic susceptibility of NO_2, has been applied to the measurement of the magnetic susceptibilities of several common gases at room and at liquid air temperatures. The following values $\times 10^6$ were obtained for the molecular suceptibilities: $\chi(H_2) = -4.0051$, $\chi(He) = -1.906$, $\chi(Ne) = -7.651$, $\chi(A) = -19.23$, $\chi(CO_2) = -20.88$, $\chi(N_2) = -11.938$ at room temperatures, and $\chi(H_2) = -3.98$, $\chi(He) = -1.91$, $\chi(Ne) = -7.64$, $\chi(A) = -19.8$, $\chi(O_2) = 11830$ at liquid air temperatures. These values are based on the provisional value of $\chi(O_2) = 3335 \times 10^{-6}$ at 20°C. The results show that the diamagnetic susceptibilities do not vary, within the experimental error, in going from room to liquid air temperatures. This is contrary to the 40 percent change observed for hydrogen by Bitter. The susceptibility of oxygen was found to follow Curie's law to within 2 percent.

INTRODUCTION

THIS series of experiments was undertaken with the hope of furnishing more accurate values for the magnetic susceptibilities of some of the more common gases which could be used for comparison with existing and future theoretical calculations. At present the agreement between the theoretical and experimental results is none too satisfactory, mainly because of the large variations among the results of different experimental observers. For example, the results of several observers for the molecular susceptibility of hydrogen range from -3.3×10^{-6} to -5.8×10^{-6}; similar differences exist for other gases. In many cases it is hard to select the experimental result which is the most reliable. It seemed necessary, in order to obtain results which could be fully relied upon, to measure susceptibilities far more accurately than had hithertofore been done. The apparatus used by the writer[1] had the merits of high sensitivity, freedom from troublesome electrostatic effects, of being very little affected by external changes of conditions, and it could be used for work at liquid air temperatures.

Several theoretical values are now available for comparison with experiment. Wang's[2] theoretical calculation, as later corrected by Van Vleck,[3] indicates that $\chi(H_2) = -4.20 \times 10^{-6}$ should be correct to within 5 percent for the molecular susceptibility of hydrogen. Mrowka's[4] recent calculation indicates nearly the same theoretical value. Slater[5] and Stoner[6] have calculated the susceptibility of helium, obtaining $\chi(He) = -1.85$ and -1.90×10^{-6}, respectively. A recent, but probably less accurate calculation has been made by Vinti,[7] who obtains $\chi(He) = -2.09 \times 10^{-6}$. Vinti[7] also obtains a value of $\chi(Ne) = -7.5 \times 10^{-6}$ for neon.

APPARATUS AND PROCEDURE

If a solid is suspended in a nonhomogeneous magnetic field, a torque is exerted by the field on each element of volume of the solid, which is proportional to the magnetic field strength, its directional derivative, and the difference between the volume susceptibility of the solid and that of the surrounding medium. Based on these facts, a method, known as the Faraday test body method, has been used and developed by Glaser,[8] Hammar,[9] Vaidyanathan,[10] Bitter,[11] and others whereby the torque on the solid due to changing the surrounding medium is measured. In a previous article[1] the writer describes several improvements he had made in the measuring ap-

[1] Glenn G. Havens, Phys. Rev. **41**, 337 (1932).
[2] Wang, Nat. Acad. Sci. Proc. **13**, 798 (1927).
[3] Van Vleck, Nat. Acad. Sci. Proc. **15**, 539 (1929).
[4] Mrowka, Zeits. f. Physik **80**, 495 (1933).
[5] Slater, Phys. Rev. **32**, 349 (1928).
[6] Stoner, Proc. Leeds Phil. Soc. **1**, 484 (1929).
[7] Vinti, Phys. Rev. **41**, 813 (1932).
[8] Glaser, Ann. d. Physik **75**, 459 (1924).
[9] Hammar, Proc. Nat. Acad. Sci. **12**, 594 (1926).
[10] Vaidyanathan, Phil. Mag. **5**, 380 (1928); Ind. J. Phys. **1**, 183 (1926–27).
[11] Bitter, Phys. Rev. **35**, 1572 (1930).

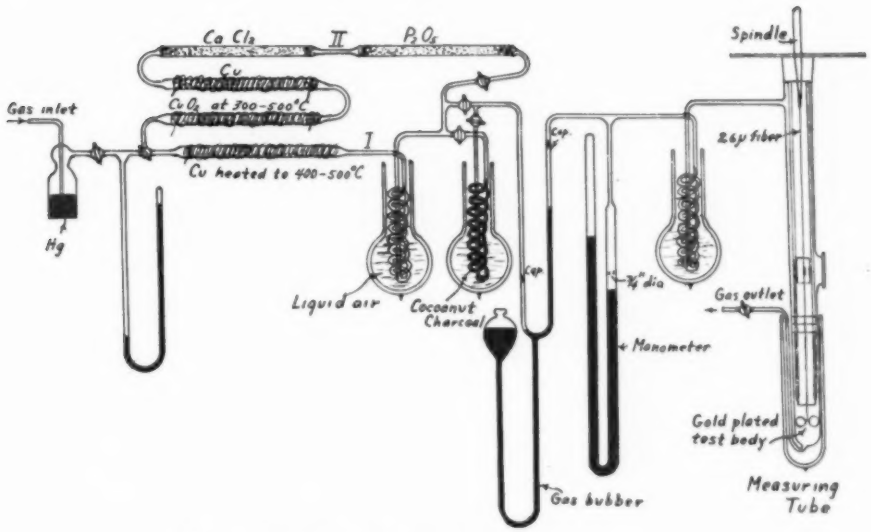


FIG. 1. Purifying and measuring apparatus.

paratus employed in the above method. The measuring apparatus has since been modified for liquid air measurements by surrounding the lower part of the tube with a Dewar flask. The flask also served to reduce the temperature variations around the test body at room temperatures.

The gas purifying apparatus is shown in Fig. 1. Hydrogen, helium, neon, and argon were all purified by being passed through arrangement I, the coil containing cocoanut charcoal, the mercury bubbler, and finally through a long spiral coil surrounded by liquid air. The bubbler was constructed to facilitate the accurate control of the rate of flow of gas into the measuring chamber and for maintaining a pressure slightly in excess of one atmosphere in the purifying tubes. This eliminated any chance of contamination of the gas by the development of small leaks in the stopcocks or glassware. The gas which entered the measuring chamber was freed from mercury and other vapors by the final coil in liquid air. Nitrogen was purified by the same procedure, except that it was not sent through the cocoanut charcoal. Carbon dioxide, which could be passed through neither the cocoanut charcoal nor the liquid air because of its high freezing point, was passed over P_2O_5 for the removal of water vapor. Oxygen was passed through the arrangement II and the coils in liquid air but not through the cocoanut charcoal.

The temperature of the measuring tube was maintained constant for measurements at room temperatures by controlling the temperature of the magnet and surrounding room. A mercury thermostat connected to the grid of a Thyratron tube controlled the heat input into the room, while a fan circulated the air. The temperature of the distilled water, which was used for cooling the magnet, was controlled by a similar Thyratron arrangement. Temperatures could be maintained constant, by these arrangements, to within a tenth of a degree.

Preliminary to starting a series of measurements, the entire system was evacuated to a pressure less than 0.1 mm of Hg and the charcoal baked out at temperatures between 400 and 500°C for a period of about twenty minutes, and while still hot, the gas to be measured was flushed through it. The coil containing the charcoal was then surrounded by liquid air, the measuring instrument was reevacuated and flushed out with the gas going through the cold charcoal. Finally, the purifying system was filled, as far as the bubbler, to a pressure slightly in excess of one atmosphere. The magnetic field was then excited and the temperature controls were allowed to operate about two hours before any measurements were began so as to allow the system to come to a steady condition. Most of the susceptibility determinations were taken with a magnetic field strength of about 7500 gauss, although some were made with weaker fields.

After steady conditions were obtained, the torsion fiber was twisted by turning the torsion head until the test body was in the position of required sensitivity. The position of the test body was observed by means of an optical lever having a ten meter light path. For the most accurate measurements, sensitivities were used such that one mm change in pressure of hydrogen would produce a deflection of from 2 to 5 cm of the light spot on the scale of the optical lever. After gas was let into the tube the fiber was twisted until the light beam of the optical lever returned to its original position.

The torque on a test body placed in a magnetic field and surrounded by a vacuum is given by[12]

$$T=(k_s/2)(1+4\pi k_s)\int[\nabla H^2, r]d\tau,$$

where k_s is the volume susceptibility of the solid, H is the magnetic field strength, r is the radius vector about the axis for which the torque is taken, and the integration is to be taken over the entire volume of the test body. If the test body is surrounded by a gas of volume susceptibility k_g, k_s is replaced in this formula by k_s-k_g. The k's used in this experiment were always less than 10^{-5}, we can therefore neglect the squared terms in k and write:

$$T=(k_s-k_g)A,$$

where A is a constant for a given test body in a given position in the magnetic field. If the pressure of the gas is changed we have:

$$\Delta T=-\Delta k_g A.$$

If k_x is the volume susceptibility of an unknown gas and k_s refers to the known gas:

$$k_x=(\Delta T_x/\Delta T_s)k_s=[(dT_x/dp)/(dT_s/dp)]\cdot k_s.$$

Thus the volume susceptibilities are proportional to the change in torques which are in turn proportional to the angles through which the suspending fiber is twisted.

Errors due to inaccurate centering

The torque on a test body due to a nonhomogeneous magnetic field varies linearly with the pressure of the surrounding gas, if the test body remains fixed relative to the field. However, unless the suspending fiber is perfectly centered at its upper end, this condition is never satisfied experimentally. If, as in the present experiment, a null method is used, as the torsion head is turned the test body will be translated around the circumference of a circle, the radius of which will depend on the accuracy of the centering; a torque will be introduced by this translation which varies, to a good approximation for small circles, as a sine function of the position of the torsion head. As an example of the magnitude of this torque, for the case of a 2.6μ fiber centered to within a radius of 10μ the test body has been observed to rotate in a direction opposite to that of the twisted fiber showing that the torque introduced by translation was larger and opposite to that produced by the twisting of the fiber.

If the fiber used is so large that the total change in the torque obtained for the gas under investigation, corresponds to only a small fraction of a revolution of the fiber (for example, A to B or A' to B' in Fig. 2, b), the sine characteristic will most likely not be recognized; however, the slope of the curve, which is proportional to the susceptibility of the gas, will depend on the portion of the sine curve for which the data are taken. For example, if one-fiftieth of a turn of the torsion head is used for one atmosphere of hydrogen it would still require sixteen turns for an atmosphere

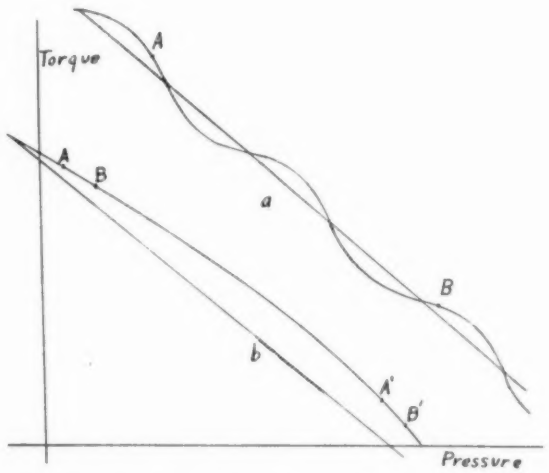


FIG. 2. The effect of inaccurate centering of small and large fibers is shown by a and b.

[12] M. Mason and W. Weaver, *The Electromagnetic Field*, p. 218.

of oxygen, and it is possible for the centering to produce a variation of a factor of two in the slope for hydrogen depending on the portion of the sine curve used, and yet not appreciably affect the slope of the oxygen curve. In case a sufficiently small fiber is used, so that the torque changes by several revolutions of the fiber (A to B in Fig. 2, a), this sine variation can be recognized and corrected for.

In the present experiment the torque due to inaccuracy of centering has been reduced to less than 1 percent of the torque caused by one atmosphere of hydrogen by finding a favorable position for the test body in the magnetic field and by centering the fiber to within a radius of 5μ.

Low pressure effects

If one observes the torque required to keep the test body in a given position, with zero magnetic field, as a function of the pressure, it is found to be constant down to a pressure of about 1 to 5 cm, below which it deviates, the sign of the deviation depending on whether the temperature of the magnet is below or above that of the room. Fig. 3. The effect increases at least down

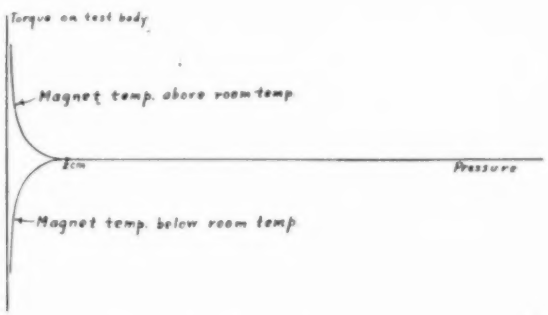


FIG. 3. An effect at low pressures.

to a pressure of 10^{-3} mm of Hg. That the effect does not enter above 5 cm is shown by the fact that the curve in Fig. 3 is horizontal above that pressure. The effect is present both with and without the liquid air flask surrounding the lower part of the tube. The behavior suggests that it is of the nature of a radiometer disturbance.

Liquid air measurements

The liquid air measurements could not be made as accurately as those at room temperatures because of convection currents around the test body. These convection currents increased with pressure and with the time after which the Dewar flask was initially filled with liquid air. This may be accounted for by assuming that the nitrogen near the surface of the liquid air boils away first thereby leaving a warmer layer richer in oxygen, this temperature difference in turn causes convection currents inside the tube. The magnitude of the disturbances depended on the gas that was being measured; hydrogen behaved the best with helium, neon, argon, and nitrogen behaving progressively worse, in fact the susceptibilities of the latter two could not be determined to within 10 percent and 50 percent respectively at liquid air temperatures.

The following reasons argue for the superiority of the present apparatus and the reliability of the results over those of previous observers:

(1) Extremely high sensitivities bring out and indicate means for the elimination of systematic errors such as constant drifts in the apparatus and centering errors.

(2) The fact that the measured susceptibilities check within the limit of error those made at room temperatures, as they should from theoretical considerations, argues for the reliability of the results.

(3) The results are reproducible from day to day.

(4) The torque on the test body in a vacuum due to the magnetic field remains very nearly the same from day to day. Other observers using the test body method have found relatively large variations from day to day.

(5) The torque on the test body in a vacuum due to the magnetic field does not vary appreciably in going from room to liquid air temperatures, showing that the torque on the test body is only slightly effected by changes in temperature. Therefore the small temperature changes at room temperatures affect the results only insofar as they affect the pressure of the gas.

(6) The torque on the test body does not vary rapidly with the change in the magnetic field. This has been brought about by reducing the resultant torque on the test body by means of compensators.

(7) The apparatus is free from electrostatic effects.

(8) The materials used in the construction of the measuring apparatus were Pyrex glass, fused quartz, platinum and gold all of which are practically inert chemically.

Discussion of Results

Hydrogen

Hydrogen has been the subject of controversy for some time. The results of recent observers have tended to increase the uncertainty of the value for the susceptibility of this gas. The author has obtained over one hundred curves for this gas, using four different types of test bodies, the recent ones had a coat of platinum or gold evaporated on them the others did not. Although many of the one hundred curves were not calibrated accurately and were used mainly to test the effect of alterations of the instrument none of them either at room or liquid air temperatures have indicated a molecular susceptibility which varied by more than a few percent from -4.0×10^{-6}. The accuracy and consistency of the results are illustrated by curves 156 and 159 in Fig. 4, and by the Tables I, III and IV. All observations were taken with increasing pressures and then decreasing pressures in the measuring tube in order that any constant drifts in the apparatus would be detected.

Table I. H_2, *curve 139.*

Pressure (cm Hg)	Torque (Rev. of fiber)	Pressure (cm Hg)	Torque (Rev. of fiber)
5.500	15.2566	37.860	12.3348
6.537	15.1708	42.010	11.9606
8.486	14.9967	46.177	11.5859
10.519	14.8114	49.888	11.2551
12.536	14.6345	54.078	10.8662
14.647	14.4388	57.152	10.5893
16.583	14.2689	59.813	10.3520
18.500	14.0919	61.865	10.1640
20.499	13.9089	63.572	10.0125
24.297	13.5616	53.144	10.9588
26.336	13.3775	44.363	11.7476
28.364	13.1987	34.576	12.6347
30.372	13.0152	24.840	13.5186
32.299	12.8405	15.450	14.3696
34.151	12.6703	5.010	15.4848
36.409	12.4733		

The results for hydrogen of Soné,[13] and Wills and Hector[14] agree within their limits of error with those obtained by the author in Table V. The results of the author for hydrogen at liquid air temperatures disagrees with the low result of 3.3×10^{-6} for the molecular susceptibility as found by Bitter[15] who averaged two determina-

[13] Soné, Phil. Mag. **39**, 305 (1920).
[14] Wills and Hector, Phys. Rev. **23**, 209 (1924).
[15] Bitter, Phys. Rev. **36**, 1648 (1930).

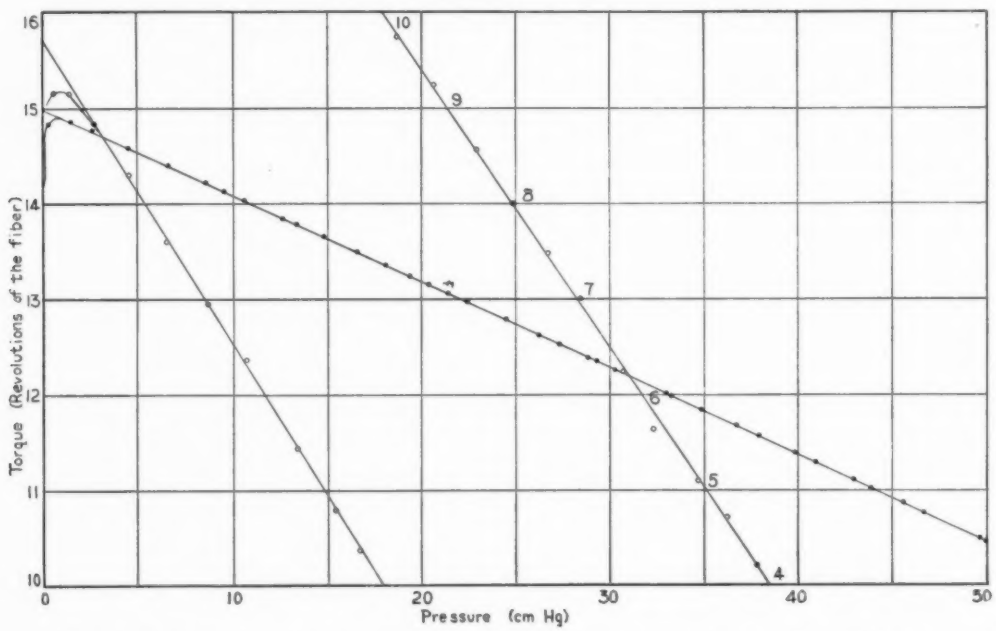


Fig. 4. Dots, hydrogen at room temperatures, curve 156; circles, hydrogen at liquid air temperatures, curve 159.

TABLE II. O_2, *curve 140.*

Pressure (cm Hg)	Torque (Rev. of fiber)	Pressure (cm Hg)	Torque (Rev. of fiber)
0.240	34.167	2.519	202.178
0.509	53.680	3.047	241.214
0.982	88.422	4.078	316.649
1.482	125.916	5.486	421.111
2.014	164.722	3.486	274.417
		1.675	141.564

tions at liquid air temperatures. The author's result agrees with Van Vleck's theoretical calculation within the theoretical uncertainty.

Oxygen

The accurate determination of the slope for oxygen was difficult because of its high molecular susceptibility which, unless an enormous number of turns of the fiber were used, limited the pressure to a few centimeters. This required the use of a mercury manometer having a large cross-sectional area and the aid of a cathotometer for the accurate measurements of the difference of level. This gas was used as a standard for the measurements of the other gases. The provisional value of 3335×10^{-6} for the molecular susceptibility at 20°C was used; should this value be found to be in error, the results for the other gases will have to be changed proportionally. The results for oxygen at liquid air temperatures show that Curie's law is satisfied to within 2 percent. The results are given in Tables II, III and IV.

Helium

The results for helium are surprisingly close to the theoretical calculation made by Stoner,[6] and agree with the experimental results of Wills

TABLE III.

Curve No.	Gas	Date of obs.	Sloz of curve in rev. of fiber/cm Hg Pres.	Obs. temp.
138	H_2	8/12/32	−0.09050 ±0.00015	24.3
139	H_2	8/14/32	−0.090531 ±0.000021	24.45
140	O_2+0.4% A	8/14/32	73.64 ±0.03	24.9
141	N_2	8/18/32	−0.2702 ±0.0005	23.9
142	CO_2	9/20/32	−0.4734 ±0.0020	21.90
145	H_2	10/ 2/32	−0.0901 ±0.0002	23.3
148	He	10/ 3/32	−0.0432 ±0.0002	21.95
151	He	10/ 8/32	−0.146 ±0.004	(−189.4)*
152	H_2	10/ 8/32	−0.306 ±0.010	(−189.4)*
153	H_2	10/24/32	−0.316 ±0.004	(−189.4)*
155	H_2	10/28/32	−0.3316 ±0.003	−189.4
156	H_2	10/29/32	−0.090946 ±0.000021	21.8
159	H_2	11/ 2/32	−0.3193 ±0.0008	−189.4
160	H_2	11/ 3/32	−0.09073 ±0.00030	21.5
164	Ne	11/13/32	−0.17299 ±0.00006	21.8
168	Ne	11/16/32	−0.609 ±0.004	−189.5
169	O_2+0.4% A	11/17/32	75.46 ±0.06	20.82
170	O_2+0.4% A	11/17/32	945.0 ±6.0	−189.7
172	H_2	11/18/32	−0.09107 ±0.00060	20.8
177	H_2	12/22/32	−0.09017 ±0.00017	22.88
180	H_2	12/28/32	−0.09010 ±0.00010	22.60
181	O_2 Elect	12/28/32	74.401 ±0.07	22.66
185	H_2	1/11/33	−0.0891 ±0.0006	22.72
186	O_2 Elect	1/11/33	73.34 ±0.15	22.23
187	H_2	1/19/33	$>$ −0.0889 ±0.0003	22.14
189	O_2 Elect	1/19/33	74.50 ±0.15	22.30
190	H_2	1/26/33	−0.09013 ±0.00050	22.16
191	N_2	1/26/33	−0.26849 ±0.00013	22.42
192	He	1/26/33	−0.04260 ±0.0003	22.47
193	H_2	2/13/33	−0.09035 ±0.0006	22.06
194	H_2	3/20/33	−0.0645 ±0.0010	22.57
195	H_2	3/20/33	−0.2304 ±0.0025	−192.2
197	He	3/22/33	−0.03125 ±0.00020	22.02
199	He	3/23/33	−0.03075 ±0.00020	22.04
201	A	3/24/33	−0.3098 ±0.0010	22.57
202	A	3/24/33	−1.16 ±0.10	−191.9

* These temperatures were estimated from later temperature determinations under practically the same conditions.

TABLE IV.

Curve No.	No. of standardizing curve	$x \times 10^6$	Assigned weights
		H_2 (Room temperatures)	
139	140	-4.008 ± 0.004	12
180	181	$-4.002 \pm .005$	8
138	140	$-4.009 \pm .006$	6
172	169	$-3.998 \pm .006$	6
177	181	$-4.008 \pm .008$	3
	Weighted ave. val.	-4.0051 ± 0.0016	
		H_2 (Room temperatures)	
160	169	-4.009 ± 0.014	8
190	189	$-4.001 \pm .020$	4
185	186	$-4.024 \pm .025$	3
193	189	$-4.01 \pm .03$	2
187	189	$-3.97 \pm .04$	1
	Weighted ave. val.	-4.008 ± 0.005	
		H_2 (Liquid air temperatures)	
159	160	-4.01 ± 0.04	25
153	156	$-3.95 \pm .06$	11
195	194	$-3.92 \pm .07$	8
155	156	$-4.14 \pm .20$	1
	Weighted ave. val.	-3.982 ± 0.019	
		He (Room temperatures)	
148	145	-1.910 ± 0.010	16
192	189	$-1.892 \pm .013$	10
197	195	$-1.94 \pm .03$	2
199	195	$-1.91 \pm .04$	1
	Weighted ave. val.	-1.906 ± 0.006	
		He (Liquid air temperatures)	
151	152	1.91 ± 0.08	
		Ne (Room temperatures)	
164	169	-7.651 ± 0.008	
		Ne (Liquid air temperatures)	
168	169	-7.64 ± 0.08	
		A (Room temperatures)	
201	194	-19.23 ± 0.20	
		A (Liquid air temperatures)	
202	201	-19.8 ± 1.4	
		N_2 (Room temperatures)	
141	140	-11.950 ± 0.026	2
191	189	-11.930 ± 0.020	3
	Weighted ave. val.	-11.938 ± 0.017	
		CO_2 (Room temperatures)	
142	145	-20.94 ± 0.09	5
142	140	$-20.79 \pm .12$	3
	Weighted ave. val.	-20.88 ± 0.08	
		O_2 (Liquid air temperatures)	
170	169	11830 ± 80 at $-189.7°C$	

TABLE V. *Comparison of the results of various observers molecular susceptibilities* $\times 10^6$.*

Observer	H_2	He	Ne	A	N_2	CO_2
Author[1]	−4.0051 ±0.0016	−1.906 ±0.006	−7.651 ±0.008	−19.23 ±0.20	−11.938 ±0.017	−20.88 ±0.08
	−3.982 ±0.019	−1.91 ±0.08	−7.64 ±0.08	−19.8 ±1.4	Liquid air measurements	
Wills and Hector[14]	−3.84 −4.04 −3.94	−1.94				
Hector[16]		−1.84 −1.91 −1.88	−7.70 −6.97 −5.63 −6.75 −6.85 −6.62 −6.03 −6.57 −6.52 ±0.21	−18.3 −17.8 −18.1	−11.96 −11.50 −11.94 −11.80	
Soné[13]†	−4.13 −3.92 −3.98 −3.84 −3.89 −4.02 −4.11 −3.85 −4.04 −4.09 −4.10 −3.96 −3.996 ±0.029				−7.22 −7.62 −7.42 −7.42 −8.04 −8.06 −8.05 −10.2 −10.0 −10.1	−18.61 −18.88 −18.30 −18.60
Lehrer[17]	−5.08 ±0.18			−20.11 ±0.32		−20.86 ±0.16
Bitter[15]	−5.8				−14.8	−24.2
	−3.3	Liquid air measurements			−14.2	
Hammar[9]‡	(−4.0)				−8.0	−19.3
Glaser[8]					−7.36	−18.9
Glaser[18]			−6.85	−20.1		−21.0±.2
Vaidyanathan[10]				−25.3	−12.9	−20.5

* The errors given for the author, are the probable errors as calculated by the rule of mean squares, likewise for Soné's results for hydrogen and Hector's results for neon.

† Soné used N_2 purified by three different methods.

‡ Hammar's values are calculated by assuming $\chi(H_2) = -4.0 \times 10^{-6}$.

[16] Hector, Phys. Rev. **24**, 418 (1924).

[17] Lehrer, Ann. d. Physik **81**, 229 (1926).

[18] Glaser, Ann. d. Physik **78**, 641 (1925); **1**, 814 (1929).

and Hector,[14] and Hector.[16] No change in the susceptibility was found in going from room temperatures to liquid air temperatures.

Neon

Only one determination was made for neon at room temperatures and one at liquid air temperatures because of the limited amount available. The curve obtained for room temperatures is one of the most accurate obtained for any of the gases. The results do not agree with either those of Hector[16] or those of Glaser.[18] However these men used neon which contained impurities of helium in excess of 1 percent which partially explains their low results. The neon used by the author was purchased from the Air Reduction Sales Company of New Jersey and was reputed to contain less than 0.1 percent impurity.

Argon

The value obtained for this gas is greater than that obtained by Hector[16] but less than those obtained by Lehrer,[17] Glaser[18] and Vaidyanathan.[10]

Nitrogen

The results for nitrogen are in good agreement with those of Hector.[16] Bitter[11] found that nitrogen originally containing an impurity of oxygen contained after passage over hot copper a small amount of NO and this might explain the low results of several observers. Soné[13] obtains three different values for three different methods of purification. Table V. The highest of these three, obtained from nitrogen purified by the reduction of air, is still below the results of Hector and the author; but he assumes this value to be too high because of the amount of argon in air; however, the amount of argon in air is known and could not account for the large differences among his values.

Carbon dioxide

The results for this gas are probably more consistent for different observers than any other gas. The author's result agrees with that of Lehrer to within 0.1 percent and with that of Glaser to within 0.6 percent.

In conclusion the author desires to express his appreciation to Dr. C. E. Mendenhall for his interest in the work and valuable suggestions, also to his brother, R. Havens for his assistance during the latter part of the work. The author wishes to express his gratitude to the Wisconsin Research foundation for its financial assistance which in a large part made these investigations possible.

Note on the Magnetic Moment of the Nitrogen Nucleus

R. F. BACHER,* *University of Michigan*

(Received April 26, 1933)

Three lines of the group $2s\,2p^4\,{}^4P-2s^2\,2p^2\,4p\,{}^4D$ of N I have been examined for hyperfine structure. They were found to be single when examined with a Fabry-Perot interferometer, using a variety of plate separations. The line widths of two of the lines were measured. The absence of hyperfine structure is attributed to the small magnetic moment (μ) of the nitrogen nucleus. It is shown that, by using the results of Goudsmit, the measured line widths lead to an unusually low magnetic moment for the nitrogen nucleus, $\mu \leqq 0.2\, eh/4\pi Mc$. Under certain assumptions about the structure of the nitrogen nucleus, this low value of the magnetic moment leads to the conclusion that the neutron has a magnetic moment in the opposite direction from its mechanical moment and about one proton magneton in magnitude.

IT is known from the alternation of intensities of the lines of the N_2^+ bands that the nitrogen nucleus has a mechanical moment of $1\times h/2\pi$. None of the stronger lines of the atomic spectrum which have been investigated,[1] however, have been observed to show any hyperfine structure. This absence of hyperfine structure may be due to one or both of the following reasons. (1) The level separations of initial and final states may be much the same and the strong lines may, therefore, all fall together. (2) The level separations themselves may be small.

In order to eliminate (1) as a possible cause of the absence of hyperfine structure, combinations of the $2s\,2p^4$ configuration (expected from the separation factors (3) to have hyperfine structure separations roughly 25 times as large as other configurations of N I) with the $2s^2\,2p^2\,4p$ were examined. Three lines in particular of this group were studied: λ5329 ($2s\,2p^4\,{}^4P_{2\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{3\frac{1}{2}}$), λ5357 ($2s\,2p^4\,{}^4P_{1\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{2\frac{1}{2}}$) and λ5373 ($2s\,2p^4\,{}^4P_{\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{1\frac{1}{2}}$). Of these the first two were more advantageous for study because of their greater intensity and comparative isolation. These lines were found to be excited with considerable intensity in the positive column of a condensed discharge. The discharge tube was designed with a section of the positive column in a U so that it could be cooled with liquid air and the tube was operated while helium containing a small quantity of nitrogen was circulated through it. The intensity of the nitrogen lines was considerably increased when the tube was cooled with liquid air. The spectrum was examined with a Fabry-Perot type interferometer with various plate separations from 5 mm to 29.5 mm used in conjunction with a spectrograph of the Hilger E_1 type (glass).

None of the lines mentioned above showed any structure even when they were examined under the high resolution of the 29.5 mm plate separation. The widths of the fringes of the lines λ5329 and λ5357 were determined from measurements on plates taken with the latter separation and were 0.055 cm^{-1} and 0.053 cm^{-1} respectively. In addition the fringes shaded off equally on both sides, while if there were any hyperfine structure as large as 0.055 cm^{-1}, an unsymmetrical broadening would be expected since a separation as large as the measured width is well within the practical resolving power of the instrument.

In order to draw any conclusions from the absence of hyperfine structure in these lines and from the measured fringe widths, it is necessary to discuss the size of the expected hyperfine structure. The hyperfine structure separation of an atomic level of quantum number J may be written in the form (1).

$$d\nu = \tfrac{1}{2}A\{F(F+1)-J(J+1)-I(I+1)\}. \quad (1)$$

I is the nuclear moment in units $h/2\pi$, A, the interval factor, is a constant which depends on the atomic state and on the magnetic moment of the nucleus, and F is the hyperfine quantum number. In case the hyperfine structure is attributed to the interaction of a single electron,

* Alfred Lloyd Fellow.

[1] Gibbs and Kruger, Phys. Rev. **37**, 1702 (1931).

the interval factor (called a in this case) may be written[2] as in (2).

$$a=\frac{R\alpha^2 Z_i Z_0^2}{n_0^3(l+\frac{1}{2})j(j+1)}\frac{g(I)}{1838}. \qquad (2)$$

$g(I)$ is the nuclear g-factor, i.e., the ratio of magnetic to mechanical moment for the nucleus, n_0 is the effective total quantum number for the electron, and Z_i and Z_0 are the effective nuclear charge near the nucleus and away from the nucleus, respectively. Using values of n_0 from the known values of the energy states and the proper values of l and j and reasonable values of Z_i and Z_0, one may find the following:

$$a(2s)=0.23g(I)\ \text{cm}^{-1},$$

$$a(3s)=0.01g(I)\ \text{cm}^{-1},$$

$$a(2p_1)=0.014g(I)\ \text{cm}^{-1},$$

$$a(2p_{1\frac{1}{2}})=0.003g(I)\ \text{cm}^{-1}.$$

The lower two are actually obtained[2] from a knowledge of the multiplet separation in $2s\ 2p^4\ {}^4P$ of N I. From these values it may immediately be seen that the effects of the others may be neglected in comparison to that of the $2s$ electron.

When it is known that the hyperfine structure may be attributed largely to the $2s$ electron, the separation factors for the three levels $2s\ 2p^4\ {}^4P_{2\frac{1}{2},\ 1\frac{1}{2},\ \frac{1}{2}}$ may easily be found.[3]

$$A(2\tfrac{1}{2})=0.046g(I)\ \text{cm}^{-1},\quad A(1\tfrac{1}{2})=0.056g(I)\ \text{cm}^{-1},$$

$$A(\tfrac{1}{2})=0.127g(I)\ \text{cm}^{-1}.$$

Fig. 1 shows the expected level separations and the resultant line structures and intensities of λ5329 and λ5357. The separations are all given in terms of $g(I)$ which for nitrogen is numerically the same as the magnetic moment in proton units since $I=1$ ($\mu=g(I)\times I$). The expected hyperfine structures are both of the flag type, shading somewhat in intensity from one side to the other. The observed fringes showed no such shading. Using the measured fringe widths as upper limits for the widths of possible hyperfine structure one may find that $g(I)\leqq 0.2$ and hence

$$\mu\leqq 0.2\ eh/4\pi Mc.$$

[2] Goudsmit, Phys. Rev. **43**, 636 (1933).

[3] Goudsmit and Bacher, Phys. Rev. **34**, 1501 (1929).

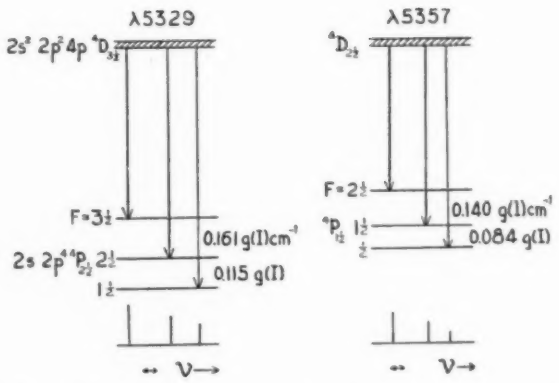


FIG. 1. Level separations and resultant hyperfine structure pattern of λ5329 and λ5357 of N I given in terms of $g(I)$; as expected theoretically. The small double arrows at the bottom give the measured line widths on the same scale as the expected separations with $g(I)=1$.

It is thus indicated that the magnetic moment of the nitrogen nucleus is considerably smaller than other nuclear magnetic moments[2] known at present since these range from 0.55 to 5.4 "proton magnetons." It is particularly interesting to see what such a small value of the nuclear magnetic moment means if one considers the structure of the nitrogen nucleus. If one assumes (a) that light nuclei contain as many α-particles as possible (or their equivalent in closed groups) then the most probable structure of the nitrogen nucleus is the following:

$$3\alpha+1\ \text{proton}+1\ \text{neutron}.$$

In addition, it is assumed that (b) the nuclear spin is the resultant of the spins of the particles[4] in the nucleus and that (c) in analogy to the electron the proton has a magnetic moment $eh/4\pi Mc$, one "proton magneton." The mechanical moment of $1\times h/2\pi$ is accounted for by the proton and the neutron both having spins[5] $\frac{1}{2}\times h/2\pi$ and having them oriented in the same direction. It is not possible, however, to get $\mu\leqq 0.2$ unless the neutron has its magnetic moment in the opposite direction from its mechanical moment and of a magnitude about one proton magneton. It must be recognized that this conclusion is dependent on hypotheses (a), (b) and (c) and while they seem reasonable at the present moment they are nevertheless speculative.

[4] The possible rôle of the positron as a nuclear constituent possessing spin is not considered.

[5] Bacher and Condon, Phys. Rev. **41**, 683 (1932).

The Theory of Depolarization, Optical Anisotropy, and the Kerr Effect

ROBERT SERBER, *University of Wisconsin*

(Received May 3, 1933)

Classical theory. The usual classical theory of optical anisotropy, based on a molecular model which considers a molecule as a set of anisotropic harmonic oscillators, leads to the formula

$$\Delta = 6\delta^2/(5+7\delta^2) \qquad (6)$$

for the depolarization of scattered light. Here δ^2 is the optical anisotropy. For the Kerr constant of nonpolar molecules it gives the formula

$$K = 3\delta^2(n^2-1)(\epsilon-1)/40\pi NkT. \qquad (9)$$

A critical examination of the theory shows that these equations can be true only in two limiting cases, in both of which it is possible to define δ^2 in a way which makes it independent of the frequency of the incident light. The first case is characterized by the condition that the index of refraction be representable by a one term Sellmeier formula. The second case arises when the anistropies of the individual absorption lines are all approximately equal. The above restriction does not seem to be generally realized; at least it is not ordinarily stated in the literature.

Quantum theory. General formulae are derived for the depolarization of light scattered by molecules, including the Raman scattering, and for the Kerr constant. It is shown that the above formulae, (6) and (9), are valid quantum-mechanically under the same conditions for which they hold classically. The quantum proof is much more general, in that no particular molecular model is assumed.

ONE of the fruitful classical attacks upon the problem of the internal structure of molecules was through the concept of "optical anisotropy." The molecule was envisaged as a collection of anisotropic harmonic oscillators, and the optical isotropy was defined in terms of the differences in the electric polarizabilities along three principal axes in the molecule. The anisotropy could be calculated from measurements of the depolarization of scattered light, or, if certain simplifying assumptions were made, from the Kerr constant. Knowledge of the anisotropy gave some clue to the arrangement of atoms within the molecule, serving as a check upon assumed molecular models. As the theory was reasonably successful, it becomes of interest to reexamine it in terms of quantum mechanics for a general molecule rather than for a collection of oscillators, to see precisely what meaning can be given to "optical anisotropy," and what approximations must be made in order to reestablish the classical formulae. In particular, it is necessary to include the effect of Raman scattering.

CLASSICAL THEORY[1]

A rapid sketch of the classical theory will serve a double purpose. The calculation, formally, closely parallels that of the quantum theory, while, because of the very simple molecular model used, the main outline stands out more clearly; we can have the woods without the trees. Also we shall have an opportunity to examine the approximations implicit in the classical treatment, and to discuss the circumstances under which these approximations are valid.

Only optically inactive molecules will be considered, and it will be supposed that the molecule is free to rotate, i.e., that it is not acted on by strong fields due to surrounding molecules. This condition will, of course, be well fulfilled in gases.

The reaction of the anisotropic oscillators of the classical model to the field of a light wave is described by a polarization tensor α_{ij}. The x' component of the induced electric moment, for example, is

$$X'^{(1)} = \alpha_{x'x'}E_{x'} + \alpha_{x'y'}E_{y'} + \alpha_{x'z'}E_{z'}.$$

The x', y', z' coordinate axes are supposed fixed in space, not in the molecule. Every α_{ij} can be written as a sum of terms each of which depends

[1] The theory was developed largely by P. Langevin, M. Born and R. Gans. For discussion of the theory and the experimental results see P. Debye, *Handbuch der Radiologie*, 6; C. V. Raman and K. S. Krishnan, Phil. Mag. **3**, 713 (1927); H. A. Stuart, Ergebnisse der Exakten Naturwissenschaften **10**, 159 (1931).

on a single fundamental frequency of one of the oscillators; thus

$$\alpha_{ij}=\sum_{n'}\alpha_{ij}^{(n')}/(\nu^2{}_{n'}-\nu^2). \tag{1}$$

In the absence of external fields the three diagonal elements of the polarization tensor, averaged over all positions of the molecule, are equal; the mean value of the nondiagonal elements is zero. This is an immediate consequence of the free rotation of the molecules, whereby all directions in space are equivalent. However the mean values of the squares of the nondiagonal elements will not vanish; this is the origin of optical anisotropy.

Depolarization of scattered light

The intensity of scattered light, polarized in a direction $\mathbf{q}$, is proportional to the square of the component, in this direction, of the induced electric moment vector, $\mathbf{P}^{(1)}$,

$$I_q=(\mathbf{P}^{(1)}\cdot\mathbf{q})^2.$$

Suppose that the incident light is directed along the z' axis, and the scattering is observed along the x' axis. Let the direction of polarization of the incident light make an angle θ with the y' axis; thus $E_{y'}=E\cos\theta$, $E_{z'}=E\sin\theta$. The intensities of scattered light polarized in the y' and z' directions are, respectively,

$$I_{y'}=\overline{(\alpha_{y'y'}E\cos\theta+\alpha_{y'z'}E\sin\theta)^2},$$
$$I_{z'}=\overline{(\alpha_{z'y'}E\cos\theta+\alpha_{z'z'}E\sin\theta)^2},$$

the bar indicating a spatial average over all positions of the molecule. If the incident light is unpolarized we must average over θ, obtaining

$$I_{y'}=\tfrac{1}{2}(\overline{\alpha_{y'y'}{}^2}+\overline{\alpha_{y'z'}{}^2})E^2,$$
$$I_{z'}=\tfrac{1}{2}(\overline{\alpha_{z'y'}{}^2}+\overline{\alpha_{z'z'}{}^2})E^2=\overline{\alpha_{y'z'}{}^2}E^2.$$

Since it is immaterial which particular directions in space we happen to label x', y', z', we must have

$$\overline{\alpha_{z'y'}{}^2}=\overline{\alpha_{z'z'}{}^2}=\overline{\alpha_{y'z'}{}^2}.$$

The depolarization of scattered light is then

$$\Delta=I_{z'}/I_{y'}=2\overline{\alpha_{y'z'}{}^2}/(\overline{\alpha_{y'y'}{}^2}+\overline{\alpha_{y'z'}{}^2}). \tag{2}$$

We must now transform from axes fixed in space to axes fixed in the molecule, and perform the spatial averaging. Calling the coordinates fixed in the molecule x, y, z, we find[2]

$$15\overline{\alpha_{y'y'}{}^2}=\sum_{xyz}\{3\alpha_{xx}{}^2+2\alpha_{xx}\alpha_{yy}+4\alpha_{xy}{}^2\}, \tag{3}$$

$$15\overline{\alpha_{y'z'}{}^2}=a_{yx}a_{yx},$$

where $a_{yx}a_{yx}=\sum_{xyz}\{\alpha_{xx}{}^2-\alpha_{xx}\alpha_{yy}+3\alpha_{xy}{}^2\}$,

and the symbol $\sum_{xyz}$ indicates a summation of the three terms obtained by a cyclic interchange of x, y, z. Introducing the further notation

$$a_{xx}=\sum_{xyz}\alpha_{xx},$$

and using (1), we readily verify that (2) reduces to

$$\Delta=\frac{6\sum_{n'n''}\dfrac{a_{yx}{}^{(n')}a_{yx}{}^{(n'')}}{(\nu^2{}_{n'}-\nu^2)(\nu^2{}_{n''}-\nu^2)}}{\sum_{n'n''}\dfrac{5a_{xx}{}^{(n')}a_{xx}{}^{(n'')}+7a_{yx}{}^{(n')}a_{yx}{}^{(n'')}}{(\nu^2{}_{n'}-\nu^2)(\nu^2{}_{n''}-\nu^2)}}. \tag{4}$$

If we use the formula

$$n^2-1=\frac{4\pi N}{3}\sum_{n'}\frac{a_{xx}{}^{(n')}}{\nu^2{}_{n'}-\nu^2}$$

for the index of refraction, the first term in the denominator of (4) becomes

$$5[3(n^2-1)/4\pi N]^2.$$

Eq. (4) is the general formula for the depolarization.

The usual method of handling (4) is to define a quantity called the "optical anisotropy" by means of the equation

$$\delta^2=\frac{\sum_{n'n''}\dfrac{a_{yx}{}^{(n')}a_{yx}{}^{(n'')}}{(\nu^2{}_{n'}-\nu^2)(\nu^2{}_{n''}-\nu^2)}}{\sum_{n'n''}\dfrac{a_{xx}{}^{(n')}a_{xx}{}^{(n'')}}{(\nu^2{}_{n'}-\nu^2)(\nu^2{}_{n''}-\nu^2)}}. \tag{5}$$

Eq. (4) can then immediately be written

$$\Delta=6\delta^2/(5+7\delta^2). \tag{6}$$

[2] It should be noted that the polarization tensor, α_{ij}, is symmetric. A proof of this is given by Debye.[1]

This procedure virtually makes (6) the defining equation for δ^2; it has the disadvantage that δ^2 so defined will in general vary with the frequency of the incident light. If optical anisotropy is to be a true molecular characteristic, rather than an empirically determined constant, that is, if it is to have any clear significance in terms of molecular structure, we should expect it to be capable of definition in terms which are independent of the incident frequency (at least when the frequency range is suitably restricted). It will, in fact, be shown in our consideration of the Kerr effect that satisfaction of this condition is necessary in order that the Kerr constant may be expressed in terms of δ^2. When this condition cannot be satisfied, (6) is the beginning and end of "optical anisotropy," and the concept leads to nothing. The term "optical anisotropy" will therefore be reserved to those cases in which it can be defined in a way which makes it independent of frequency; the definition (5) will no longer be used. We must now consider the circumstances under which this is possible.

(a) Optical anisotropy can be defined for molecules whose index of refraction is representable by a one term Sellmeier formula.[3] The fact that only one term is required implies that all the important absorption lines lie in a narrow region of the spectrum, that is, that their frequency differences, $\nu_{n'}-\nu_{n''}$, are small compared to $\nu_{n'}-\nu$. An absorption line is called unimportant if $\alpha_{xx}^{(n')}$ is very small, or if $\nu_{n'}$ is very large. In this case we can make what we shall call the centroid approximation, replacing all the important $\nu_{n'}$'s by a centroid frequency $\bar{\nu}$, and neglecting all the unimportant lines. The optical anisotropy can then be defined by

$$\delta^2=\sum_{n'n''}{}^{*}\, a_{yz}^{(n')}a_{yz}^{(n'')}\Big/\sum_{n'n''}{}^{*}\, a_{xz}^{(n')}a_{xz}^{(n'')}.$$

The asterisk indicates that the summation is to be taken only over the important terms. With this definition of δ^2, (4) reduces directly to (6).

(b) For molecules to which the centroid approximation is not applicable we can define the anisotropy for each pair of absorption lines,

$$\delta^2(n'n'')=a_{yz}^{(n')}a_{yz}^{(n'')}/a_{xz}^{(n')}a_{xz}^{(n'')}.$$

It may happen that all the $\delta^2(n'n'')$ of important lines are approximately equal. In this case the concept of optical anisotropy again acquires a useful meaning. We can write $\delta^2(n'n'')=\delta^2$ for all important n' and n'', and (4) again reduces to (6).[4]

Kerr effect

The discussion of the Kerr effect will be limited to nonpolar molecules. Since an external electric field is applied we can no longer consider all directions in space as equivalent, and the mean values of the diagonal elements of the polarization tensor will no longer be equal. In fact, if the electric field is directed along the z' axis, the Kerr constant is proportional to the difference between the mean z' and x' elements,[5] i.e.,

$$K=\frac{2\pi N}{E^2}\frac{\int(\alpha_{z'z'}-\alpha_{x'x'})e^{\mathring{a}_{z'z'}E^2/kT}d\omega}{\int d\omega},$$

with the integration taken over all positions of the molecule. Here $\mathring{\alpha}_{z'z'}$ is the polarizability for a static field, i.e., for $\nu=0$. Expanding the exponential in powers of $1/kT$ we find

$$K=(2\pi N/E^2)(\overline{\alpha_{z'z'}}-\overline{\alpha_{x'x'}})+(\pi N/kT)(\overline{\mathring{\alpha}_{z'z'}\alpha_{z'z'}}-\overline{\mathring{\alpha}_{z'z'}\alpha_{x'x'}}). \quad (7)$$

Since the Stark effect is small compared to kT, higher order (saturation) terms can be neglected. Now it is a peculiarity of the particular molecular model (harmonic oscillators) which we are using that the elements of the polarization tensor are independent of the static field E.[6] This would not be true with a more general model, for example anharmonic oscillators. Thus in taking the spatial averages indicated in (7) we can proceed as if the external field were absent, that is, we can again regard all directions as equivalent. The

[3] Of course this will never be possible for all frequencies. However in practice only a comparatively small frequency range will be in question (usually the visible), while all the important $\nu_{n'}$'s may lie far in the ultraviolet.

[4] A modification of case (b) arises when the absorption frequencies all cluster about two or more "centroid" frequencies, $\bar{\nu}_1, \cdots, \bar{\nu}_i, \cdots$. Here δ_i^2 is defined, for each cluster of lines, by a formula analogous to that used in case (a), and, when all the δ_i^2 are equal, δ^2 can be defined as their common value.

[5] M. Born and P. Jordan, *Elementare Quantenmechanik*, p. 262.

[6] The effect of the static field is merely to shift the equilibrium positions of the vibrating particles. The point is that it does not affect the stiffness coefficients.

first term in (7) is immediately seen to be zero, as $\overline{\alpha_{x'x'}}=\overline{\alpha_{x'x'}}$. Further, since there is no preferential direction in space it must be a matter of indifference in what direction we choose our coordinate axes. The quantity $\overline{\mathring{\alpha}_{x'x'}\alpha_{x'x'}}$ must therefore be a rotational invariant, i.e., if we choose a new set of axes x_1, y_1, z_1, we must have

$$\overline{\mathring{\alpha}_{z_1z_1}\alpha_{z_1z_1}}=\overline{\mathring{\alpha}_{z'z'}\alpha_{z'z'}}.$$

Choosing $z_1=(\frac{1}{2})^{\frac{1}{2}}(z'+x')$, and expressing $\alpha_{z_1z_1}$ in terms of the α_{ij}'s ($i, j=x', y', z'$), we find

$$\overline{\mathring{\alpha}_{z'z'}\alpha_{z'z'}}-\overline{\mathring{\alpha}_{z'z'}\alpha_{x'x'}}=2\overline{\mathring{\alpha}_{z'x'}\alpha_{z'x'}}.$$

As

$$\overline{\mathring{\alpha}_{z'x'}\alpha_{z'x'}}=\overline{\mathring{\alpha}_{y'x'}\alpha_{y'x'}},$$

we can write

$$K=\frac{2\pi N}{kT}\overline{\mathring{\alpha}_{y'x'}\alpha_{y'x'}}=\frac{2\pi N}{15kT}\sum_{n'n''}\frac{a_{yx}^{(n')}a_{yx}^{(n'')}}{\nu^2_{n''}(\nu^2_{n'}-\nu^2)}, \quad (8)$$

the final form being written in terms of axes fixed in the molecule by means of (3) and (1).

The conditions under which this equation can be expressed in terms involving the optical anisotropy are exactly the same as those discussed in connection with the depolarization. Defining δ^2 as before, for either possible case, (*a*) or (*b*), we have

$$K=\frac{2\pi N\delta^2}{15kT}\sum_{n''}{}^{*}\frac{a_{xx}^{(n'')}}{\nu^2_{n''}}\sum_{n'}{}^{*}\frac{a_{xx}^{(n')}}{\nu^2_{n'}-\nu^2},$$

or, in terms of the index of refraction n, and the dielectric constant ϵ,

$$K=3\delta^2(n^2-1)(\epsilon-1)/40\pi NkT. \quad (9)$$

Eqs. (6) and (9) are the fundamental equations in the theory of the optical anisotropy of nonpolar molecules.

If, as often tacitly done in the literature, we had allowed δ^2 to be a function of ν, defining it by (5), it would have been necessary in order to reduce (8) to (9) to make the approximation (due to Gans)

$$\frac{\sum_{n'n''}\dfrac{a_{yx}^{(n')}a_{yx}^{(n'')}}{\nu^2_{n''}(\nu^2_{n'}-\nu^2)}}{\sum_{n'n''}\dfrac{a_{yx}^{(n')}a_{yx}^{(n'')}}{(\nu^2_{n''}-\nu^2)(\nu^2_{n'}-\nu^2)}}=\frac{(\epsilon-1)}{(n^2-1)}.$$

But this can be valid for a range of values of ν only if one or the other of our two cases, (a) or (b), holds. Thus our restrictions are unavoidable.

Quantum Theory

Notation. The übermatrix notation will be used,[7] whereby the symbol $a^{(nn')}$ denotes a matrix of all the elements of **a** having all the quantum numbers in the initial state n the same except j and m, and all the quantum numbers in the final state n' the same except j' and m'. Sp_{jm} is an abbreviation for spur, i.e., a summation over j and m of the diagonal elements of a matrix.[8] Sp_m will be used to designate a sum over m only, the value of j being fixed. Thus

$$\mathrm{Sp}_m(a^{(nn')}b^{(n'n)})^{(ij)}=\sum_{mj'm'}a^{(nn')}_{jm;\ j'm'}b^{(n'n)}_{j'm';\ jm}=\sum_{mj'm'}a_{njm;\ n'j'm'}b_{n'j'm';\ njm}; \quad (10)$$

$$\mathrm{Sp}_{jm}(a^{(nn')}b^{(n'n)})=\sum_{jmj'm'}a_{njm;\ n'j'm'}b_{n'j'm';\ njm}.$$

Depolarization of scattered light

The calculation is very similar to that already given under "Classical Theory," only now the quantities α_{ij} can be explicitly expressed in terms of amplitudes (matrix elements of the electric moment) by means of the Kramers dispersion formula. In place of (2) we have for the depolarization of light scattered by a molecule in a state (n, j)[9]

[7] Born and Jordan,[5] p. 27.

[8] In case there is degeneracy other than the spatial degeneracy, Sp_{jm} is to be understood as including a sum over all the degenerate states, as well as over j and m. The same remark applies to Sp_m.

[9] Eq. (11) is readily obtained from the expression given by Born and Jordan,[5] p. 281, or from the one given by P. A. M. Dirac, *The Principles of Quantum Mechanics*, p. 235.

$$\Delta = \frac{2\mathrm{Sp}_m(\alpha^{(nn)}_{x'y'}\alpha^{(nn)}_{y'x'})^{(jj)}}{\mathrm{Sp}_m(\alpha^{(nn)}_{x'x'}\alpha^{(nn)}_{x'x'}+\alpha^{(nn)}_{x'y'}\alpha^{(nn)}_{y'x'})^{(jj)}}, \tag{11}$$

with

$$(\alpha_{ik})^{(nn)}_{jm;\ j''m''} = \sum_{n'j'm'}\left[\frac{P^{(nn')}_{ijm;\ j'm'}P^{(n'n)}_{kj'm';\ j''m''}}{\nu+\nu_{n'j';\ nj''}} - \frac{P^{(nn')}_{kjm;\ j'm'}P^{(n'n)}_{ij'm';\ j''m''}}{\nu-\nu_{n'j';\ nj}}\right]. \tag{11a}$$

Here the P_i denote the components, X', Y', Z', of the electric moment. The contribution of Raman lines which involve a change in j (rotational fine structure) is included in (11); the term in (11a) for which $j''=j$ gives the effect of Raleigh scattering, the terms $j''\neq j$ give that of the Raman.

Unless the incident frequency lies very close to an absorption band we can neglect the separations between rotational levels, compared to $\nu-\nu_{n'j';\ nj}$ and replace $\nu_{n'j';\ nj}$ by a mean frequency $\nu_{n'n}$. Eq. (11) then becomes

$$\Delta = \frac{2\sum_{n'n''}\tau_{n'n}\tau_{n''n}\mathrm{Sp}_m(X'Y'^*Y'X'+X'Y'^*X'Y')^{(ii)}}{\sum_{n'n''}\tau_{n'n}\tau_{n''n}\mathrm{Sp}_m(2X'X'^*X'X'+X'Y'^*Y'X'+X'Y'^*X'Y')^{(ii)}} + \frac{2\nu^2\sum_{n'n''}\dfrac{\mathrm{Sp}_m(X'Y'^*Y'X'-X'Y'^*X'Y')^{(ii)}}{(\nu^2_{n'n}-\nu^2)(\nu^2_{n''n}-\nu^2)}}{+\nu^2\sum_{n'n''}\dfrac{\mathrm{Sp}_m(X'Y'^*Y'X'-X'Y'^*X'Y')^{(ii)}}{(\nu^2_{n'n}-\nu^2)(\nu^2_{n''n}-\nu^2)}}, \tag{12}$$

with the abbreviations

$$\tau_{n'n}=\nu_{n'n}/(\nu^2_{n'n}-\nu^2), \qquad X'Y'^*Y'X'=X'^{(nn')}Y'^{(n'n)}Y'^{(nn'')}X'^{(n''n)}. \tag{13}$$

Eq. (12) can be simplified by use of the relations (14, II). The II is used to designate equations in the following paper. These relations, (14, II), apply to any polyatomic or diatomic molecule. If we employ the notation

$$C(n'n'')=\tfrac{1}{2}\{[[XY^*YX]]+[[XY^*XY]]\}$$

the formula

$$[[XX^*XX]]+C(n'n'')=(5/3)\sum_{xyz}(1/g)\mathrm{Sp}\{XX^*XX+XX^*YY+YY^*XX\}+(7/3)C(n'n'')$$

is readily verified. The right side of this equation can be written[10]

$$(5/3)P(n'n)^2P(n''n)^2+(7/3)C(n'n''), \quad \text{with} \quad P(n'n)^2=(1/g)\sum_{xyz}\mathrm{Sp}\{XX\}.$$

The term in (12) having the factor ν^2 is zero, since $[[XY^*YX]]=[[XY^*XY]]$, a relation evident from (15, II) if we note that quantities like $X^{(nn')}$ are real.

Thus (12) finally reduces to

$$\Delta = 6\sum_{n'n''}\tau_{n'n}\tau_{n''n}C(n'n'')/[5\sum_{n'n''}\tau_{n'n}\tau_{n''n}P(n'n)^2P(n''n)^2+7\sum_{n'n''}\tau_{n'n}\tau_{n''n}C(n'n'')]. \tag{14}$$

[10] This is obvious for nonlinear polyatomic molecules, and for diatomic and linear polyatomic molecules in Σ states, since here the normal state of the stationary molecule is nondegenerate. It also holds for diatomic and linear polyatomic molecules not in Σ states, for if we choose for "internal" wave functions the ones which are the "zeroth approximation" for Λ-type doubling it is seen that in products such as

$$X^{(nn')}_{\alpha\alpha'}X^{(n'n)}_{\alpha'\alpha'''}X^{(nn'')}_{\alpha'''\alpha''}X^{(n''n)}_{\alpha''\alpha},$$

where α designates the component of the Λ-type doublet, we must have $\alpha'''=\alpha$, and further, the product has the same value for $\alpha=+$ as for $\alpha=-$.

It will be observed that the depolarization is independent of j.[11]

Just as in the classical theory, the first term in the denominator of (14) can be written in terms of the index of refraction, which now is given by[12]

$$n^2-1=(8\pi N/3h)\sum_{n'}\tau_{n'n}P(n'n)^2. \tag{15}$$

The conditions under which (14) can be reduced to (6) are similar to those required to reduce (4) to (6). In the centroid case, (a), p. 1005, we define the optical isotropy by

$$\delta^2=\sum_{n'n''}{}^{*}\,C(n'n'')/\sum_{n'n''}{}^{*}\,P(n'n)^2P(n''n)^2. \tag{16a}$$

Otherwise the anisotropy of the levels n', n'' is defined by

$$\delta^2(n'n'')=C(n'n'')/P(n'n)^2P(n''n)^2, \tag{16b}$$

and, just as in the classical theory, (6) is valid only to the approximation that we can write $\delta^2(n'n'')=\delta^2$ for the important terms.

Inclusion of vibrational Raman lines. So far we have supposed that in measuring the depolarization the vibrational Raman lines are resolved, although the rotational fine structure of the undisplaced line is not. Eq. (14) is readily modified to include the case in which the vibrational lines are not resolved. The restriction that ν is not too close to resonance with absorption lines must be made more stringent; we must suppose that the frequencies $\nu_{n'v'j';\ nvj}$ (writing in the vibrational quantum numbers explicitly) can be replaced by a centroid frequency $\nu_{n'n}$. In order to include the vibrational Raman effect we have only to interpret the spur notation as meaning

$$\mathrm{Sp}_m(a^{(nn')}b^{(n'n)})^{(ii)}=\sum_{mv'j'm'}a_{nvjm;\ n'v'j'm'}b_{n'v'j'm';\ nvjm}.$$

This differs from (10) in that a summation over v' is included. The analysis which leads to (14) is unchanged, but now quantities like XY^*YX mean[13]

$$XY^*YX=\sum_{v'v''v'''}X^{(nv;\ n'v')}Y^{(n'v';\ nv'')}Y^{(nv'';\ n''v''')}X^{(n''v'''';\ nv)},$$

instead of (13). Expressions of this form can be handled by Van Vleck's general method of hybrid matrix multiplication (of which Neissen's method is a special case).[14] Following Van Vleck's notation, we can write

$$XY^*YX=(X_{nn'}Y_{n'n}Y_{nn''}X_{n''n})(v;v).$$

The advantage of this form is that knowledge of the vibrational wave function of the normal state alone is required, although, of course, all the orbital wave functions must be known.

Kerr effect

In the following paper it is shown that the Kerr constant in molecular gases is given by a formula of the form

$$K=\alpha+(1/T)(\beta+\beta')+(1/T^2)\gamma,$$

[11] Since the depolarization is independent of j, the statistical distribution of molecules among the normal states will obviously not affect our results. If we had considered the total scattering, rather than the scattering from the state (n, j), i.e., if we had multiplied the numerator of (11) by $e^{-W_{nj}/kT}$ and summed over j, and done likewise to the denominator, (14) would have remained unaltered, a factor $\Sigma_j(2j+1)e^{-W_{nj}/kT}$ cancelling from the numerator and denominator. We are supposing that the separations of the excited states from the normal state are large compared to kT, so that there is no appreciable concentration of molecules in excited states.

[12] J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities*, p. 362.

[13] It should be noted that because of the summation over intermediate values of v, the quantity $P(n'n)^2\,P(n''n)^2$ appearing in (14) can no longer be interpreted as the product of the separate factors $P(n'n)^2$ and $P(n''n)^2$.

[14] J. H. Van Vleck, Nat. Acad. Sci. **15**, 754 (1929).

where α, β, β', γ are functions only of the frequency of the incident light. In the coefficient of $1/T$, β represents the nonpolar terms, i.e., the terms which do not vanish for nonpolar molecules, β' the polar terms. The quantity γ is purely polar, and α contains both polar and nonpolar terms. Explicit expressions for α, β, β', γ are given in (25, II). Terms corresponding to α and β' do not appear in the classical theory because of the special model (harmonic oscillators) there used[15]; they arise from the perturbation of the amplitudes and the frequencies by the static electric field, i.e., from the first term in (7).

Another treatment of the Kerr effect has been given by Born and Jordan.[16] Their results can be written in the above form, but with α, β, $\beta,'$ γ replaced by quantities C_0, D_0, C_1, D_1 which are functions of temperature. Born and Jordan's statement that C_0, D_0, C_1, D_1 vary only slowly with temperature is not correct, because of the effects of the "pairing" of low frequency terms (see the following paper). In fact it is easily seen that C_0 contains some terms which vary as $1/T^2$ and some which vary as $1/T$, and D_0 and C_1 contain terms proportional to $1/T$. Similarly, the effect of low frequency pairing on the frequency dependence is not adequately considered. Born and Jordan's equation, in short, corresponds to an intermediate stage in the calculation of (25, II); a stage in which some, but not all, of the small ν's have been removed from the denominators, and in which the equation has not yet been put in a rotationally invariant form.

Relative magnitudes of the terms. The following paper shows that any statistical mean value based upon the Boltzmann distribution function can be expanded as a power series in the field strength λ,

$$\bar{u} = u^{(0)} + \lambda[u_0^{(1)} + (u_1^{(1)}/kT)] + \lambda^2[u_0^{(2)} + (u_1^{(2)}/kT) + (u_2^{(2)}/k^2T^2)] + \cdots,$$

where $u_0^{(1)}, \cdots$, are independent of temperature. It will be seen from (12, II), and, independently of this, it is fairly obvious from dimensional considerations, that a factor $1/kT$ always replaces a factor $1/h\nu_{n'n}$, e.g., every term in $u_0^{(1)}$ contains a factor $1/h\nu_{n'n}$, while $u_1^{(1)}$ contains no such factors; every term in $u_0^{(2)}$ contains a factor $1/h^2\nu_{n'n}\nu_{n''n}$, while $u_1^{(2)}$ contains only $1/h\nu_{n'n}$, and $u_2^{(2)}$ has no frequencies in the denominators. Thus if the spurs appearing in $u_0^{(1)}$ are of the same order of magnitude as the spurs in $u_1^{(1)}$, $u_1^{(1)}/kT$ is of the order $h\nu_{n'n}/kT$ times as large as $u_0^{(1)}$. At ordinary temperatures this ratio is very large, of the order of several hundred in fact. Thus the term independent of temperature is exceedingly small compared to the $1/T$ term.

This is very well illustrated by the Langevin-Debye formula, $\chi = N[(\mu^2/3kT) + \alpha]$, for paramagnetic susceptibilities. It is well known that α is very much smaller than $\mu^2/3kT$. On the other hand, for electric susceptibilities α is often comparable with the polar term. The explanation is that here the condition that the spurs involved in $u_1^{(1)}$ are of the same magnitude as the spurs in $u_0^{(1)}$ is not fulfilled. The permanent moment involves only low frequency elements of the electric moment, $P_{nj';\,nj}$, while α involves high frequency elements, $P_{n'j';\,nj}$, and, as $|P_{nj';\,nj}|^2$ is much smaller than $|P_{n'j';\,nj}|^2$, this largely offsets the factor $h\nu_{n'n}/kT$.

Similarly, if the spurs are of the same magnitude, $u_1^{(2)}/kT$ is of order $h\nu_{n'n}/kT$ as large as $u_0^{(2)}$, and $u_2^{(2)}/k^2T^2$ and $u_1^{(2)}/kT$ are related in the same way.

Returning to the Kerr effect, we will first consider the terms α and β/kT. It will be seen from (25, II) that the condition on the spurs is satisfied, as the dominant terms of α and β consist of products of high frequency amplitude elements alone, and so are of the same magnitude. Thus α is much smaller than β/kT, of about the same relative importance as the $N\alpha$ term compared to the $N\mu^2/3kT$ term in paramagnetic susceptibilities. The terms of γ, however, all have two low frequency amplitude elements as factors, so the relative magnitude of β/kT and γ/k^2T^2 is roughly the same as that of $N\alpha$ and $N\mu^2/3kT$ in the electric susceptibilities. The terms of β' all have at least one low frequency amplitude element as a factor; very roughly, we can expect β' to be smaller than β by a factor of magnitude $P_{nj';\,nj}/P_{n'j';\,nj}$.

[15] See Debye[1] for the classical treatment of the polar term proportional to $1/T^2$.

[16] Born and Jordan,[5] p. 259.

Kerr effect in nonpolar molecules. Since α is small in comparison with β/kT, the Kerr constant for nonpolar molecules, as given by (25, II), is

$$K=\frac{4\pi N}{15h^2kT}\sum_{n'n''}{}'\frac{\tau_{n''}c(n'nn'')}{\nu_{n'n}}. \tag{17}$$

In virtue of the relation (16, II), $\tau_{n''}c(n'nn'')$ can be rewritten in the notation of the section on depolarization as $2\tau_{n''n}C(n'n'')$. When condition (a) or (b), p. 1005, is satisfied, (17) becomes

$$K=\frac{8\pi N\delta^2}{15h^2kT}\sum_{n'}{}'^{*}\frac{P(n'n)^2}{\nu_{n'n}}\sum_{n''}{}^{*}\tau_{n''n}P(n''n)^2, \tag{18}$$

where δ^2 is defined by (16a) or (16b). Expressing this in terms of the index of refraction, given by (15), and the dielectric constant, obtained from (15) by putting $\nu=0$, we find that (18) reduces to (9).

Thus the formulas (6) and (9), expressing the relations between optical anisotropy and the depolarization of scattered light or the Kerr effect, have exactly the same validity in the quantum theory that they had in the classical physics. Of course the quantum proof is much more general, as it does not depend on the assumption of a particular molecular model.

I wish to thank Professor J. H. Van Vleck for his very kind advice and guidance.

The Calculation of Statistical Averages for Perturbed Systems

ROBERT SERBER, *University of Wisconsin*
(Received May 3, 1933)

A general expression is derived for the statistical mean value of any function of dynamical variables, averaged over all the molecules of a gas in the presence of external fields. This expression is in the form of a multiple Taylor's series in powers of $1/kT$ and of the field strengths. Explicit formulas, valid for diatomic and polyatomic molecules, are given for the coefficients in this series, up to and including the cubic terms. The general formula is then applied in the solution of two specific problems: (1) Effect of a magnetic field on the electric susceptibility of a molecule. (2) Theory of the Kerr effect in diatomic and polyatomic molecules. The concluding sections give applications of the formulas developed in §1 to the perturbation theory of conservative and non-conservative systems.

IN many problems in quantum mechanics we are concerned with finding the value of a function of dynamical variables, $u(p, q)$, averaged over a large number of molecules, when the molecules are subject to an external field λ. This statistical mean value is given by

$$\bar{u}=\frac{N\,\mathrm{Sp}(ue^{-H/kT})}{\mathrm{Sp}(e^{-H/kT})} \tag{1}$$

where $H=H(p, q, \lambda)$ is the Hamiltonian function, and N is the number of molecules per cubic centimeter. Letters such as H, H_0, u,v, w, W, with no indices attached, are to be understood as denoting matrices. The exponential function is, of course, defined by a power series. In writing (1) it is not necessary to specify the system of representation used, as the spur of a matrix is invariant of a canonical transformation.[1]

The usual procedure is to employ a system of representation in which H is diagonal, then to express the diagonal elements of H in terms of the matrix elements of H in an unperturbed system of representation, i.e., one in which the external field is zero, by means of perturbation theory. Except in the simplest cases this procedure has serious disadvantages. For one thing, the quantities which we are forced to evaluate are not invariant of a rotation of coordinate axes; thus sum rules and symmetry properties cannot be employed. This condition can be remedied by Van Vleck's method of "pairing" terms,[2] provided all frequencies of the unperturbed molecule can be classified as "high" or "low," that is, as large or small compared to kT/h. After "pairing" only rotational invariants appear in our equations. However, although this method works beautifully for the term linear in the field strength, it becomes very difficult to apply to higher order terms. The calculation is not straightforward, but demands considerable ingenuity on the part of the calculator. A second difficulty lies in the increasing complexity in the pertubation formulas as we go to higher order terms. The equations become very cumbersome, and of course the "pairing" is thereby made more difficult.

In the following section a new method of handling such problems is given, which involves exactly the same approximations as the perturbation method, but which has great practical advantages. Both the above-mentioned difficulties disappear; complicated perturbation formulas are not needed, and the terms always automatically appear properly "paired."

[1] This fact has been utilized by F. Bloch, Zeits. f. Physik **74**, 295 (1932), and by E. Wigner, Phys. Rev. **40**, 749 (1932). Their use of it differs from ours in that they employ representations in which coordinates or momenta are diagonal, while we diagonalize the energy.

[2] J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities*, p. 191.

§1. General Method of Solution

Let us suppose, for simplicity, that the Hamiltonian function in the presence of the external field, λ, is of the form

$$H=H_0(p, q)+\lambda v(p, q).$$

Hamiltonian functions in which higher order terms in λ appear can readily be handled by an obvious extension of the method which we use for this most simple, and most important, case. We will employ a system of representation in which H_0 is diagonal[3]; this will be indicated by writing $H_0=W$. The first step is to expand the matrix $e^{-(W+\lambda v)/kT}$, appearing in (1), as a power series in λ. The elements of this matrix are

$$(e^{-(W+\lambda v)/kT})_{n'n}=\delta_{n'n}+\sum_{r=1}^{\infty}\frac{(-1)^r}{r!}\frac{[(W+\lambda v)^r]_{n'n}}{(kT)^r}. \tag{2}$$

Multiplying out the factor $(W+\lambda v)^r$ we find

$$[(W+\lambda v)^r]_{n'n}=W_n{}^r\delta_{n'n}+\lambda v_{n'n}\sum_{s=0}^{r-1}W_{n'}{}^{r-s-1}W_n{}^s+\lambda^2\sum_{n''}v_{n'n''}v_{n''n}\sum_{s=0}^{r-2}\sum_{t=0}^{r-2-s}W_{n'}{}^{r-2-s-t}W_{n''}{}^sW_n{}^t+\cdots$$

$$=W_n{}^r\delta_{n'n}+\lambda v_{n'n}\left[\frac{W_{n'}{}^r}{h\nu_{n'n}}+\frac{W_n{}^r}{h\nu_{nn'}}\right]$$

$$+\lambda^2\sum_{n''}v_{n'n''}v_{n''n}\left[\frac{W_{n'}{}^r}{h^2\nu_{n'n''}\nu_{n'n}}+\frac{W_{n''}{}^r}{h^2\nu_{n''n'}\nu_{n''n}}+\frac{W_n{}^r}{h^2\nu_{nn'}\nu_{nn''}}\right]+\cdots. \tag{3}$$

The latter form is obtained on summing the geometric series involved in the former. Substitution of this in (2) gives[4]

$$(e^{-(W+\lambda v)/kT})_{n'n}=e^{-W_n/kT}\delta_{n'n}+\lambda v_{n'n}\left[\frac{e^{-W_{n'}/kT}}{h\nu_{n'n}}+\frac{e^{-W_n/kT}}{h\nu_{nn'}}\right]$$

$$+\lambda^2\sum_{n''}v_{n'n''}v_{n''n}\left[\frac{e^{-W_{n'}/kT}}{h^2\nu_{n'n''}\nu_{n'n}}+\frac{e^{-W_{n''}/kT}}{h^2\nu_{n''n'}\nu_{n''n}}+\frac{e^{-W_n/kT}}{h^2\nu_{nn'}\nu_{nn''}}\right]+\cdots. \tag{4}$$

The general term in this expansion is

$$\lambda^{\mu}\sum_{n''\cdots n^{(\mu)}}v_{n'n''}\cdots v_{n^{(\mu)}n}\left[\frac{e^{-W_{n'}/kT}}{h^{\mu}\nu_{n'n''}\cdots\nu_{n'n}}+\cdots+\frac{e^{-W_n/kT}}{h^{\mu}\nu_{nn'}\cdots\nu_{nn^{(\mu)}}}\right], \tag{5}$$

as is shown in the appendix. If two or more of the indices in (5) are the same, say $n''=n'''=\cdots=n^{(s)}$ the bracketed factor is an indeterminate form. In this case the bracketed factor is to be understood as meaning its limit as $\nu_{n''n'''}, \cdots, \nu_{n^{(s)}n''}\to 0$. This limit always exists.[5] The advantage of writing these "degenerate" terms as indeterminate forms is that (5), so written, is symmetric in "degenerate" and "low frequency" elements, that is, in elements for which the ν's in the denominator actually vanish, and those in which the ν's are small. The whole aim of Van Vleck's method of "pairing" is to achieve this symmetry.

---

[3] This representation, in general, is not unique, but any representation which makes H_0 diagonal will do.

[4] The term linear in λ has recently been given by I. Waller, Zeits. f. Physik **79**, 370 (1932).

[5] Eq. (8) serves as a simple example. It will be observed that a confluence of two W's always leads to an indeterminate form because of the reversal of indices of the ν's in passing from one term to another.

Using (4), we find

$$\mathrm{Sp}(ue^{-(W+\lambda v)/kT}) = \sum_n \left\{ u_{nn} e^{-W_n/kT} + \lambda \sum_{n'} u_{nn'} v_{n'n} \left[\frac{e^{-W_{n'}/kT}}{h\nu_{n'n}} + \frac{e^{-W_n/kT}}{h\nu_{nn'}} \right] \right.$$
$$\left. + \lambda^2 \sum_{n'n''} u_{nn'} v_{n'n''} v_{n''n} \left[\frac{e^{-W_{n'}/kT}}{h^2 \nu_{n'n''}\nu_{n'n}} + \frac{e^{-W_{n''}/kT}}{h^2 \nu_{n''n'}\nu_{n''n}} + \frac{e^{-W_n/kT}}{h^2 \nu_{nn'}\nu_{nn''}} \right] + \cdots \right\}. \quad (6)$$

Our considerations thus far have been perfectly general. In the remainder of this section two assumptions will be made[6]:

(a) It will be supposed that intermolecular forces are negligible, so that we have complete spatial degeneracy.

(b) It will be supposed that all frequencies of the unperturbed molecule which appear in (6) are large or small compared to kT/h.

Each state will be designated by (n, j, m), where n represents the high frequency quantum numbers, j and m the rotational and axial quantum numbers respectively, which can be considered constants of the motion in consequence of (a).

The term of (6) linear in λ can now be written

$$\lambda \sum_{\substack{njm \\ n'm'j'}}{}' u_{njm;\ n'j'm'} v_{n'j'm';\ njm} \left[\frac{e^{-W_{n'j'}/kT}}{h\nu_{n'j';\ nj}} + \frac{e^{-W_{nj}/kT}}{h\nu_{nj;\ n'j'}} \right] + \lambda \sum_{\substack{njm \\ j'm'}} u_{njm;\ nj'm'} v_{nj'm';\ njm} \left[\frac{e^{-W_{nj'}/kT}}{h\nu_{nj';\ nj}} + \frac{e^{-W_{nj}/kT}}{h\nu_{nj;\ nj'}} \right]. \quad (7)$$

The prime on the first summation sign indicates that $n' = n$ is to be omitted from the summation. On expanding $e^{-W_{nj'}/kT} = e^{-(W_{nj} + h\nu_{nj';\ nj})/kT}$ in powers of $h\nu_{nj';\ nj}/kT$ we find that

$$\left[\frac{e^{-W_{nj'}/kT}}{h\nu_{nj';\ nj}} + \frac{e^{-W_{nj}/kT}}{h\nu_{nj;\ nj'}} \right] = -\frac{1}{2kT} \left[e^{-W_{nj'}/kT} + e^{-W_{nj}/kT} \right] \quad (8)$$

plus terms of order $(h\nu_{nj';\ nj}/kT)^3$ and higher. Since $h\nu_{nj';\ nj}$ is small compared to kT, these high-order terms will be neglected. It will be observed that the above relation holds exactly for the degenerate terms, i.e., for $\nu_{nj';\ nj} \to 0$. Substituting (8) in (7), we obtain, after a simple relabeling of indices,

$$-\lambda \mathrm{Sp} \left\{ \sum_{n'j'm'}{}' \frac{\left[u_{njm;\ n'j'm'} v_{n'j'm';\ njm} + v_{njm;\ n'j'm'} u_{n'j'm';\ njm} \right]}{h\nu_{n'j';\ nj}} e^{-W_{nj}/kT} \right.$$
$$\left. + (1/2kT) \sum_{j'm'} \left[u_{njm;\ nj'm'} v_{nj'm';\ njm} + v_{njm;\ nj'm'} u_{nj'm';\ njm} \right] e^{-W_{nj}/kT} \right\}. \quad (9)$$

The frequencies $\nu_{n'j';\ nj}$ can, with good approximation, be replaced by centroid frequencies $\nu_{n'n}$. As the number of molecules in excited states will be negligible, we can replace Sp by Sp_{jm}.[7] If we introduce the übermatrix notation,[7] we can now rewrite (9) as

$$-\lambda \left\{ \sum_{n'}{}' \frac{\mathrm{Sp}_{jm} \{ [u^{(nn')} v^{(n'n)} + v^{(nn')} u^{(n'n)}] e^{-W^{(n)}/kT} \}}{h\nu_{n'n}} \right.$$
$$\left. + (1/2kT) \mathrm{Sp}_{jm} \{ [u^{(nn)} v^{(nn)} + v^{(nn)} u^{(nn)}] e^{-W^{(n)}/kT} \} \right\}.$$

Higher order terms in (6) can be handled in the same way. In writing the results it is convenient to introduce the abbreviation

$$2\Re \mathrm{Sp}_{jm} [a^{(nn')} b^{(n'n'')} \cdots c^{(n'''n^{IV})} d^{(n^{IV})} e^{-W^{(n)}/kT}]$$
$$= \mathrm{Sp}_{jm} \{ [a^{(nn')} b^{(n'n'')} \cdots c^{(n'''n^{IV})} d^{(n^{IV}n)} + d^{(nn^{IV})} c^{(n^{IV}n''')} \cdots b^{(n''n')} a^{(n'n)}] e^{-W^{(n)}/kT} \}. \quad (10)$$

[6] If these assumptions are not made, we obtain a result which, formally, is only trivially different from (12) (see §5).

[7] This notation is explained immediately above (10) in the preceding paper.

If $a, b, \cdots, c, d$ are Hermitian, $\Re$ means simply "the real part of". Rather than write in all the indices, we shall use the symbol * to indicate an excursion of the indices from the normal state to excited states and back to the normal state again. Thus

$$abcd=a^{(nn')}b^{(n'n''')}c^{(n''n''')}d^{(n'''n)},\ ab*cd=a^{(nn')}b^{(n'n)}c^{(nn'')}d^{(n''n)},\ ab*c*d=a^{(nn')}b^{(n'n)}c^{(nn)}d^{(nn)},\quad (11)$$

and so forth. Carrying out the calculation, we find

$$\begin{aligned}\bar{u}=B\Re\Big[&\mathrm{Sp}_{jm}\{u*e^{-W/kT}\}-\lambda\Big\{\frac{1}{kT}\mathrm{Sp}_{jm}\{u*v*e^{-W/kT}\}+2\sum_{n'}{}'\frac{\mathrm{Sp}_{jm}\{uv*e^{-W/kT}\}}{h\nu_{n'n}}\Big\}\\
&+\lambda^2\Big\{(1/6k^2T^2)\mathrm{Sp}_{jm}\{(2u*v*v+v*u*v)*e^{-W/kT}\}\\
&+\sum_{n'}{}'\Big[\frac{1}{kTh\nu_{n'n}}-\frac{1}{h^2\nu^2_{n'n}}\Big][\mathrm{Sp}_{jm}\{(u*vv+uv*v+v*uv)*e^{-W/kT}\}]+\sum_{n'n''}{}'\frac{\mathrm{Sp}_{jm}\{(2uvv+vuv)*e^{-W/kT}\}}{h^2\nu_{n'n}\nu_{n''n}}\Big\}\\
&-\lambda^3\Big\{(1/12k^3T^3)\mathrm{Sp}_{jm}\{(u*v*v*v+v*u*v*v)*e^{-W/kT}\}+\frac{2}{3}\sum_{n'}{}'\Big[\frac{1}{2k^2T^2h\nu_{n'n}}-\frac{1}{kTh^2\nu^2_{n'n}}+\frac{1}{h^3\nu^3_{n'n}}\Big]\\
&\times[\mathrm{Sp}_{jm}\{(uv*v*v+vu*v*v+v*uv*v+u*vv*v+u*v*vv+v*u*vv)*e^{-W/kT}\}]\\
&+\sum_{n'n''}{}'\Big[\frac{1}{kTh^2\nu_{n'n}\nu_{n''n}}-\frac{\nu_{n'n}+\nu_{n''n}}{h^3\nu^2_{n'n}\nu^2_{n''n}}\Big][\mathrm{Sp}_{jm}\{(uvv*v+u*vvv+uv*vv+v*uvv+vuv*v+vu*vv)*e^{-W/kT}\}]\\
&+2\sum_{n'n''n'''}{}'\frac{\mathrm{Sp}_{jm}\{(uvvv+vuvv)*e^{-W/kT}\}}{h^3\nu_{n'n}\nu_{n''n}\nu_{n'''n}}\Big\}+\cdots\Big],\end{aligned}\quad (12)$$

where

$$\begin{aligned}B=N[\mathrm{Sp}\{e^{-(W+\lambda v)/kT}\}]^{-1}&=N\Big[Z-(\lambda/kT)\mathrm{Sp}_{jm}\{v*e^{-W/kT}\}\\
&\quad+\frac{\lambda^2}{2kT}\Big\{\frac{1}{kT}\mathrm{Sp}_{jm}\{v*v*e^{-W/kT}\}+2\sum_{n'}{}'\frac{\mathrm{Sp}_{jm}\{vv*e^{-W/kT}\}}{h\nu_{n'n}}\Big\}+\cdots\Big]^{-1}\\
&=NZ\Big[1+(\lambda/kT)Z\,\mathrm{Sp}_{jm}\{v*e^{-W/kT}\}+(\lambda^2/k^2T^2)Z^2(\mathrm{Sp}_{jm}\{v*e^{-W/kT}\})^2\\
&\quad-\frac{\lambda^2}{2kT}Z\Big\{\frac{1}{kT}\mathrm{Sp}_{jm}\{v*v*e^{-W/kT}\}+2\sum_{n'}{}'\frac{\mathrm{Sp}_{jm}\{vv*e^{-W/kT}\}}{h\nu_{n'n}}\Big\}+\cdots\Big],\end{aligned}\quad (13)$$

with $Z=[\mathrm{Sp}_{jm}\{e^{-W/kT}\}]^{-1}$.

§2. Sum Rules for Spurs for Diatomic and Polyatomic Molecules

Calculations using (12) and (13) are greatly aided by the fact that the spurs appearing in these equations are rotational invariants. For example, symmetry properties can frequently be used to show that certain terms are zero. Moreover, sum rules are often available for evaluating such spurs. When we deal with diatomic and polyatomic molecules Niessen's sum rules[8] are very useful. These rules hold only with neglect of the high frequency terms in the Hamiltonian function which lead to rotational

[8] K. F. Niessen, Phys. Rev. **34**, 253 (1929).

distortion, Λ type doubling in diatomic molecules, and such small effects, but the influence of these effects on a statistical mean value is generally of negligible importance.

The sum rules for products of two vectors are well known, and need not be repeated. We shall be particularly interested in the sum rules for products of four vectors. If x', y', z' be a system of axes fixed in space, and x, y, z a system fixed in the molecule, and if **R**, **S**, **T**, **U** are any four vectors with components $R_{x'}$, $R_{y'}$, $R_{z'}$, etc., then Niessen's rules show that

$$15\mathrm{Sp}_m(R_{\alpha'}{}^{(nn')}S_{\beta'}{}^{(n'n'')}T_{\gamma'}{}^{(n''n''')}U_{\delta'}{}^{(n'''n)})^{(jj)}=(2j+1)g[[R_\alpha{}^{(nn')}S_\beta{}^{(n'n'')}T_\gamma{}^{(n''n''')}U_\delta{}^{(n'''n)}]], \quad (14)$$

where g is the statistical weight of the normal state of the stationary molecule, and the double bracket symbol is defined by

$$[[R_\alpha S_\beta T_\gamma U_\delta]]=(1/g)\sum_{xyz}\mathrm{Sp}\{aR_xS_xT_xU_x+b[R_xS_xT_yU_y+R_yS_yT_xU_x] \\ +c[R_xS_yT_xU_y+R_yS_xT_yU_x]+d[R_xS_yT_yU_x+R_yS_xT_xU_y]\}, \quad (15)$$

where

$a=1$, $b=-\frac{1}{2}$, $c=2$, $d=-\frac{1}{2}$ when $\alpha=\gamma=x$, $\beta=\delta=z$;

$a=1$, $b=-\frac{1}{2}$, $c=-\frac{1}{2}$, $d=2$ " $\alpha=\delta=x$, $\beta=\gamma=z$;

$a=1$, $b=2$, $c=-\frac{1}{2}$, $d=-\frac{1}{2}$ " $\alpha=\beta=x$, $\gamma=\delta=z$;

$a=3$, $b=1$, $c=1$, $d=1$ " $\alpha=\beta=\gamma=\delta=z$.

In accordance with the convention (11), we have omitted all the indices in writing (15). The symbol $\sum_{xyz}$ indicates a sum of the three terms obtained by cyclic interchange of x, y, z. Quantities like $R_x{}^{(nn')}$ are the matrix elements (or teilmatrices, if the states of the stationary molecule are degenerate) of **R** referred to axes fixed in the molecule. The spur is to be taken only over the normal state.[9]

The relation

$$[[R_xS_xT_xU_x]]-[[R_xS_xT_xU_x]]=[[R_xS_xT_xU_x]]+[[R_xS_xT_xU_x]] \quad (16)$$

follows immediately from (15). This can also be proved directly from the symmetry properties of the spurs by an argument similar to that used in obtaining (8) in the preceding paper.

The utility of sum rules such as (14) is that they enable us to eliminate the Boltzmann factors in (12) and (13). For example

$$\frac{\mathrm{Sp}_{jm}\{R_{\alpha'}S_{\beta'}T_{\gamma'}U_{\delta'}e^{-W/kT}\}}{\mathrm{Sp}_{jm}\{e^{-W/kT}\}}=\frac{\sum_j\mathrm{Sp}_m(R_{\alpha'}{}^{(nn')}S_{\beta'}{}^{(n'n'')}T_{\gamma'}{}^{(n''n''')}U_{\delta'}{}^{(n'''n)})^{(jj)}e^{-W_j/kT}}{g\sum_j(2j+1)e^{-W_j/kT}} \\ =(1/15)[[R_\alpha S_\beta T_\gamma U_\delta]]. \quad (17)$$

All terms in (12) and (13) can be handled in this way.

§3. Influence of a Magnetic Field on the Electric Susceptibility

The analysis of §1 can readily be extended to the case in which two or more external fields are applied simultaneously. If the Hamiltonian function is

$$H=H_0(p,q)+\lambda v(p,q)+\gamma w(p,q),$$

(4) must be modified by replacing

$$\lambda v_{n'n} \quad \text{by} \quad [\lambda v_{n'n}+\gamma w_{n'n}],$$

$$\lambda^2 v_{n'n''}v_{n''n} \quad \text{by} \quad [\lambda^2 v_{n'n''}v_{n''n}+\lambda\gamma(v_{n'n''}w_{n''n}+w_{n'n''}v_{n''n})+\gamma^2 w_{n'n''}w_{n''n}],$$

[9] The normal states of unsymmetrical polyatomic molecules will of course be nondegenerate, so the spur reduces to a single term. The states of diatomic and linear polyatomic molecules will be doubly degenerate, here $g=2$, and Sp means a sum over $+\Lambda$ and $-\Lambda$, or $+\Omega$ and $-\Omega$.

and so forth. The formula corresponding to (12) can be immediately written down if we note that in the reordering of products such as $u_{nn'}v_{n'n''}w_{n''n}$ in passing from (6) to (12) the cyclic order of the factors u, v, w is preserved.

This method will now be applied in calculating the effect of a magnetic field on the electric susceptibility. First let us suppose that the electric and magnetic fields are both directed along the z' axis. Denoting the components of electric and magnetic moment by X', Y', Z', $M_{x'}$, $M_{y'}$, $M_{z'}$ respectively, we have $u=Z'$, $\lambda v=-EZ'$, $\gamma w=-HM_{z'}$. Many terms of (12) and (13) (generalized, of course, as explained above) are immediately seen to vanish. The term independent of E and H in (12) and the terms linear in E and H in (13) are zero, since obviously $\mathrm{Sp}_m\{Z'^{(ni;\ nj)}\}=0$; $\mathrm{Sp}_m\{M_{z'}^{(ni;\ nj)}\}=0$. The term in (12) linear in H is zero because $Z'_{njm;\ n'j'm'}M_{z'n'j'm';\ njm}$ is invariant of a reflection of coordinates in the origin, and under such a reflection this term goes into its own negative. The terms in (12) proportional to H^3 and E^2H, and the term in (13) proportional to EH vanish for the same reason. The terms in (12) proportional to E^2, EH, and H^2 are zero because spurs like $\mathrm{Sp}_m(Z'^{(nn')}M_{z'}^{(n'n'')}Z'^{(n''n)})^{(jj)}$ are rotational invariants, and go into their negatives under a rotation in which $z'\rightarrow -z'$. There remains an expression of the form

$$\chi_E E=\overline{\overline{Z'}}=N[\chi^0{}_E E+bEH^2+cE^3+\cdots]. \tag{18}$$

The quantity $\chi^0{}_E$ is given by the well-known Langevin-Debye equation,[10]

$$\chi^0{}_E=\frac{\mu^2{}_E}{3kT}+\frac{2}{3}\sum_{n'}\frac{P(n'n)^2}{h\nu_{n'n}}. \tag{19}$$

To find b, we have only to replace, in the coefficient of λ^3 in (12), each term, $\{uvvv\}$ for example, by three terms:

$$\{uvvv\}\rightarrow\{uvvw+uvwv+uwvv\}. \tag{20}$$

There is also, in b, one term contributed by B, which is easily seen to be $-(1/2kT)\chi^0{}_E\chi^0{}_M$, with $\chi^0{}_M$ representing the magnetic analogue of (19).

After using (14) and (17) we find

$$\begin{aligned}
b_i=(1/2k^3T^3)&[[Z{*}Z{*}M_i{*}M_i]]\\
&+2\sum_{n'}{}'\left\{\frac{1}{2k^2T^2h\nu_{n'n}}-\frac{1}{kTh^2\nu^2{}_{n'n}}+\frac{1}{h^3\nu^3{}_{n'n}}\right\}\{[[ZZ{*}M_i{*}M_i]]+4[[Z{*}ZM_i{*}M_i]]+[[Z{*}Z{*}M_iM_i]]\}\\
&+2\sum_{n'n''}{}'\left\{\frac{1}{kTh^2\nu_{n'n}\nu_{n''n}}-\frac{\nu_{n'n}+\nu_{n''n}}{h^3\nu^2{}_{n'n}\nu^2{}_{n''n}}\right\}\{2[[ZZM_i{*}M_i]]+2[[Z{*}ZM_iM_i]]+[[ZM_iZ{*}M_i]]\\
&\qquad+[[Z{*}M_iZM_i]]+[[ZZ{*}M_iM_i]]+[[ZM_i{*}M_iZ]]+[[ZM_i{*}ZM_i]]\}\\
&+2\sum_{n'n''n'''}{}'\frac{1}{h^3\nu_{n'n}\nu_{n''n}\nu_{n'''n}}\{2[[ZZM_iM_i]]+2[[ZM_iZM_i]]+[[ZM_iM_iZ]]+[[M_iZZM_i]]\}\\
&\qquad-(1/2kT)\chi^0{}_E\chi^0{}_M,
\end{aligned} \tag{21}$$

where $i=z$. In order to indicate that the electric and magnetic fields are in the same direction we have written $b=b_z$. In the same way it can be shown that when the electric and magnetic fields are applied perpendicular to each other b is given by (21) with $i=x$.

If the magnetic field is applied at an angle θ with the electric field, we have

$$w=-(M_{z'}\cos\theta+M_{x'}\sin\theta).$$

[10] J. H. Van Vleck,[2] p. 186. The quantity $P(n'n)^2$ is defined on p. 1007 of the preceding paper.

Then in (20)

$$ww = \{M_{z'}M_{z'}\cos^2\theta + M_{x'}M_{x'}\sin^2\theta + (M_{z'}M_{x'}+M_{x'}M_{z'})\sin\theta\cos\theta\}.$$

The last term contributes no nonvanishing terms to b, in consequence of the rotational invariance of the spurs, so the general expression for b is simply

$$b = b_z\cos^2\theta + b_x\sin^2\theta.$$

If we neglect all matrix elements of the magnetic moment involving excited states (i.e., all elements except $M^{(nn)}$) it is readily shown that this expression for b is exactly the same as that derived classically by Van Vleck.[11] For a further discussion of the magnitude of the effect and the dependence upon the angle between the fields the reader is referred to the work of Professor Van Vleck.[11]

The quantity c, the saturation term in (18), can be obtained by replacing M_i by Z throughout (21), and $\chi^0{}_M$ by $\chi^0{}_E$, and dividing all except the term $-(1/2kT)\chi^0{}_E{}^2$ by three. This factor three arises because in c only the one term on the left in (20) appears, rather than the three terms on the right. The expression for c obtained in this way agrees with that previously given by Niessen.[8]

The formula for the magnetic moment in the presence of an electric field,

$$\chi_M H = \overline{\overline{M_{z'}}} = N[\chi^0{}_M H + b'E^2H + c'H^3 + \cdots],$$

is found by simply interchanging the rôles of Z and M_i, E and H, in the proceeding calculation.[12] Since (21) is symmetric in the electric and magnetic moments, we obviously have $b' = b$.

§4. Theory of the Kerr Effect

A further modification of the method of §1 must be made when u is an explicit function of λ, $u = u(p, q, \lambda)$. Then u must be expanded in a power series in λ by methods analogous to those used in expanding $e^{-H/kT}$. Eq. (12) now holds with this power series substituted for u.

This situation is encountered when we seek to calculate the Kerr constant. If the static electric field is directed along the z' axis, the Kerr constant is given by[13]

$$K = (2\pi/hE^2)(\overline{\overline{u^{(z)}}} - \overline{\overline{u^{(x)}}}),$$

where, in a system of representation in which $H = H_0 - EZ'$ is diagonal,

$$u^{(z)}{}_{nn} = \sum_{n'} \frac{\nu_{n'n}}{\nu^2{}_{n'n} - \nu^2} Z'_{nn'}Z'_{n'n}, \qquad u^{(x)}{}_{nn} = \sum_{n'} \frac{\nu_{n'n}}{\nu^2{}_{n'n} - \nu^2} X'_{nn'}X'_{n'n}, \tag{22}$$

ν being the frequency of the incident light. The $\nu_{n'n}$ are explicit functions of E. The method of §1 cannot be applied directly, as $u^{(i)}(p, q, E)$, $(i = Z, X)$, is not given, but only its diagonal elements in a particular representation. Our object is thus to determine two functions $u^{(i)}(p, q, E)$ such that, in a representation in which H is diagonal, their diagonal elements are the same as (22). The theorem of spur invariance then assures us that if $u^{(i)}(p, q, E)$ is any such function, the expression (1), with $u = u^{(i)}(p, q, E)$, has the correct value whatever the representation.

Let us suppose, for the moment, that ν is larger than any of the $\nu_{n'n}$. Then $u^{(i)}{}_{nn}$ can be written

$$u^{(i)}{}_{nn} = -\sum_{r=0}^{\infty}\sum_{n'} P^{(i)}{}_{nn'}P^{(i)}{}_{n'n}\frac{\nu_{n'n}{}^{2r+1}}{\nu^{2r+2}} = -\sum_{r=0}^{\infty}\sum_{t=0}^{2r+1}(-1)^t\binom{2r+1}{t}\frac{\{\sum_{n'}P^{(i)}{}_{nn'}P^{(i)}{}_{n'n}H_{n'}{}^{2r+1-t}H_n{}^t\}}{h^{2r+1}\nu^{2r+2}}, \tag{23}$$

[11] J. H. Van Vleck,[2] p. 113.

[12] The expression for c' obtained in this way is correct if the spin multiplets are large compared to kT/h. If the spin multiplets are small, c' gives the contribution of the orbital part of the moment correctly, but the spin moment must be treated separately. The contribution of the spin moment is representable by a Brillouin function (see reference 2, p. 257; or reference 8).

[13] Born and Jordan, *Elementare Quantenmechanik*, p. 262.

since

$$\nu_{n'n}{}^{2r+1}=\frac{1}{h^{2r+1}}[H_{n'}-H_n]^{2r+1}=\sum_{t=0}^{2r+1}(-1)^t\binom{2r+1}{t}\frac{H_{n'}{}^{2r+1-t}H_n{}^t}{h^{2r+1}}.$$

Here $P^{(Z)}=Z'$, $P^{(X)}=X'$. The factor in the curly bracket can be written in the symmetrical form

$$\tfrac{1}{2}(H^tP^{(i)}H^{2r+1-t}P^{(i)}+P^{(i)}H^{2r+1-t}P^{(i)}H^t)_{nn}.$$

Thus the desired functions, $u^{(i)}(p, q, E)$, are given by

$$u^{(i)}=-\tfrac{1}{2}\sum_{r=0}^{\infty}\sum_{t=0}^{2r+1}(-1)^t\binom{2r+1}{t}\frac{[H^tP^{(i)}H^{2r+1-t}P^{(i)}+P^{(i)}H^{2r+1-t}P^{(i)}H^t]}{h^{2r+1}\nu^{2r+2}}.$$

We can now return to a representation in which H_0 is diagonal. In this unperturbed representation $u^{(i)}$ can be expanded as a power series in E; using (3), and reversing the steps of (23), we find

$$u^{(i)}{}_{nn'}=\tfrac{1}{2}\sum_{n''}[\tau_{n''n}+\tau_{n''n'}]P^{(i)}{}_{nn''}P^{(i)}{}_{n''n'}-\frac{E}{2}\sum_{n''n'''}\left\{\left[\frac{\tau_{n''n}-\tau_{n''n'''}}{h\nu_{nn'''}}\right]Z'{}_{nn'''}P^{(i)}{}_{n'''n''}P^{(i)}{}_{n''n'}\right.$$

$$+\left[\frac{\tau_{n''n'}-\tau_{n''n'''}}{h\nu_{n'n'''}}\right]P^{(i)}{}_{nn''}P^{(i)}{}_{n''n'''}Z'{}_{n'''n'}+\left[\frac{\tau_{n''n'}-\tau_{n'''n'}+\tau_{n''n}-\tau_{n'''n}}{h\nu_{n''n'''}}\right]P^{(i)}{}_{nn''}Z'{}_{n''n'''}P^{(i)}{}_{n'''n'}\Big\}$$

$$+\delta_{nn'}E^2\Re\sum_{n''n'''n\mathrm{IV}}\left\{\left[\frac{\tau_{n''n\mathrm{IV}}}{h^2\nu_{n\mathrm{IV}n'''}\nu_{n\mathrm{IV}n}}+\frac{\tau_{n''n'''}}{h^2\nu_{n'''n'}\nu_{n'''n}}+\frac{\tau_{n''n}}{h^2\nu_{nn'''}\nu_{n\,n\mathrm{IV}}}\right]P^{(i)}{}_{nn''}P^{(i)}{}_{n''n\mathrm{IV}}Z'{}_{n\mathrm{IV}n'''}Z'{}_{n'''n}\right.$$

$$+\left[\frac{\tau_{n''n}}{h^2\nu_{n''n\mathrm{IV}}\nu_{n''n'''}}+\frac{\tau_{n\mathrm{IV}n}}{h^2\nu_{n\mathrm{IV}n''}\nu_{n\mathrm{IV}n'''}}+\frac{\tau_{n'''n}}{h^2\nu_{n'''n''}\nu_{n'''n\mathrm{IV}}}\right]P^{(i)}{}_{nn''}Z'{}_{n''n\mathrm{IV}}Z'{}_{n\mathrm{IV}n'''}P^{(i)}{}_{n'''n}$$

$$\left.\left.+\left[\frac{\tau_{n''n\mathrm{IV}}-\tau_{n''n}+\tau_{n'''n}-\tau_{n'''n\mathrm{IV}}}{h^2\nu_{n''n'''}\nu_{n\mathrm{IV}n}}\right]P^{(i)}{}_{nn''}Z'{}_{n''n'''}P^{(i)}{}_{n'''n\mathrm{IV}}Z'{}_{n\mathrm{IV}n}\right\}+\cdots,\quad(24)\right.$$

where

$$\tau_{n''n}=\nu_{n''n}/(\nu^2{}_{n''n}-\nu^2).$$

Eq. (24) gives the nondiagonal elements of $u^{(i)}$ to terms in E, the diagonal elements to terms in E^2. The $\nu_{n'n}$'s appearing in this equation refer to the unperturbed molecule, whereas those in (22) refer to the molecule in the presence of the electric field. The remark made under (5) concerning degenerate terms also applies to (24). As before, we can treat degenerate and low frequency terms similarly, and remove all small ν's from the denominators of (24) by expanding difference quotients such as

$$(\tau_{n''j'';\,nj}-\tau_{n''j'';\,nj'''})/\nu_{nj;\,nj'''}$$

as power series in $\nu_{nj;\,nj'''}/(\nu^2{}_{n''j'';\,nj}-\nu^2)$. The resulting expressions are to be substituted in (12). The calculations are rather lengthy, but we finally find

$$K=\frac{4\pi N}{15}\left\{\sum_{n''}[\tau_{n''}A_{n''}+\omega_{n''}B_{n''}+\chi_{n''}C_{n''}]+\frac{1}{kT}\sum_{n''}[\tau_{n''}D_{n''}+\omega_{n''}E_{n''}]+\frac{1}{k^2T^2}\sum_{n''}\tau_{n''}F_{n''}\right\},\quad(25)$$

where

$$\tau_{n''}=\frac{\nu_{n''n}}{\nu^2{}_{n''n}-\nu^2};\qquad\omega_{n''}=\frac{\nu^2{}_{n''n}+\nu^2}{(\nu^2{}_{n''n}-\nu^2)^2};\qquad\chi_{n''}=\frac{\nu_{n''n}(\nu^2{}_{n''n}+3\nu^2)}{(\nu^2{}_{n''n}-\nu^2)^3};$$

$$A_{n''}=\frac{1}{h^3}\sum_{n'}'\left\{\sum_{n'''}'\left[\frac{2c(n'n'''n'')+a(n'n''n''')}{\nu_{n'''n}\nu_{n'n}}+\frac{2a(n''n'n''')+a(n'n''n''')}{\nu_{n''n'}\nu_{n'''n''}}+\frac{b(n'n'''n'')+b(n'n''n''')}{\nu_{n'''n''}\nu_{n'n}}\right]\right.$$
$$\left.-\frac{2a(n'n''n'')+a(n''n'n'')}{\nu^2_{n'n''}}-\frac{2c(nn'n'')+c(n'nn'')}{\nu^2_{n'n}}\right\};$$

$$B_{n''}=\frac{1}{h^3}\sum_{n'}'\left\{\frac{2b(n'n''n'')-a(nn''n')-c(n'nn'')-c(nn'n'')}{\nu_{n'n}}\right.$$
$$\left.+\frac{b(nn'n'')+b(nn''n')-a(n'n''n'')-\frac{1}{2}a(n''n'n'')}{\nu_{n''n'}}\right\};$$

$$C_{n''}=(1/h^3)[c(nnn'')+a(n''n''n'')-2b(nn''n'')];$$

$$D_{n''}=\frac{1}{h^2}\sum_{n'}'\left\{\frac{c(n'nn'')+c(nn'n'')}{\nu_{n'n}}+\frac{b(nn''n')+b(nn'n'')}{\nu_{n'n''}}\right\};$$

$$E_{n''}=(1/h^2)[b(nn''n'')-c(nnn'')];\qquad F_{n''}=(1/6h)[2c(nnn'')+b(nn''n)];$$

$$a(n'n''n''')=[[ZZZZ]]-[[ZXXZ]];\qquad b(n'n''n''')=[[ZZZZ]]-[[ZXZX]];$$
$$c(n'n''n''')=[[ZZZZ]]-[[ZZXX]].$$

Although in the derivation of (25) it was formally necessary to suppose ν larger than any of the $\nu_{n'n}$, the actual restriction on ν is far less stringent. The expansion of

$$\frac{\nu_{n''n}}{\nu^2_{n''n}-\nu^2}=\frac{\nu^{(0)}{}_{n''n}+\nu^{(2)}{}_{n''n}E^2}{(\nu^{(0)}{}_{n''n}+\nu^{(2)}{}_{n''n}E^2)^2-\nu^2}$$

as a power series in E really requires only that $\nu^{(0)}{}_{n''n}-\nu$ be large compared to the Stark effect. In removing the small ν's from the denominators of (24) and replacing the $\nu_{n'j';\,nj}$ by centroid frequencies, $\nu_{n'n}$, we introduce the further restriction that $\nu_{n'j';\,nj}-\nu$ be large compared to $\nu_{n'j';\,n'j''}$. Thus (25) is valid whenever ν is not too close to any absorption line of the molecule. If, among the excited states, there are overlapping band systems, we must be more cautious in defining our centroid frequencies, as factors $1/\nu_{n'n''}$ appear in (25). Here the method of dividing all states into "groups," previously used by the writer,[14] must be employed. This entails no change in the equations, but merely a modification in the interpretation of the notation.

A further discussion of (25) will be found in the preceding paper.

The Faraday effect can easily be handled by the same method. The results are identical with those already found by the writer, using perturbation theory.[14]

§5. Perturbation Theory of Conservative Systems

A general formula for the diagonal elements of u in a system of representation in which H is diagonal, in terms of the unperturbed matrix elements of u and v, can be obtained by using the results of §1. By "unperturbed" matrix elements we mean, of course, the elements in a representation in which H_0 is diagonal. We will suppose the unperturbed system nondegenerate; the extension to degenerate systems proceeds as in the usual perturbation theory. Consider the quantity $\mathrm{Sp}\{ue^{-H/kT}\}$ in a representation in which H is diagonal. If we expand u and the energy as power series in λ, we have

[14] R. Serber, Phys. Rev. **41**, 489 (1932).

$$\mathrm{Sp}\{ue^{-H/kT}\}=\sum_n [u_{nn}{}^{(0)}+\lambda u_{nn}{}^{(1)}+\lambda^2 u_{nn}{}^{(2)}+\cdots]e^{-[W_n+\lambda W_n{}^{(1)}+\lambda^2 W_n{}^{(2)}+\cdots]/kT}$$

$$=\sum_n [u_{nn}{}^{(0)}+\lambda u_{nn}{}^{(1)}+\lambda^2 u_{nn}{}^{(2)}+\cdots]\left[1-\lambda\frac{W_n{}^{(1)}}{kT}+\lambda^2\left(\frac{W_n{}^{(1)2}}{2k^2T^2}-\frac{W_n{}^{(2)}}{kT}\right)+\cdots\right]e^{-W_n/kT}$$

$$=\sum_n [u_{nn}{}^{(0)}+\lambda u_{nn}{}^{(1)}+\lambda^2 u_{nn}{}^{(2)}+\cdots+\sum_{s=1}^{\infty}(kT)^{-s}(\cdots)]e^{-W_n/kT}. \tag{26}$$

But in §1 we have evaluated this same quantity in terms of the matrix elements of u and v in an unperturbed system of representation. The result, in fact, is given by the series (12) if we set $B=1$, interpret quantities like $u^{(nn')}$ as matrix elements, rather than teilmatrices, and replace Sp_{jm} by a summation over n. It is readily seen that (12), with these modifications, follows rigorously from (6). Since the identity of (26) and (12) holds for all values of λ and T, and since $W_{n'}\neq W_n$ when $n'\neq n$, we see that $u_{nn}{}^{(r)}$ is the coefficient of $\lambda^r e^{-W_n/kT}$ in the series (12). The general term of (12) depends on λ and T through the factor $\lambda^r e^{-W_n/kT}/(kT)^s$; it must be understood that "the coefficient of $\lambda^r e^{-W_n/kT}$" refers only to the term which has $s=0$.

This result can readily be extended to the case where two or more fields are applied simultaneously, by using the method described in §3.

The perturbed values of the energy can of course be found by setting $u=H$. However it is much simpler to consider

$$\mathrm{Sp}\{e^{-[W_n+\lambda W_n{}^{(1)}+\cdots]/kT}\}=\sum_n[1-(1/kT)(\lambda W_n{}^{(1)}+\lambda^2 W_n{}^{(2)}+\cdots)+\sum_{s=2}^{\infty}(kT)^{-s}(\cdots)]e^{-W_n/kT},$$

and compare this with (13). The modifications in the interpretation of (12) mentioned above also apply to (13). Thus we find that $-W_n{}^{(r)}$ is the coefficient of $\lambda^r e^{-W_n/kT}/kT$ in the series for $\mathrm{Sp}\{e^{-(W+\lambda v)/kT}\}$ given implicitly in (13). In this way one can readily check the formula for $W_n{}^{(3)}$ given by Born, Heisenberg and Jordan,[15] and the formula for $W_n{}^{(4)}$ given by Niessen.[8]

§6. Perturbation Theory of Non-Conservative Systems

An expansion similar to (4) is useful in the solution of problems of the following type. Consider a dynamical system, whose Hamiltonian function is $H_0(p, q)$, in a stationary state ψ_n. Suppose the system is suddenly subjected to a perturbation at time $t=0$, such that, at all subsequent times, the Hamiltonian function is $H=H_0(p, q)+\lambda v(p, q)$. We wish to find the transformation matrix which gives the state of the system, $\psi_n(t)$, at time t, in terms of the eigenfunctions of the system at time $t=0$; that is, the matrix $S(t)$ satisfying the boundary condition $S(0)=1$, and

$$\psi_n(t)=\sum_{n'} S_{n'n}(t)\psi_{n'}(0),$$

where the $\psi_{n'}(0)$ are solutions of

$$H_0\psi_{n'}=W_{n'}\psi_{n'}.$$

The transformation matrix $S(t)$ satisfies the Schrödinger equation

$$(h/2\pi i)\dot{S}+HS=0,$$

in a system of representation in which H_0 is diagonal. It is easily verified that a solution of this equation, which obviously satisfies the boundary condition, is

$$S(t)=e^{-xH}=e^{-x(W+\lambda v)},$$

[15] Born, Heisenberg, and Jordan, Zeits. f. Physik **35**, 557 (1926).

where

$$\chi = 2\pi i t/h.$$

Replacing $1/kT$ by χ in (4), we obtain an expansion of $S(t)$ as a power series in λ.[16]

Appendix. Proof of (5)

To prove (5) we must evaluate the coefficient of λ^μ in $[(W+\lambda v)^r]_{n'n}$. This coefficient is

$$\lambda^\mu \sum_{n'', n''', \cdots, n^{(\mu)}} v_{n'n''} v_{n''n'''} \cdots v_{n^{(\mu)}n} \left\{ \sum_{s=0}^{j} \sum_{t=0}^{j-s} \cdots \sum_{z=0}^{j-s-t-\cdots-y} W_{n'}{}^{s} W_{n''}{}^{t} \cdots W_{n^{(\mu)}}{}^{z} W_{n}{}^{j-s-t-\cdots-z} \right\}, \tag{27}$$

where $j = r-\mu$. It will be shown, by induction, that the bracketed factor is given by

$$\{\quad\} = \frac{W_{n'}{}^{j+\mu}}{h^\mu v_{n'n''} v_{n''n'''} \cdots v_{n'n}} + \frac{W_{n''}{}^{j+\mu}}{h^\mu v_{n''n'} v_{n''n'''} \cdots v_{n''n}} + \cdots + \frac{W_{n}{}^{j+\mu}}{h^\mu v_{nn'} v_{nn''} \cdots v_{nn^{(\mu)}}}. \tag{28}$$

Suppose this holds when μ is replaced by $\mu-1$. Then the left side of (28) equals

$$\sum_{s=0}^{j} W_{n'}{}^{s} \left[\frac{W_{n''}{}^{j+\mu-1-s}}{h^{\mu-1} v_{n''n'''} \cdots v_{n''n}} + \cdots + \frac{W_{n}{}^{j+\mu-1-s}}{h^{\mu-1} v_{nn''} \cdots v_{nn^{(\mu)}}} \right] = -W_{n'}{}^{j+1} \left\{ \frac{W_{n''}{}^{\mu-1}}{h^\mu v_{n''n'} v_{n''n'''} \cdots v_{n''n}} + \cdots \right.$$

$$\left. + \frac{W_{n}{}^{\mu-1}}{h^\mu v_{nn'} v_{nn''} \cdots v_{nn^{(\mu)}}} \right\} + \frac{W_{n''}{}^{j+\mu}}{h^\mu v_{n''n'} v_{n''n'''} \cdots v_{n''n}} + \cdots + \frac{W_{n}{}^{j+\mu}}{h^\mu v_{nn'} v_{nn''} \cdots v_{nn^{(\mu)}}}, \tag{29}$$

the latter form being found on summing the geometric series involved in the former.

The equation

$$\left\{ \frac{W_{n''}{}^{\mu-1}}{h^\mu v_{n''n'} \cdots v_{n''n}} + \cdots + \frac{W_{n}{}^{\mu-1}}{h^\mu v_{nn'} \cdots v_{nn^{(\mu)}}} \right\} = -\frac{W_{n'}{}^{\mu-1}}{h^\mu v_{n'n''} v_{n'n'''} \cdots v_{n'n}}$$

follows from the fact that the term on the right, considered as a function of $W_{n'}$, is a rational function, and has simple poles at the points $W_{n'} = W_j$, $(j \neq n')$. Thus (29) is the same as the right side of (28). Since (28) holds for $\mu = 2$, as is shown by (3), the proof is complete. Substituting (28) in (27), and (27) in (2), and remembering that $j+\mu = r$, we obtain (5).

In this proof we have supposed $W_i \neq W_j$, $(i \neq j)$, but (28) is still valid, if interpreted as indicated under (5), when $W_j = W_k = \cdots = W_l = W'$, as the left side of (28) is a continuous function of the W_i.

I am indebted to Professor J. H. Van Vleck for many helpful discussions.

[16] The analogy between $1/kT$ and χ has been pointed out by F. Bloch.[1] Formulae for $S^{(1)}$ and $S^{(2)}$ are given by Born and Jordan,[13] p. 238.

Magnetic Dipole Energy in Homogeneously Strained Cubic Crystals

L. W. McKeehan, *Sloane Physics Laboratory, Yale University*

(Received March 27, 1933)

The magnetic potential energy per dipole in homogeneously strained, originally perfect, cubic arrays of parallel equal dipoles is computed in two ways. Results for a few of these cases, previously treated by Becker and by Akulov in essentially these two ways, are corrected. A check formula, due to Powell, is given in general terms and used in proving the precision of the final results.

IN two previous papers[1] we have discussed magnetic fields and magnetic potential energy in cubic arrays of parallel equal dipoles and in arrays derived from these by a small translation of a definite group of dipoles. It now remains to consider the fields which arise when an originally perfect cubic array is homogeneously strained in any way. Parts of this problem have already been solved by Akulov,[2] Becker[3] and Powell,[4] but we will find it necessary to correct slightly all the numerical results so far published.

Just as we found two methods available in computing the magnetic field in an unstrained cubic array of dipoles we find two methods applicable under the new conditions. Becker's method is analogous to that of Lorentz. It depends upon the evaulation of sums which are only conditionally convergent, though this was not apparently recognized at first. Akulov's method is derived directly from Kornfeld's development[5] of Ewald's method. Powell has shown that the results obtained by Becker and Akulov are not inconsistent if proper account is taken of boundary conditions which are only implicitly contained in their equations. We will see that the agreement is better when both types of computation are carried out more precisely.

Becker's Method

The crystal specimen is originally a sphere, so that it becomes an ellipsoid when homogeneously strained. The magnetic potential energy factor per dipole increases on account of the strain by an amount Δu which may be written

$$\Delta u = \sum_{ij} C_{ij} A_{ij} p_i p_j, \tag{1}$$

in which $\mathbf{p}$ is a unit vector parallel to the magnetization, the A_{ij} are the components of the strain tensor, and the C_{ij} are numerical coefficients characteristic of the array, but dependent upon the boundary conditions. Becker shows that for the boundary conditions here imposed

$$\Delta u = C\{\sum_i A_{ii}(1-3p_i^2)+\sum_{i\neq j} A_{ij}(2p_ip_j)\}, \tag{2}$$

so that but one coefficient is needed for each array. This coefficient

$$C = \tfrac{3}{4}\sum{}'[r^{-3}-5r_1{}^4r^{-7}] \tag{3}$$

is a summation over all dipoles (except for the dipole at the origin) which lay within a convenient sphere in the unstrained crystal. In using (2) we must remember that $\sum_{i\neq j} A_{ij}(2p_ip_j)$ includes two terms for each pair i, j, e.g., $A_{12}(2p_1p_2)$ and $A_{21}(2p_2p_1)$. It is also to be noted that $A_{ij}=A_{ji}$, so that there are but 6 independent tensor components. In using (3) we remember that $\mathbf{r}$ is expressed in terms of a_0 and that selection rules given in Table I of our first paper[1] govern the appropriate values of r and r_1 in each of the arrays (S), (I) and (F).

The cubic symmetry of the $\mathbf{r}$ lattice allows us to reduce the number of separately computed terms in C. Putting n_r for the number of dipoles which have r_i of a given form, so that n_r is 6, 8, 12, 24 or 48, we obtain

[1] L. W. McKeehan, Phys. Rev. [2] **43**, 913–930 (1933).
[2] N. S. Akulov, Zeits. f. Physik **52**, 389–405 (1928).
[3] R. Becker, Zeits. f. Physik **62**, 253–269 (1930).
[4] F. C. Powell, Proc. Camb. Phil. Soc. **27**, 561–569 (1931).
[5] H. Kornfeld, Zeits. f. Physik **22**, 27–43 (1924).

TABLE I. *Summations by Becker's method.*

(S)			(I)			(F)		
r^2	ΣN	C	$4r^2$	ΣN	C	$4r^2$	ΣN	C
1	6	−0.37500	3	8	0.51320	2	12	0.53033
2	18	− .30871	4	14	.13820	4	18	.15533
3	26	− .24456	8	26	.20449	6	42	.35945
4	32	− .29143	11	50	.13381	8	54	.42575
5	56	− .31827	12	58	.19796	10	78	.21704
6	80	− .29275	16	64	.15109	12	86	.28119
8	92	− .28446	19	88	.20487	14	134	.39573
9	122	− .27160	20	112	.17804	16	140	.34885
10	146	− .29769	24	136	.20356	18	176	.29138
11	170	− .30653	27	168	.16695	20	200	.26455
12	178	− .29851	32	180	.17524	22	224	.33206
13	202	− .29643	35	228	.18186	24	248	.35758
14	250	− .28211	36	258	.19472	26	320	.32906
16	256	− .28797	40	282	.16863	30	368	.28768
17	304	− .29226	43	306	.18718	32	380	.29596
18	340	− .29945	44	330	.17835	34	428	.33042
19	364	− .29272	48	338	.18637	36	458	.34328
20	388	− .29608	51	386	.16949	38	530	.33061
21	436	− .29756	52	410	.17158	40	554	.30452
22	460	− .28912	56	458	.18589	42	602	.32656
24	484	− .28593	59	530	.18530	44	626	.31773
25	514	− .28711	64	536	.17944	46	674	.30788
26	586	− .29067	67	560	.18103	48	682	.31590
27	618	− .29525	68	608	.17673	50	766	.32604
29	690	− .29049	72	644	.16955	52	790	.32813
30	738	− .29566	75	700	.17937	54	886	.31357
32	750	− .29462	76	724	.18609	56	934	.32789
33	798	− .29214	80	748	.18274	58	958	.31853
34	846	− .28783	83	820	.18057	62	1054	.32469
35	894	− .28701	84	868	.17909	64	1060	.31883
36	924	− .28540	88	892	.18753	66	1156	.31285
37	948	− .28619	91	940	.17355	68	1202	.30856
38	1020	− .29277	96	964	.17674	70	1250	.32817
40	1044	− .29603	99	1036	.18440	72	1286	.32098
			100	1066	.18322	74	1406	.31728
	Limit	−0.29310	104	1138	.17966	76	1430	.32401
			107	1210	.18286	78	1478	.33272
			108	1242	.17828	80	1502	.32936
			115	1290	.18074	82	1550	.31283
			116	1362	.18550	84	1598	.31135
			120	1410	.18033	86	1718	.30672
			123	1458	.17754	88	1742	.31516
			128	1470	.17858	90	1862	.32231
			131	1590	.17427	94	1958	.32356
			132	1638	.17675	96	1982	.32675
			136	1686	.18106	98	2090	.32768
			139	1758	.18285	100	2120	.32651
			140	1806	.18367	102	2168	.31997
			144	1836	.18528	104	2240	.31641
			147	1892	.18319	106	2312	.31455
			148	1916	.17740	108	2344	.30998
			152	1988	.17582	110	2488	.31072
			155	2084	.18110	114	2584	.32550
			160	2108	.17784	116	2656	.33026
						118	2728	.32889
				Limit	0.17995	120	2776	.32372
						122	2896	.32667
						126	3040	.31425
						128	3052	.31529
						130	3100	.31106
						132	3148	.31354
						134	3316	.31405
						136	3364	.31836
						138	3460	.32044
						140	3508	.32127
						142	3556	.32763
						144	3586	.32924
						146	3778	.32696
						148	3802	.32117
						150	3922	.32181
						152	3994	.32023
						154	4090	.31613
						158	4186	.31799
						160	4210	.31473
							Limit	0.31998

$$C=\tfrac{1}{4}\sum{}'n_r[3r^{-3}-5(r_1^4+r_2^4+r_3^4)r^{-7}], \quad (4)$$

where $r_1r_2r_3$ are particular values of r_i for one of the n_r points of a given form.

Table I gives the values of C which are obtained as we enlarge the sphere within which the summation is carried out. The limits to which C converges as this sphere increases still further are also given. They are derived from the results by Akulov's method, discussed in the next section, using formulae derived by Powell.

Becker gives, for (I), $S=2C=0.4$ and, for (F), $S=2C=0.6$. He does not state how many dipoles he summed over in either case.

AKULOV'S METHOD

The crystal specimen is now supposed to extend to infinity in all directions. (It may be remarked that the results also hold for an infinite cylinder with its axis parallel to **p**.) We rewrite Akulov's formula in the following form:

$$\Delta u=\sum_i A_{ii}(C_0+C_1p_i^2)+\sum_{i\neq j} A_{ij}(2C_0p_ip_j). \quad (5)$$

As in (2) terms for A_{ij} and A_{ji} both occur in the second summation.

We break up C_0 and C_1 into several terms each:

$$C_0=C_{0r4}+C_{0r2}+C_{0q4}+C_{0q2}, \quad (6)$$

$$C_1=C_{1r4}+C_{1r2}+C_{1q4}+C_{1q2}. \quad (7)$$

Each term is a convergent summation.

$$C_{0r4}=\epsilon^7\sum{}'r_i^2r_j^2g_3(\epsilon r), \quad (8)$$

$$C_{0r2}=-\epsilon^5\sum{}'r_i^2g_2(\epsilon r), \quad (9)$$

$$C_{1r4}=\epsilon^7\sum{}'r_i^4g_3(\epsilon r)-C_{0r4}, \quad (10)$$

$$C_{1r2}=2C_{0r2}, \quad (11)$$

$$C_{0q4}=8\pi\sum{}'q_i^2q_j^2q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2)\cos 2\pi\{q_1s_1+q_2s_2+q_3s_3\}, \tag{12}$$

$$C_{0q2}=-4\pi\sum{}'q_i^2q^{-2}\exp(-\pi^2q^2/\epsilon^2)\cos 2\pi\{q_1s_1+q_2s_2+q_3s_3\}, \tag{13}$$

$$C_{1q4}=8\pi\sum{}'q_i^4q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2)\cos 2\pi\{q_1s_1+q_2s_2+q_3s_3\}-C_{0q4}, \tag{14}$$

$$C_{1q2}=2C_{0q2}. \tag{15}$$

In these expressions all symbols have the meanings explained previously.[1] Exactly as before we reduce the labor of computation by introducing the same selection rules for q_i and in these cases one value of $\mathbf{s}$ is always zero so that (12), (13), (14) reduce to

$$C_{0q4}=8\pi N\sum{}'fq_i^2q_j^2q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2), \tag{16}$$

$$C_{0q2}=-4\pi N\sum{}'fq_i^2q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{17}$$

$$C_{1q4}=8\pi N\sum{}'fq_i^4q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2)-C_{0q4}. \tag{18}$$

The cubic symmetry of both $\mathbf{r}$ and $\mathbf{q}$ groups permits further simplification. If n_r and n_q are put for the number of points for constant r and constant q respectively we get

$$C_{0q4}=(8\pi N/3)\sum{}'fn_q(q_2^2q_3^2+q_3^2q_1^2+q_1^2q_2^2)q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2), \tag{19}$$

$$C_{0q2}=-(4\pi N/3)\sum{}'fn_q\exp(-\pi^2q^2/\epsilon^2), \tag{20}$$

$$C_{1q4}=(8\pi N/3)\sum{}'fn_q(q_1^4+q_2^4+q_3^4)q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2)-C_{0q4}, \tag{21}$$

$$C_{0r4}=(\epsilon^7/3)\sum{}'n_r(r_2^2r_3^2+r_3^2r_1^2+r_1^2r_2^2)g_3(\epsilon r), \tag{22}$$

$$C_{0r2}=-(\epsilon^5/3)\sum{}'n_rr^2g_2(\epsilon r), \tag{23}$$

$$C_{1r4}=(\epsilon^7/3)\sum{}'n_r(r_1^4+r_2^4+r_3^4)g_3(\epsilon r)-C_{0r4}. \tag{24}$$

Table II contains the results for arrays (S), (I), (F) and (D). The values of ϵ used in computation were $2^{\frac{1}{2}}$ and 2 or 2 and 3. Akulov's values for (S) and (I) are as follows in our notation:

For (S)	$2C_0=-2.189$,	$2C_1=19.05$,	$4C_0=-4.387$,
For (I)	$2C_0=7.86$,	$2C_1=1.35$,	$4C_0=15.84$.

TABLE II. *Summations by Akulov's method.*

Array	C_0	C_1	C
(S)	−1.07819	9.51775	−0.29310
(I)	3.95289	0.70768	0.17995
(F)	7.58638	2.37364	0.31998
(D)	27.71070	−32.86662	2.20720

Two values are quoted for C_0 because Akulov computed the third coefficient in our Eq. (7) without observing that it should have been exactly twice as great as the first coefficient. The other cases have not previously been treated.

The following equation, due to Powell,[4] affords excellent checks on the values of C_0 and C_1, which were computed independently, and yields better values of C than can be obtained by Becker's method.

$$C_0-2\pi N/5=4\pi N/15-C_1/3=8C. \tag{25}$$

The values of C calculated from this formula (both results agree to one unit in the fifth decimal place) are included in Table II.

Magnetic Dipole Energy in Hexagonal Crystals

L. W. McKEEHAN, *Sloane Physics Laboratory, Yale University*

(Received March 27, 1933)

The magnetic fields at dipole positions in two hexagonal arrays of parallel dipoles are computed by two methods. One of these methods is also used to find the field in hexagonal close packing at a point on the hexagonal axis near a vacated dipole position. The hexagonal close-packed arrangement with axial ratio $c=2\times 6^{\frac{1}{2}}/3$ (as for spheres) is found to be magnetically stable by a small margin when magnetization is along the hexagonal axis. The general expressions for the energy per dipole in homogeneously strained hexagonal arrays are solved and applied to two arrays, one simple hexagonal (*H*), the other close-packed hexagonal (*HC*), both with $c=2\times 6^{\frac{1}{2}}/3$. The results are applicable in cases where c differs slightly from this value.

IN previous papers[1] we have considered the magnetic fields in cubic or nearly cubic arrays of parallel dipoles. We here extend the discussion to certain hexagonal arrays. These are of particular interest in view of the magnetic behavior of hexagonal cobalt crystals, in which the sixfold axis ($\mathbf{a}_3$) is the only easy direction of magnetization.

In unstrained arrays the method of Lorentz is at once applicable in the form already presented, but some assumption must be made regarding the relative dimensions of the unit of structure. This unit is a right prism with rhombic base, the internal angles of the rhombus being 120° and 60°. The origin is taken at an obtuse corner of the base, $\mathbf{a}_1$ and $\mathbf{a}_2$ are edges of the base and $\mathbf{a}_3$ is the altitude of the prism. In order to include the atomic arrangement of hexagonal cobalt, which is, within experimental error, that of a close-packing of spheres, without special assumptions at a later stage we choose the ratio $a_3/a_1=2\times 6^{\frac{1}{2}}/3$. With this particular choice we can if desired use a rectangular system of coordinate axes in fixing dipole positions, and can express their coordinates in this system as an integral multiple r_i'' of a pseudo-cubic parameter $a_0''=a_1\,2^{\frac{1}{2}}/6$. These rectangular axes, $\mathbf{a}_i''$, are equally inclined to $\mathbf{a}_3$ and lie in planes perpendicular to $\mathbf{a}_1$, $\mathbf{a}_2$, and $-(\mathbf{a}_1+\mathbf{a}_2)$. The last of these vectors is an auxiliary axis in the basal plane, which is used in designating planes by Miller-Bravais indices.

With respect to the rectangular axes just mentioned the selection rule for the points of a simple hexagonal lattice (*H*), for which $N=1$, is as follows:

$$r_i''=3n_i+4;\quad \sum_i r_i''=12n\geqq 0, \tag{1}$$

$$r_i''=3n_i-4;\quad \sum_i r_i''=12n<0. \tag{2}$$

The additional points for hexagonal close-packing (*HC*), $N=2$, which does not constitute a single space-lattice, are obtained by adding to these all points for which

$$r_i''=3n_i+4;\quad \sum_i r_i''=6n>0, \tag{3}$$

$$r_i''=3n_i-4;\quad \sum_i r_i''=6n<0. \tag{4}$$

The easy direction of magnetization, here most naturally assumed for the direction of $\mathbf{p}$, has pseudo-cubic indices [111].

It is no longer necessary, as in cubic arrays, that the local field vanish at a dipole position, so we proceed to calculate $\sum'\delta h_i''$ for the origin as well as for a nearby point on the principal axis. Table I shows how much the sums fluctuate as the limiting sphere increases. It is not possible, from these computations, to fix even the sign of the limiting value with any certainty though the difference between nearby points converges well enough. It is clear therefore that the method of Ewald[1] must here be relied upon. Its application requires the construction of new formulae based on those furnished by Kornfeld[2] for the general

[1] L. W. McKeehan, Phys. Rev. [2] **43**, 913–923, 924–930, 1025–1029 (1933).

[2] H. Kornfeld, Zeits. f. Physik **22**, 27–43 (1924).

TABLE I. *Hexagonal close-packing (HC).*

For $c=2\times 6^{\frac{1}{2}}/3$ $\quad p_1=p_2=0,\ p_3=1$

$3r^2$	$\Sigma' N$	$\Sigma'\delta h_i$ at 0,0,0	$\Sigma'\delta h_i$ at 0,0,$(0.055\times 6^{\frac{1}{2}})$	Difference
3	12	0	0.34558	0.34558
6	18	0	0.20080	.20080
8	20	1.49997	1.76311	.26314
9	38	−1.64273	−1.39386	.24887
11	50	1.65571	1.89742	.24171
12	56	0.43097	0.69745	.26648
15	68	− .62066	−0.35761	.26305
17	80	− .02249	0.21703	.22952
18	86	− .46694	−0.22709	.23985
19	92	.66547	0.92123	.25576
20	104	.89315	1.13313	.23998
21	128	− .92070	−0.66652	.25418
22	134	− .20305	.05456	.25761
25	146	.74187	.99822	.25633
27	158	.09674	.35744	.26070
29	182	− .12809	.12305	.25114
30	194	− .62384	− .37074	.25310
31	206	− .18614	.06287	.24901
32	208	.00136	.25229	.25093
33	220	− .43811	− .18543	.25268
34	226	− .28705	− .03627	.25078
35	250	.41545	.66946	.25401
36	256	.17975	.43536	.25561
37	268	.38762	.63992	.25230
39	292	− .38420	− .12792	.25628
41	298	− .12405		
43	304	− .07787		
44	328	.17586		
45	340	− .11647		
46	352	− .05971		
Limit		0.00225		

case, and we must abandon pseudo-cubic axes $\mathbf{a}_i''$ in favor of hexagonal axes $\mathbf{a}_i$.

It follows from symmetry that for $\mathbf{p}$ along $\mathbf{a}_3$ any resultant local field also lies along $\mathbf{a}_3$. It will therefore be sufficient to compute the magnetic potential energy per dipole, from which we can easily derive the magnitude of the local field. We will, however, leave $\mathbf{p}$ unrestricted for the present. As usual we may eliminate the dimensions of the particular lattice in question by putting $a_1=a_2=1, a_3=c$. Dipole positions are defined by vectors $\mathbf{r}$ and the selection rule for (H) would then be

$$r_1=n_1;\quad r_2=n_2;\quad r_3c=n_3c. \tag{5}$$

We use r_3c for the component along $\mathbf{a}_3$ so as to keep all r_i integral. When we try to construct the reciprocal or $\mathbf{q}$ lattice, however, we find its fundamental vectors have very inconvenient forms. It is easier in finding them to make temporary use of an orthorhombic system of axes for both $\mathbf{r}$ and $\mathbf{q}$. We put

$$\mathbf{a}_1'''=\mathbf{a}_1,\quad \mathbf{a}_2'''=-\tfrac{1}{2}\mathbf{a}_1+(3^{\frac{1}{2}}/2)\mathbf{a}_2,\quad \mathbf{a}_3'''=\mathbf{a}_3, \tag{6}$$

and the selection rule for orthorhombic components in (H) is

$$r_1'''=r_1-r_2/2,\quad r_2'''=r_2\,3^{\frac{1}{2}}/2,\quad r_3'''c=r_3c, \tag{7}$$

where r_1, r_2, r_3c are the corresponding hexagonal components. In the reciprocal lattice the points are selected by

$$q_1'''=q_1,\quad q_2'''=(q_1+2q_2)/3^{\frac{1}{2}},\quad q_3'''/c=q_3/c, \tag{8}$$

where $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3/c$ are the fundamental hexagonal vectors in the reciprocal lattice. In r_1''' and r_2''' the integer r_2 must be the same, in q_1''' and q_2''', q_1 must be the same. Otherwise $r_1r_2r_3$, $q_1q_2q_3$ are unrestricted integers, except that $q_1=q_2=q_3=0$ is excluded.

The expressions for r^2 and q^2 are now easily seen to be

$$r^2=(r_1^2-r_1r_2+r_2^2)+r_3^2c^2, \tag{9}$$

$$q^2=(4/3)(q_1^2+q_1q_2+q_2^2)+q_3^2/c^2, \tag{10}$$

and we will have no further use for the orthorhombic axes. For abbreviation in our final equations we will put

$$r_{12}^2=(r_1^2-r_1r_2+r_2^2), \tag{11}$$

$$q_{12}^2=(4/3)(q_1^2+q_1q_2+q_2^2). \tag{12}$$

The energy factor per dipole becomes

$$u=u_0+u_3p_3^2 \tag{13}$$

and the coefficients u_0, u_3 are each composed of several convergent summations

$$u_0=u_{00}+u_{0r0}+u_{0r2}+u_{0q2}, \tag{14}$$

$$u_3=u_{3r2}+u_{3q2}, \tag{15}$$

wherein

$$u_{00}=-4\epsilon^3/3\,\pi^{\frac{1}{2}}, \tag{16}$$

$$u_{0r0}=\epsilon^3\textstyle\sum' n_r g_1(\epsilon r), \tag{17}$$

$$u_{0r2}=-(\epsilon^5/2)\textstyle\sum' n_r r_{12}^2 g_2(\epsilon r), \tag{18}$$

$$u_{3r2}=-c^2\epsilon^3\textstyle\sum' n_r r_3^2 g_2(\epsilon r)-u_{0r2}, \tag{19}$$

$$u_{0q2}=\pi\,2^{\frac{1}{2}}\textstyle\sum' n_q q_{12}^2 q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{20}$$

$$u_{3q2}=(2\pi\,2^{\frac{1}{2}}/c^2)\textstyle\sum' n_q q_3^2 q^{-2}\times\exp(-\pi^2q^2/\epsilon^2)-u_{0q2}. \tag{21}$$

The factors n_r and n_q are the number of points with a given form $r_1r_2r_3$ and $q_1q_2q_3$, respectively. The possible values are 2, 6, 12 or 24 depending upon the form according to the following scheme:

$$\begin{aligned} n_r = \ & 2 \text{ if } \{r_1=r_2=0;\ r_3\neq 0\},\\ & 6 \text{ if } \{r_1=0;\ r_2\neq 0;\ r_3=0\} \quad \text{or if} \quad \{r_1=r_2\neq 0;\ r_3=0\},\\ & 12 \text{ if } \{r_1=0;\ r_2\neq 0;\ r_3\neq 0\} \quad \text{or if} \quad \{r_1=r_2\neq 0;\ r_3\neq 0\},\\ & 24 \text{ if } \{r_1\neq 0;\ r_2\neq 0;\ r_1\neq r_2;\ r_3\neq 0\}. \end{aligned}$$

The same conditions hold if we substitute n_q for n_r and $q_1q_2q_3$ for $r_1r_2r_3$.

In (*HC*) Eqs. (16), (17), (18), (19) retain their form but we get an additional set of terms in each summation due to a change in the selection rule for r_i which may now take fractional values. The complete selection rule is:

$$r_i=n \quad \text{and} \quad r_1=n_1+\tfrac{2}{3};\quad r_2=n_2+\tfrac{1}{3};\quad r_3=n_3+\tfrac{1}{2}.$$

It is now possible for n_r to take the value 3, this occurring if $r_2=r_1/2\neq n$; $r_3=0$. An additional case for $n_r=6$ is obtained if $r_2=r_1/2\neq n$; $r_3\neq 0$.

Eqs. (20) and (21) take new forms

$$u_{0q2}=\pi N 2^{\frac{1}{2}}\sum{}' f n_q q_{12}^2 q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{22}$$

$$u_{3q2}=(2\pi N 2^{\frac{1}{2}}/c^2)\sum{}' f n_q q_3^2 q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{23}$$

where f depends upon the form $q_1q_2q_3$ as shown below.

$4q_1+2q_2+3q_3$	f
$6n$	1
$6n\pm 1$	3/4
$6n\pm 2$	1/4
$6n+3$	0.

It will be noticed that, as usual, the number of terms in series depending on q is reduced by making the dipole array more complex.

TABLE II. *Coefficients for Eq.* (13) *in unstrained arrays with* $c=2\times 6^{\frac{1}{2}}/3$.

Array	N	u_0	u_3	$2\pi N\,2^{\frac{1}{2}}/3$	$u_0+2\pi N\,2^{\frac{1}{2}}/3$
(*H*)	1	−5.51483	7.65872	2.96192	−2.55291
(*HC*)	2	−5.92046	−0.01014	5.92384	0.00338

The computations, carried out for two choices of ϵ, *viz.*, $\epsilon=3^{\frac{1}{2}}$ and $\epsilon=2$, give the results presented in Table II. Besides u_0 and u_3 this table gives the correction $2\pi N\,2^{\frac{1}{2}}/3$ which compensates the effect of remote dipoles in the infinite array and the part of the energy due to the local field, $u_0+2\pi N\,2^{\frac{1}{2}}/3$ which does not depend upon the direction of **p**. For $p_3=1$, the case treated by the method of Lorentz, we get, for (*HC*), $u=0.00338-0.01014=-0.00676$ showing that the assumed direction of magnetization is stable, but only by a small amount. The fluctuations to be expected in a real crystal would completely mask so small a stabilizing field. In (*H*) on the other hand this direction for **p** gives a high maximum for u, corresponding to great instability.

Vibrations restricted to the hexagonal axis should help to stabilize **p** along the observed easy direction of magnetization, as is seen by considering the positive field factor values in the last column of Table I.

There seems to be only one statement in the literature of this subject in regard to dipole fields in hexagonal arrays. This appears in a note (in German) by Akulov[3] appended to a longer paper[4] (in Russian). The statement is that the Lorentz formula for the field $(4\pi I/3)$ at a dipole also holds for hexagonal close packing with an accuracy of one part in a thousand. Neither the note nor the longer paper contains any evidence for the statement which is, however, almost exactly correct, since the maximum deviation from the Lorentz energy factor, 5.92384, is 0.00676, or 0.1141 percent.

The energy per dipole in homogeneously strained hexagonal arrays (*H*) and (*HC*) has also been computed. The necessary formulae are derived in the usual way from those given by Kornfeld,[2] which are applicable to any crystal system.

The magnetic energy per dipole due to straining is

[3] N. S. Akulov, *Problèmes modernes de l'électromagnétisme. Recueil consacré au dixième anniversaire du laboratoire de magnetisme de Moscou*, p. 44, 1931.

[4] N. S. Akulov, reference 3, pp. 37–43.

$$u=\sum_{ij} H_{ij}A_{ij}, \tag{24}$$

where A_{ij} are the tensor components of the strain. For convenience we impose the condition $j\geqq i$. The coefficients H_{ij} are then as follows:

$$H_{11}=H_{110}+H_{1111}p_1^2+H_{1133}p_3^2, \tag{25}$$

$$H_{12}=H_{1212}p_1p_2, \tag{26}$$

$$H_{13}=H_{1313}p_1p_3, \tag{27}$$

$$H_{33}=H_{330}+H_{3333}p_3^2. \tag{28}$$

Not all of these coefficients are independent, for

$$H_{1111}=H_{1212}=2H_{110}. \tag{29}$$

In equations for H_{22}, H_{23}, analogous to (25), (27) appear the following coefficients, related as indicated to those just defined:

$$H_{220}=H_{110};\quad H_{2222}=H_{1111};\quad H_{2233}=H_{1133};$$
$$H_{2323}=H_{1313}. \tag{30}$$

Each H_{ii0}, H_{iijj} and H_{ijij} consists of several convergent summations according to the equations:

$$H_{110}=-H_{1r}+H_{12r}-H_{1q}+H_{12q}, \tag{31}$$

$$H_{1133}=-H_{12r}+H_{13r}+H_{1q}-H_{3q}-H_{12q}+H_{13q}, \tag{32}$$

$$H_{1313}=-H_{1r}-H_{3r}+2H_{13r}-H_{1q}-H_{3q}+2H_{13q}, \tag{33}$$

$$H_{330}=-H_{3r}+H_{13r}-H_{1q}+H_{13q}, \tag{34}$$

$$H_{3333}=-2H_{3r}-H_{13r}+H_{33r}-3H_{3q}-H_{13q}+H_{33q}, \tag{35}$$

the component summations in which are:

$$H_{1r}=(\epsilon^5/2)\sum{}'n_r r_{12}^2 g_2(\epsilon r), \tag{36}$$

$$H_{3r}=c^2\epsilon^5\sum{}'n_r r_3^2 g_2(\epsilon r), \tag{37}$$

$$H_{12r}=(\epsilon^7/8)\sum{}'n_r r_{12}^4 g_3(\epsilon r), \tag{38}$$

$$H_{13r}=(c^2\epsilon^7/2)\sum{}'n_r r_{12}^2 r_3^2 g_3(\epsilon r), \tag{39}$$

$$H_{33r}=c^4\epsilon^7\sum{}'n_r r_3^4 g_3(\epsilon r), \tag{40}$$

$$H_{1q}=\pi N\,2^{\frac{1}{2}}\sum{}'f n_q q_{12}^2 q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{41}$$

$$H_{3q}=(2\pi N\,2^{\frac{1}{2}}/c^2)\sum{}'f n_q q_3^2 q^{-2}\exp(-\pi^2q^2/\epsilon^2), \tag{42}$$

$$H_{12q}=\pi N\,2^{\frac{1}{2}}\sum{}'f n_q q_{12}^4 q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2), \tag{43}$$

$$H_{13q}=(4\pi N\,2^{\frac{1}{2}}/c^2)\sum{}'f n_q q_{12}^2 q_3^2 q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2), \tag{44}$$

$$H_{33q}=(8\pi N\,2^{\frac{1}{2}}/c^4)\sum{}'f n_q q_3^4 q^{-4}\{(\pi^2q^2/\epsilon^2)+1\}\exp(-\pi^2q^2/\epsilon^2). \tag{45}$$

The factor f in Eqs. (41) to (45) inclusive is 1 for (H) and has values 1, 3/4, 1/4, or 0 for (HC) according to the tabulation following Eqs. (22) and (23). The results of computation for $\epsilon=3$ and $\epsilon=2$ are given in Table III, all the values in which agreed to less than half a unit in the fifth decimal place for the two values of ϵ.

It is obvious that the results here presented may be applied to unstrained arrays (H) and (HC) in which $c\sim 2\times 6^{\frac{1}{2}}/3$ for any such array may be derived from the case here treated by regard-

TABLE III. *Homogeneously strained hexagonal arrays*

Coefficient—Eqs. (25)–(28)	(H)	(HC)
H_{110}	4.14282	3.86345
H_{1111}	8.28564	7.72690
H_{1133}	−11.82832	−1.54571
H_{1212}	8.28564	7.72690
H_{1313}	− 7.71230	4.62534
H_{330}	− 0.02679	2.30760
H_{3333}	8.96613	10.84874

ing the crystal as strained, with $A_{33}=3c/2\times 6^{\frac{1}{3}}-1$ as the only strain component different from zero. If A_{33}, so computed, is small enough to justify the assumption that effects of strain are linear this does not invalidate Eq. (24) by which we must find the correction term for Eq. (13). As already mentioned, c for cobalt has so nearly the theoretical value for close-packed spheres that the artifice is unnecessary in this case of greatest interest.

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

The Diamagnetic Susceptibilities and Polarizabilities of Ions

In a recent paper, J. E. Mayer and M. G. Mayer[1] have recalculated the polarizabilities of gaseous ions of rare gas structure by the Born-Heisenberg method, and in the course of their paper they indicate qualitatively how the polarizabilities of ions in crystals and solutions are likely to differ from the results for free ions. The writer has recently been engaged on some calculations of diamagnetic susceptibilities of atoms and ions of rare gas structure in relation to their refractivities and polarizabilities and it is of interest to compare the results with those obtained by J. E. and M. G. Mayer for free ions. At the same time it is also of interest to test an equation derived by Kirkwood[2] and by Vinti[3] connecting diamagnetic susceptibilities and ionic refractivities; this equation can be written as follows:

$$\chi_A = -(Ne^2/4mc^2)(a_0 n\alpha)^{\frac{1}{3}}. \quad (1)$$

χ_A is the gram-atomic susceptibility and α the polarizability of a spherically symmetrical atom containing n electrons; N is Avogadro's number, a_0 is the radius of the one quantum orbit of hydrogen, and the other symbols have their usual significance. In place of α, we may put $3R_\infty/4\pi N$, where R_∞ is the refractivity for infinite waves. Substituting numerical values, we obtain

$$\chi_A = -3.09\times10^6(n\alpha)^{\frac{1}{3}} \quad (2)$$

and

$$R_\infty = 2.54\times10^{24}\cdot\alpha. \quad (3)$$

TABLE I.

1	2 R_∞ (Heydweiller)	3 $-\chi_A\times10^6$ (from R_∞)	4 $-\chi_A\times10^6$ (from exp.)	5 $\alpha\times10^{24}$ (from R_∞)	6 $\alpha\times10^{24}$ (from $\chi_{exp.}$)	7 $\alpha\times10^{24}$ J. E. Mayer and M. G. Mayer
He	0.518	1.97	1.90	0.20	0.19	
Ne	0.995	6.12	6.7	0.39	0.47	
A	4.14	16.7	18.1 (18.8)	1.63	1.90	
Kr	6.26	29.3	(31.8)	2.46	(2.94)	
Xe	10.18	45.5	(47.9)	4.01	(4.44)	
F^-	2.17	9.0	13.	0.85	1.7$_6$	
Cl^-	8.22	23.6	24.3, 22.6	3.24	3.43, 2.97	
Br^-	11.60	39.7	37.7, 35.4	4.57	4.12, 3.64	
I^-	17.53	59.7	54.5, 52.2	6.90	5.75, 5.28	
Li^+	0.12	0.9	0.7	0.05	0.03	0.025
Na^+	0.65	5.0$_5$	5.7	0.26	0.34	0.17
K^+	2.71	13.5	13.6	1.15	1.07	0.80
Rb^+	4.10	23.6	23.9	1.61	1.65	1.5 ?
Cs^+	6.71	36.9	36.8	2.64	2.62	2.35
Mg^{+2}	—	—	4.0	—	0.17	0.10
Ca^{+2}	1.60	10.4	10.3	0.63	0.62	0.54
Sr^{+2}	2.56	18.7	19.1	1.01	1.06	1.0
Ba^{+2}	5.00	31.9	29.2	1.97	1.65	—

The results are set out in Table I. In column (2) values are given of R_∞ derived by Heydweiller from experimental data for very dilute solutions; the values for the rare gases are obtained from data by Cuthbertson. In column (3) are given the values of χ_A calculated from R_∞ by means of Eqs. (2) and (3). The latter are to be compared with the values of χ_A in column (4) obtained by the writer from

[1] J. E. Mayer and M. G. Mayer, Phys. Rev. **43**, 605 (1933).

[2] J. G. Kirkwood, Phys. Zeits. **33**, 57 (1932).

[3] J. P. Vinti, Phys. Rev. **41**, 813 (1932).

experimental data for the susceptibilities of salts in solution as given by the most recent determinations by Ikenmeyer, Hocart and others. The values for A, Kr and Xe in brackets are calculated from Slater's approximate wave functions. For Cl^-, Br^- and I^- it has been found that χ_A has a larger value in univalent than in bivalent solutions. The agreement between columns (3) and (4) is remarkably good and indicates that the Kirkwood-Vinti equation is probably a very good approximation to the truth; moreover, the values of α in columns (5) and (6) derived from these values of R_∞ and χ_A should be fairly reliable. In column (7) are given the values of α obtained by J. E. and M. G. Mayer for *free* ions. The latter are smaller than the former which is in accordance with the general argument given by the Mayers.

G. W. BRINDLEY

University of Leeds,
Leeds, England,
May 14, 1933.

On a New Type of Reasoning and Some of Its Possible Consequences

A. On a principle of flexibility of scientific truth

From a deeper scrutiny of the foundations of scientific truth it follows that every scientific statement referring to observations should possess a certain minimum degree of flexibility. In other words, no set of two-valued truths can be established with the expectation that this set ultimately will stand the test of experience. Formulations of scientific truth intrinsically must be *many-valued*.

I cannot here give a more complete justification of the above contention, except the mention, that any closed set of truths can be stated only after the adoption of a certain definite set of rules describing the type of thinking which is to be used. As *a priori* there are many such sets of rules, we cannot hope to embrace any part of nature completely by restricting ourselves to a definite set. We therefore must require of every scientific statement that it be in accord with what might be called the *principle of flexibility of scientific truth* if we wish to be in an organic relation to experience. The study of this principle must not at all be thought of as an interesting but useless pastime. On the contrary, this study suggests a new and so far unexploited type of reasoning concerning especially the fundamental concepts in physics. If properly handled our principle promises to lead the way to many new discoveries. The new type of reasoning which I propose may be compared in some respects to the systematic negation of postulates and the construction of more general sets of truths, a procedure well known in mathematics, but it goes deeper, in that the underlying "truth" necessarily is many-valued.

If then in physics we are confronted with any statement which has the appearance of a well-established absolute truth, we must question it, guess at more general possibilities in accordance with many-valued logics and consult the experiments. In particular, it may be anticipated that no set of nonstatistical truths will stand the test of experience. Passing from generalities to definite applications I give a few examples.

B. On the continuity of time and space

One statement in physics which has the appearance of an absolute truth refers to the simultaneous "in between" relation connecting the coordinates of time and space. Take for instance an electron which enters and leaves a lead plate at the times t_1, t_2 at the points P_1, P_2, respectively, without crossing the boundaries at any other points. We then say that all processes which have happened to the electron between P_1 and P_2 must also have happened at a time between t_1 and t_2. Our principle of flexibility negates this statement. This means that there must exist processes which for happenings between P_1 and P_2 produce corresponding times t which may be both inside or outside of the interval (t_1t_2), this no matter how we may define our measurements of time and space. It is of great importance

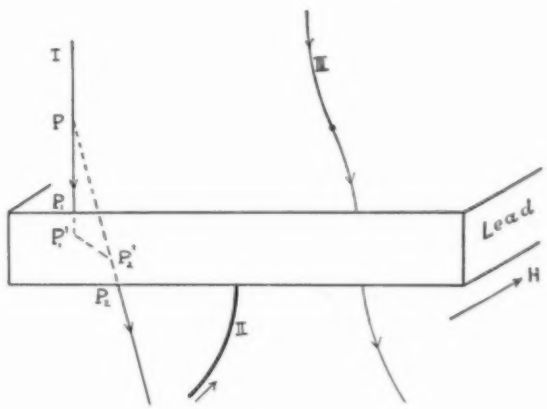


FIG. 1.

that electron tracks as shown in Fig. 1 have actually been observed[1] and that they indicate the existence of effects which according to the customary definition of time occasionally *produce* a *reversal of time*.

If one wants to explain track (*I*) with our ordinary notions in physics, one must assume that a double scattering at some points P_1' and P_2' has taken place. This, however, is irreconcilable with the fact that the occurrence of single scatterings through angles as shown at P_1' is not correspondingly more frequent than the double scattering. We therefore conclude that there exists a *single* process, so far unknown which produces a parallel displacement of the electron track, as well as an apparent reversal of time. It is a problem for further investigation to determine what kind of a process this is. As a very tentative suggestion the idea might be advanced that a fast electron, on a hard impact stores itself with its total energy in another elementary

[1] C. D. Anderson, Phys. Rev. **41**, 405 (1932). Fig. 23 of this paper shows an electron track of the type I.

particle and by some kind of a resonance effect (similar to the exchange of electrons) jumps sidewise and continues its path in the shown direction.

The claim that measured time can reverse means that in no given system of coordinates time is flowing continually in the same direction along the paths of all elementary particles. If the spatial length on a world line increases, time need not necessarily increase simultaneously, but may occasionally decrease, depending on the interaction with other world lines. This leads to a many-valued truth in contradistinction to a two-valued truth which answers the law of the excluded middle. This law says that an electron undergoes some process between P_1 and P_2 at a time between t_1 and t_2 *or* it undergoes a process outside of (P_1P_2) at a time outside of (t_1t_2). In our new way of thinking additional values of the truth must be introduced, namely, that the electron makes some process *between* P_1 and P_2 at a time *outside* of the interval (t_1t_2) or *vice versa*. The statistical distribution over the four values of the truth depends of course on the interaction between the electrons with other particles.

C. Degrees of freedom

Light quanta may change their number of degrees of freedom by combination with other quanta. However, elementary particles of matter are not supposed to change the number of their degrees of freedom. If this assertion could be verified experimentally with certainty we would be in possession of an absolute truth. The principle of flexibility therefore suggests that we negate the statement of the constancy of degrees of freedom for material particles and that we search for phenomena verifying this negation. One way out is, that matter might be annihilated and transformed into radiation. The only direct indication for the possibility of annihilation is the fact that the incoherent scattering of γ-rays from nuclei seems to contain two components of energies approximately equal to mc^2 and $2mc^2$ where m is the mass of the electron.

Another absolute statement is that of the constancy of the electric charge and the magnetic moment of elementary particles. This statement also implies the possibility of determining *simultaneously* the mass and the charge of a particle. From our principle of flexibility we must negate the statement and assume that the charge and the magnetic moment may occasionally change. Dr. C. Anderson has called my attention to a heavily ionized cloud chamber track (type II in Fig. 1) which might be due to an α-particle ejected by cosmic rays and moving in the direction of the arrow. However if from the curvature and the magnetic field its energy is computed it follows that the range should be smaller than the length of the observed track. One might then assume that the energy of the α-particle is high enough to explain the observed range. However in this case the applied magnetic field can produce the observed curvature only if the α-particle has suffered several collisions. One must further assume that the α-particle *accidentally* lands normal to the lead. This explanation of track II rests on two highly improbable assumptions. Therefore I think it must be discarded. I suggest instead that we are confronted with the track of a particle of about the mass of an electron emerging from the plate. In order to explain the high ionization the particle must have a charge several times that of an ordinary electron which would be a verification of our prediction.[2]

The change of sign of a charge might in some cases find an interpretation as a complete exchange of energy of a positive and a negative particle. Such a process would produce tracks of the type III in the figure. Tracks of this kind also occur in cloud chamber photographs. I think they imply that it is not only impossible to identify each of two electrons during a respective impact but that the same is true for a positive and a negative electron.

Another statement which deals with numbers only is the so-called exclusion principle in quantum mechanics. I am inclined to think that the negation of this statement might prove fruitful in the construction of an adequate theory of the nucleus.

D. The electric flux

The total flux of the electric field through a surface enclosing a finite material system has always been regarded as one of the most fundamental and indestructible quantities in physics. But according to our principle of flexibility we must even doubt this assertion and search for processes which destroy electric charges individually (not only in pairs as the recently proposed mutual annihilation of positive and negative electron). The existence of this process might prove to be a necessity in order to balance the electric budget in the universe. As a special difficulty I mention that if cosmic rays contained only negative electrons the earth would shortly charge up to potentials which would prevent the arrival of more electrons. Possible ways out are the assumption that positive and negative electrons arrive in equal numbers and equal speeds or the assumption that charges get annihilated. It will be of importance to test these questions experimentally.

E. Concluding remarks

I emphasize once more that the principle of flexibility of scientific truth refers to any type of symbolic statements of truth as truth can be secured by *measurements*. Any other type of truth does not fall within the realm of present natural science and is therefore not subject to our discussion. The special predictions which I have derived are possibilities among others and not at all unambiguous, although I feel that most of them have a fair chance to be verified experimentally in the near future. There are of course many still more fundamental concepts in physics such as reproducibility, description by arithmetical numbers, etc., which some day will no longer be adequate rules of scientific thinking. However it must be kept in mind that it is not necessarily fruitful to negate all at once all of the absolute postulates in science. Indeed a hundred years ago it would have been of no avail to negate the constancy of mass, the possibility of identification of particles, etc.,

[2] M. Dehlbrueck, Nature **130**, 629 (1932), for entirely different reasons, has also postulated the existence of particles with a mass similar to that of the electron but of a charge greater than that of the electron.

as at that time there was probably no chance at all to verify the consequences of such negations. The principle of flexibility must be handled with great care, having always in mind the possibility of experimental verification. It seems to me that the time for an extended application of the principle is ripe now. The conceptional difficulties in quantum mechanics may be interpreted as due to the peculiar inconsistencies of this theory which in certain respects conforms with our principle of flexibility, whereas in other respects, some of which we mentioned in this paper, quantum mechanics and the relativity theory are based on very antiquated notions. It should also be clear from our discussion that the recent controversies regarding the absolute truth of uncertainty principle *versus* causality are quite futile, as scientific truth intrinsically cannot be absolute.

Finally I hope that the principle of flexibility of scientific truth is itself flexible enough so as not to annihilate itself through its own tools after the fashion of Epimenides, the Cretan.

I wish to thank Professor E. T. Bell for many discussions.

F. Zwicky

California Institute of Technology,
Pasadena,
May 17, 1933.

Remarks on the Preceding Note on Many-Valued Truths

The new type of reasoning suggested in Professor Zwicky's note appears to be closely related to some of the projects which have occupied workers in the foundations of mathematics during the past two decades (since L. E. J. Brouwer's rejection of the law of excluded middle as a universally valid law of reasoning), and more particularly in the last five years. For this reason, Professor Zwicky's totally independent approach should be of interest to pure mathematicians. Conversely, some of the recent work in the foundations of mathematics may be of interest to those concerned with the foundations of theoretical physics. That Professor Zwicky arrived independently at his conclusions, gives a new interest to the mathematics and suggests further mathematical investigations. Professor Zwicky's principle of flexibility obviously has a wider scope than the new mathematics.

Many-valued logics have been created and studied in considerable detail by Tarski and Lukasiewicz, and their followers. There is a readable popular exposition of some of this work in a paper in the *Monist*, October, 1932, by Professor C. I. Lewis. The paper contains examples of such logics.

It will be noticed that one of Professor's Zwicky's suggestions challenges the universal applicability to physics of the law of identity. This law has also been scrutinized and rejected in some recent work in mathematics, and in quantum mechanics, where, however, only the identifiability of particles of the same type has been questioned. There thus remains of the Aristotelian system only the law of contradiction. This, so far as I know, has not been challenged outright, although its statement in a many-valued logic must be modified.

If many-valued logics become current, one statement of Professor Zwicky's is likely to receive prompt confirmation from the mathematical side. He predicts that description by "arithmetical numbers" will some day cease to be an adequate rule of scientific thinking. So far as those properties of integers which are independent of order relations are concerned, it may be reasonably doubted now whether "arithmetical numbers" are either necessary or sufficient in any reasoning. For (as shown by the present writer in a paper in the *Transactions of the American Mathematical Society for 1927*), common arithmetic is abstractly identical, except for order relations, with the common two-valued logic of classes. This extends to the logic of relations. By a remarkable coincidence, the problem of extending this to a many-valued logic, and hence getting a fundamentally new generalization of the concept of number, was already under way when I first heard of Professor Zwicky's similar idea. The generalization was proposed solely for its intrinsic interest as an extension of the classical theory of ideals in arithmetic, without any notion that it might have a scientific interpretation. Instead of the integers 0, 1 (for true, false, respectively), of Boolean algebra, we may have some or all of the rational numbers in the interval 0 to 1 as truth values of propositions, and a given truth value may be interpreted as a probability. The last, however, is merely one possibility of interpretation, and does not affect the abstract formulation of the generalized arithmetic.

There is another point in illustration of Professor Zwicky's principle of flexibility. If it is true that the theory of general relativity eliminates the observer (through the principle of covariance), and if it is true that the quantum theory retains the observer (through the indeterminacy principle, or whatever physical imagery is supposed to justify this principle), then a unification of relativity and quantum mechanics transcends a two-valued logic because it controverts the law of the excluded middle. To effect any unification which shall be more than superficial algebra, one or other of the theories to be unified must be radically changed, or resort must be made to a more than two-valued logic.

From the considerations adduced in Professor Zwicky's note, it appears that the time is now ripe for the adoption of the theory of many-valued truths as a working hypothesis.

E. T. Bell

Department of Mathematics,
California Institute of Technology,
May 17, 1933.

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

The Diamagnetic Susceptibilities and Polarizabilities of Ions

In a recent paper, J. E. Mayer and M. G. Mayer[1] have recalculated the polarizabilities of gaseous ions of rare gas structure by the Born-Heisenberg method, and in the course of their paper they indicate qualitatively how the polarizabilities of ions in crystals and solutions are likely to differ from the results for free ions. The writer has recently been engaged on some calculations of diamagnetic susceptibilities of atoms and ions of rare gas structure in relation to their refractivities and polarizabilities and it is of interest to compare the results with those obtained by J. E. and M. G. Mayer for free ions. At the same time it is also of interest to test an equation derived by Kirkwood[2] and by Vinti[3] connecting diamagnetic susceptibilities and ionic refractivities; this equation can be written as follows:

$$\chi_A = -(Ne^2/4mc^2)(a_0 n\alpha)^{\frac{1}{2}}. \quad (1)$$

χ_A is the gram-atomic susceptibility and α the polarizability of a spherically symmetrical atom containing n electrons; N is Avogadro's number, a_0 is the radius of the one quantum orbit of hydrogen, and the other symbols have their usual significance. In place of α, we may put $3R_\infty/4\pi N$, where R_∞ is the refractivity for infinite waves. Substituting numerical values, we obtain

$$\chi_A = -3.09\times10^6(n\alpha)^{\frac{1}{2}} \quad (2)$$

and

$$R_\infty = 2.54\times10^{24}\cdot\alpha. \quad (3)$$

TABLE I.

1	2 R_∞ (Heydweiller)	3 $-\chi_A\times10^6$ (from R_∞)	4 $-\chi_A\times10^6$ (from exp.)	5 $\alpha\times10^{24}$ (from R_∞)	6 $\alpha\times10^{24}$ (from $\chi_{exp.}$)	7 $\alpha\times10^{24}$ J. E. Mayer and M. G. Mayer
He	0.518	1.97	1.90	0.20	0.19	
Ne	0.995	6.12	6.7	0.39	0.47	
A	4.14	16.7	18.1 (18.8)	1.63	1.90	
Kr	6.26	29.3	(31.8)	2.46	(2.94)	
Xe	10.18	45.5	(47.9)	4.01	(4.44)	
F^-	2.17	9.0	13.	0.85	1.7_6	
Cl^-	8.22	23.6	24.3, 22.6	3.24	3.43, 2.97	
Br^-	11.60	39.7	37.7, 35.4	4.57	4.12, 3.64	
I^-	17.53	59.7	54.5, 52.2	6.90	5.75, 5.28	
Li^+	0.12	0.9	0.7	0.05	0.03	0.025
Na^+	0.65	5.0_5	5.7	0.26	0.34	0.17
K^+	2.71	13.5	13.6	1.15	1.07	0.80
Rb^+	4.10	23.6	23.9	1.61	1.65	1.5 ?
Cs^+	6.71	36.9	36.8	2.64	2.62	2.35
Mg^{+2}	—	—	4.0	—	0.17	0.10
Ca^{+2}	1.60	10.4	10.3	0.63	0.62	0.54
Sr^{+2}	2.56	18.7	19.1	1.01	1.06	1.0
Ba^{+2}	5.00	31.9	29.2	1.97	1.65	—

The results are set out in Table I. In column (2) values are given of R_∞ derived by Heydweiller from experimental data for very dilute solutions; the values for the rare gases are obtained from data by Cuthbertson. In column (3) are given the values of χ_A calculated from R_∞ by means of Eqs. (2) and (3). The latter are to be compared with the values of χ_A in column (4) obtained by the writer from

[1] J. E. Mayer and M. G. Mayer, Phys. Rev. **43**, 605 (1933).

[2] J. G. Kirkwood, Phys. Zeits. **33**, 57 (1932).

[3] J. P. Vinti, Phys. Rev. **41**, 813 (1932).

experimental data for the susceptibilities of salts in solution as given by the most recent determinations by Ikenmeyer, Hocart and others. The values for A, Kr and Xe in brackets are calculated from Slater's approximate wave functions. For Cl^-, Br^- and I^- it has been found that χ_A has a larger value in univalent than in bivalent solutions. The agreement between columns (3) and (4) is remarkably good and indicates that the Kirkwood-Vinti equation is probably a very good approximation to the truth; moreover, the values of α in columns (5) and (6) derived from these values of R_∞ and χ_A should be fairly reliable. In column (7) are given the values of α obtained by J. E. and M. G. Mayer for *free* ions. The latter are smaller than the former which is in accordance with the general argument given by the Mayers.

G. W. BRINDLEY

University of Leeds,
Leeds, England,
May 14, 1933.

On a New Type of Reasoning and Some of Its Possible Consequences

A. On a principle of flexibility of scientific truth

From a deeper scrutiny of the foundations of scientific truth it follows that every scientific statement referring to observations should possess a certain minimum degree of flexibility. In other words, no set of two-valued truths can be established with the expectation that this set ultimately will stand the test of experience. Formulations of scientific truth intrinsically must be *many-valued*.

I cannot here give a more complete justification of the above contention, except the mention, that any closed set of truths can be stated only after the adoption of a certain definite set of rules describing the type of thinking which is to be used. As *a priori* there are many such sets of rules, we cannot hope to embrace any part of nature completely by restricting ourselves to a definite set. We therefore must require of every scientific statement that it be in accord with what might be called the *principle of flexibility of scientific truth* if we wish to be in an organic relation to experience. The study of this principle must not at all be thought of as an interesting but useless pastime. On the contrary, this study suggests a new and so far unexploited type of reasoning concerning especially the fundamental concepts in physics. If properly handled our principle promises to lead the way to many new discoveries. The new type of reasoning which I propose may be compared in some respects to the systematic negation of postulates and the construction of more general sets of truths, a procedure well known in mathematics, but it goes deeper, in that the underlying "truth" necessarily is many-valued.

If then in physics we are confronted with any statement which has the appearance of a well-established absolute truth, we must question it, guess at more general possibilities in accordance with many-valued logics and consult the experiments. In particular, it may be anticipated that no set of nonstatistical truths will stand the test of experience. Passing from generalities to definite applications I give a few examples.

B. On the continuity of time and space

One statement in physics which has the appearance of an absolute truth refers to the simultaneous "in between" relation connecting the coordinates of time and space. Take for instance an electron which enters and leaves a lead plate at the times t_1, t_2 at the points P_1, P_2, respectively, without crossing the boundaries at any other points. We then say that all processes which have happened to the electron between P_1 and P_2 must also have happened at a time between t_1 and t_2. Our principle of flexibility negates this statement. This means that there must exist processes which for happenings between P_1 and P_2 produce corresponding times t which may be both inside or outside of the interval (t_1t_2), this no matter how we may define our measurements of time and space. It is of great importance

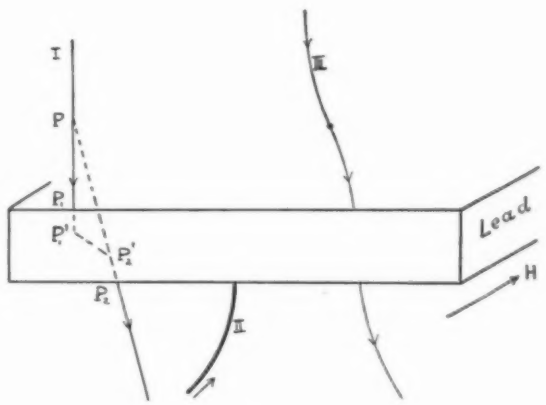


FIG. 1.

that electron tracks as shown in Fig. 1 have actually been observed[1] and that they indicate the existence of effects which according to the customary definition of time occasionally *produce* a *reversal of time*.

If one wants to explain track (*I*) with our ordinary notions in physics, one must assume that a double scattering at some points P_1' and P_2' has taken place. This, however, is irreconcilable with the fact that the occurrence of single scatterings through angles as shown at P_1' is not correspondingly more frequent than the double scattering. We therefore conclude that there exists a *single* process, so far unknown which produces a parallel displacement of the electron track, as well as an apparent reversal of time. It is a problem for further investigation to determine what kind of a process this is. As a very tentative suggestion the idea might be advanced that a fast electron, on a hard impact stores itself with its total energy in another elementary

[1] C. D. Anderson, Phys. Rev. **41**, 405 (1932). Fig. 23 of this paper shows an electron track of the type I.

particle and by some kind of a resonance effect (similar to the exchange of electrons) jumps sidewise and continues its path in the shown direction.

The claim that measured time can reverse means that in no given system of coordinates time is flowing continually in the same direction along the paths of all elementary particles. If the spatial length on a world line increases, time need not necessarily increase simultaneously, but may occasionally decrease, depending on the interaction with other world lines. This leads to a many-valued truth in contradistinction to a two-valued truth which answers the law of the excluded middle. This law says that an electron undergoes some process between P_1 and P_2 at a time between t_1 and t_2 *or* it undergoes a process outside of (P_1P_2) at a time outside of (t_1t_2). In our new way of thinking additional values of the truth must be introduced, namely, that the electron makes some process *between* P_1 and P_2 at a time *outside* of the interval (t_1t_2) or *vice versa*. The statistical distribution over the four values of the truth depends of course on the interaction between the electrons with other particles.

C. Degrees of freedom

Light quanta may change their number of degrees of freedom by combination with other quanta. However, elementary particles of matter are not supposed to change the number of their degrees of freedom. If this assertion could be verified experimentally with certainty we would be in possession of an absolute truth. The principle of flexibility therefore suggests that we negate the statement of the constancy of degrees of freedom for material particles and that we search for phenomena verifying this negation. One way out is, that matter might be annihilated and transformed into radiation. The only direct indication for the possibility of annihilation is the fact that the incoherent scattering of γ-rays from nuclei seems to contain two components of energies approximately equal to mc^2 and $2mc^2$ where m is the mass of the electron.

Another absolute statement is that of the constancy of the electric charge and the magnetic moment of elementary particles. This statement also implies the possibility of determining *simultaneously* the mass and the charge of a particle. From our principle of flexibility we must negate the statement and assume that the charge and the magnetic moment may occasionally change. Dr. C. Anderson has called my attention to a heavily ionized cloud chamber track (type II in Fig. 1) which might be due to an α-particle ejected by cosmic rays and moving in the direction of the arrow. However if from the curvature and the magnetic field its energy is computed it follows that the range should be smaller than the length of the observed track. One might then assume that the energy of the α-particle is high enough to explain the observed range. However in this case the applied magnetic field can produce the observed curvature only if the α-particle has suffered several collisions. One must further assume that the α-particle *accidentally* lands normal to the lead. This explanation of track II rests on two highly improbable assumptions. Therefore I think it must be discarded. I suggest instead that we are confronted with the track of a particle of about the mass of an electron emerging from the plate. In order to explain the high ionization the particle must have a charge several times that of an ordinary electron which would be a verification of our prediction.[2]

The change of sign of a charge might in some cases find an interpretation as a complete exchange of energy of a positive and a negative particle. Such a process would produce tracks of the type III in the figure. Tracks of this kind also occur in cloud chamber photographs. I think they imply that it is not only impossible to identify each of two electrons during a respective impact but that the same is true for a positive and a negative electron.

Another statement which deals with numbers only is the so-called exclusion principle in quantum mechanics. I am inclined to think that the negation of this statement might prove fruitful in the construction of an adequate theory of the nucleus.

D. The electric flux

The total flux of the electric field through a surface enclosing a finite material system has always been regarded as one of the most fundamental and indestructible quantities in physics. But according to our principle of flexibility we must even doubt this assertion and search for processes which destroy electric charges individually (not only in pairs as the recently proposed mutual annihilation of positive and negative electron). The existence of this process might prove to be a necessity in order to balance the electric budget in the universe. As a special difficulty I mention that if cosmic rays contained only negative electrons the earth would shortly charge up to potentials which would prevent the arrival of more electrons. Possible ways out are the assumption that positive and negative electrons arrive in equal numbers and equal speeds or the assumption that charges get annihilated. It will be of importance to test these questions experimentally.

E. Concluding remarks

I emphasize once more that the principle of flexibility of scientific truth refers to any type of symbolic statements of truth as truth can be secured by *measurements*. Any other type of truth does not fall within the realm of present natural science and is therefore not subject to our discussion. The special predictions which I have derived are possibilities among others and not at all unambiguous, although I feel that most of them have a fair chance to be verified experimentally in the near future. There are of course many still more fundamental concepts in physics such as reproducibility, description by arithmetical numbers, etc., which some day will no longer be adequate rules of scientific thinking. However it must be kept in mind that it is not necessarily fruitful to negate all at once all of the absolute postulates in science. Indeed a hundred years ago it would have been of no avail to negate the constancy of mass, the possibility of identification of particles, etc.,

---

[2] M. Dehlbrueck, Nature **130**, 629 (1932), for entirely different reasons, has also postulated the existence of particles with a mass similar to that of the electron but of a charge greater than that of the electron.

as at that time there was probably no chance at all to verify the consequences of such negations. The principle of flexibility must be handled with great care, having always in mind the possibility of experimental verification. It seems to me that the time for an extended application of the principle is ripe now. The conceptional difficulties in quantum mechanics may be interpreted as due to the peculiar inconsistencies of this theory which in certain respects conforms with our principle of flexibility, whereas in other respects, some of which we mentioned in this paper, quantum mechanics and the relativity theory are based on very antiquated notions. It should also be clear from our discussion that the recent controversies regarding the absolute truth of uncertainty principle *versus* causality are quite futile, as scientific truth intrinsically cannot be absolute.

Finally I hope that the principle of flexibility of scientific truth is itself flexible enough so as not to annihilate itself through its own tools after the fashion of Epimenides, the Cretan.

I wish to thank Professor E. T. Bell for many discussions.

F. ZWICKY

California Institute of Technology,
Pasadena,
May 17, 1933.

Remarks on the Preceding Note on Many-Valued Truths

The new type of reasoning suggested in Professor Zwicky's note appears to be closely related to some of the projects which have occupied workers in the foundations of mathematics during the past two decades (since L. E. J. Brouwer's rejection of the law of excluded middle as a universally valid law of reasoning), and more particularly in the last five years. For this reason, Professor Zwicky's totally independent approach should be of interest to pure mathematicians. Conversely, some of the recent work in the foundations of mathematics may be of interest to those concerned with the foundations of theoretical physics. That Professor Zwicky arrived independently at his conclusions, gives a new interest to the mathematics and suggests further mathematical investigations. Professor Zwicky's principle of flexibility obviously has a wider scope than the new mathematics.

Many-valued logics have been created and studied in considerable detail by Tarski and Lukasiewicz, and their followers. There is a readable popular exposition of some of this work in a paper in the *Monist*, October, 1932, by Professor C. I. Lewis. The paper contains examples of such logics.

It will be noticed that one of Professor's Zwicky's suggestions challenges the universal applicability to physics of the law of identity. This law has also been scrutinized and rejected in some recent work in mathematics, and in quantum mechanics, where, however, only the identifiability of particles of the same type has been questioned. There thus remains of the Aristotelian system only the law of contradiction. This, so far as I know, has not been challenged outright, although its statement in a many-valued logic must be modified.

If many-valued logics become current, one statement of Professor Zwicky's is likely to receive prompt confirmation from the mathematical side. He predicts that description by "arithmetical numbers" will some day cease to be an adequate rule of scientific thinking. So far as those properties of integers which are independent of order relations are concerned, it may be reasonably doubted now whether "arithmetical numbers" are either necessary or sufficient in any reasoning. For (as shown by the present writer in a paper in the *Transactions of the American Mathematical Society for 1927*), common arithmetic is abstractly identical, except for order relations, with the common two-valued logic of classes. This extends to the logic of relations. By a remarkable coincidence, the problem of extending this to a many-valued logic, and hence getting a fundamentally new generalization of the concept of number, was already under way when I first heard of Professor Zwicky's similar idea. The generalization was proposed solely for its intrinsic interest as an extension of the classical theory of ideals in arithmetic, without any notion that it might have a scientific interpretation. Instead of the integers 0, 1 (for true, false, respectively), of Boolean algebra, we may have some or all of the rational numbers in the interval 0 to 1 as truth values of propositions, and a given truth value may be interpreted as a probability. The last, however, is merely one possibility of interpretation, and does not affect the abstract formulation of the generalized arithmetic.

There is another point in illustration of Professor Zwicky's principle of flexibility. If it is true that the theory of general relativity eliminates the observer (through the principle of covariance), and if it is true that the quantum theory retains the observer (through the indeterminacy principle, or whatever physical imagery is supposed to justify this principle), then a unification of relativity and quantum mechanics transcends a two-valued logic because it controverts the law of the excluded middle. To effect any unification which shall be more than superficial algebra, one or other of the theories to be unified must be radically changed, or resort must be made to a more than two-valued logic.

From the considerations adduced in Professor Zwicky's note, it appears that the time is now ripe for the adoption of the theory of many-valued truths as a working hypothesis.

E. T. BELL

Department of Mathematics,
California Institute of Technology,
May 17, 1933.

Positrons from Gamma-Rays

A report of the direct observation of positrons ejected from lead by hard gamma-rays has been published.[1] A source of radiothorium with its disintegration products was used. To obtain a well-defined beam the gamma-rays were collimated by allowing them to pass through a half inch hole in a series of lead blocks totaling 18 inches in length. That the beam actually was well defined is shown by the fact that most of the observed tracks appeared to originate within a rather narrow region, about 3 cm wide. In this series of photographs the beam passed vertically downward through the side wall of a Wilson chamber across which were placed a 2 mm lead sheet, and below this a 0.5 mm sheet of aluminum. A magnetic field of 430 gauss normal to the face of the chamber and uniform to within 10 percent was used.

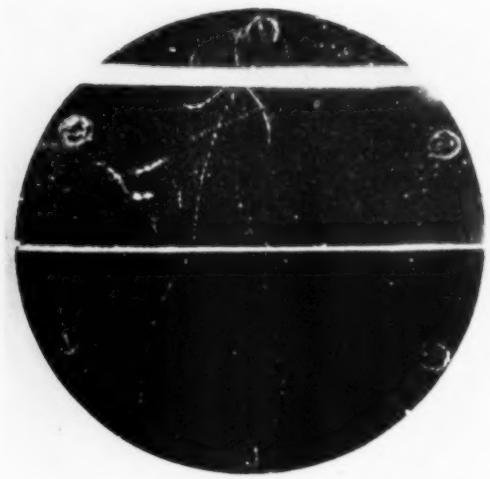


FIG. 1. A positron ejected from a lead plate by gamma-rays and passing through a 0.5 mm aluminum plate, showing a loss in energy. On the assumption that its mass is the same as that of a free electron the energy before penetrating the aluminum plate is 820,000 volts and afterward 520,000 volts.

A statistical study of the energy distribution of the positives and negatives produced by the gamma-radiation will throw considerable light on the nature of the processes involved in their production since the quantum energies of the photons are in this case well known. One of the most striking phenomena which have been observed in this experiment is the occasional simultaneous appearance of paired tracks consisting of one positive particle and one negative with a common point of origin. As a preliminary part of this study we have to date measured the energies of 22 such pairs ejected by the gamma-radiation, which has a strong band at 2.6×10^6 volts. Of the 22 pairs, 13 have kinetic energies (if the positron mass is assumed equal to that of the electron) from 10^6 volts to 1.6×10^6; and only one pair has an energy greater than this, namely 1.8×10^6. Since isolated cases cannot be regarded with absolute certainty as associated pairs we may state the above result as statistical evidence suggesting a cut-off in the neighborhood of 1.6×10^6 volts.

The process which gives rise to the positrons is at present not known, but so far as these data go they are in accord with the view expressed by Blackett and Occhialini[2] that the two particles may be formed by a process in which the energy of approximately one million volts required for the formation of a positron and a negative electron is supplied by the impinging radiation. Further, the energies of 13 single positively curved particles with which there is no trace of an associated negative have been measured. If we suppose that in each of these cases a pair was created but the positron happened to get most of the kinetic energy, then the maximum kinetic energy of these single particles should have the same upper limit as that of the pairs. Of the 13 there were 8 with energies from 10^6 to 1.5×10^6, and none higher than the latter value. If, however, further work should reveal the presence of positrons whose energy differs from that of the incident photons by less than a million volts the above interpretation must be abandoned.

Skobelzyn[3] in a detailed investigation of the space and energy distribution of the electrons ejected by hard gamma-rays found results not completely in agreement with the Klein-Nishina formula. Gray and Tarrant[4] in studying the scattering of hard gamma-rays report the presence of a fluorescence radiation whose quantum energy apparently is confined to two bands in the neighborhood of 0.5×10^6 volts and 0.9×10^6 volts. Blackett and Occhialini have suggested that the origin of this fluorescence radiation may be the simultaneous annihilation of an electron and a positron. The effects observed in these two experiments increased with the hardness of the gamma-rays used; and they might be ascribed to the formation of positrons and electrons rather than to any inherent incorrectness in the Klein-Nishina formula, which considers scattering only from free negative electrons.

Since hard gamma-rays do produce positrons, the positrons observed by Chadwick, Blackett and Occhialini,[5] and by Meitner and Philipp,[6] who used a beam of neutrons with accompanying hard gamma-rays may be due only to the accompanying gamma-rays and not the neutrons. In fact Curie and Joliot[7] by studying the yield of positrons as a function of the amount of filtering of the combined neutron and gamma-ray beam have arrived at just this conclusion.

CARL D. ANDERSON
SETH H. NEDDERMEYER
California Institute of Technology,
Pasadena, California,
May 18, 1933.

[1] Anderson, Science **77**, 432 (1933).
[2] Blackett and Occhialini, Proc. Roy. Soc. **A139**, 699 (1933).
[3] Skobelzyn, Comptes Rendus **194**, 1914 (1932).
[4] Gray and Tarrant, Proc. Roy. Soc. **A136**, 662 (1932).
[5] Chadwick, Blackett and Occhialini, Nature **131**, 473 (1933).
[6] Meitner and Philipp, Naturwiss. **21**, 286 (1933).
[7] Curie and Joliot, Comptes Rendus **196**, 1105 (1933).

Partial Resolution of the Be I Line λ4572.69 and Probable Spin of the Be^9 Nucleus

Because of the interest associated with the nuclear spin of Be^9, a further attempt has been made to detect hyperfine structure in the atomic spectrum of beryllium. A liquid air cooled discharge tube similar in design to that used by Ritschl[1] has been used as the source of the beryllium spectrum. The cathode was made entirely of copper and in the bottom of the tube several chunks of beryllium metal were placed. Purified helium was admitted to the tube to a pressure of 1.5 mm of mercury. The tube was supplied with potential from a 1000 volt d.c. generator and after a brief period of running a very pure spectrum of helium and beryllium was obtained. The spectrum was examined with a dispersion of slightly less than 3A/mm and it was found to be free of lines in the vicinity of the Be I line λ4572.69. Hence this line could be examined by using a Hilger E-3 spectrograph in conjunction with a quartz Lummer-Gehrcke plate which is 20 cm long and 0.433 cm thick (theoretical resolving power 700,000). The light incident on the slit of the spectrograph was polarized by means of a Nicol prism and the interferometer was placed between the collimating lens and the prism of the spectrograph. Eastman 40 plates were exposed for ten minutes with a current of 0.5 ampere passing through the tube. These plates were developed in a fine grain developer (Eastman's D-76) in order to reduce the fluctuations in the microphotometer traces which were taken with a Koch-Goos recording microphotometer on the 40 : 1 ratio.

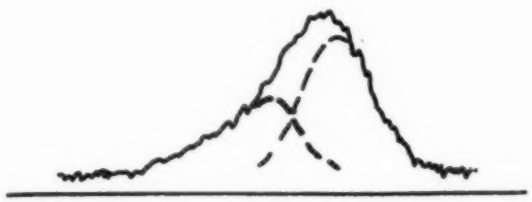


FIG. 1. Microphotometer trace of an interference fringe of Be λ4572.69 and proposed analysis.

In Fig. 1 which is a reproduction of the photometer trace for a single fringe of the interference pattern, the asymmetry is quite evident. Assuming that the asymmetry is due to unresolved hyperfine structure, the components indicated by the dotted lines will account for the asymmetry of the fringe. For the time being we will consider that this assumption is valid. In as much as the components are not resolved we must determine the nuclear spin from the intensity measurements we get upon analyzing the shape of the trace. This is most conveniently done by treatment of the integrated intensity curve on probability paper.* (Cf. F. N. D. Kurie's work on the ranges of α-particles from U I and U II.[2])

Application of this method to the microphotometer traces of the λ4572.69 Be I line has shown that there is quite a definite separation of the line into two components of which the weaker contains 36 ± 4 percent of the total intensity. This line is a $2^1P_1-3^1D_2$ transition. Assuming that the splitting in the D state is negligible, we obtain from the selection rules that for $I=\frac{1}{2}$ there should be two components while for higher values of I there should be three components. Now our resolution of the trace into two components by means of probability paper is not sufficient to conclude that $I=\frac{1}{2}$, as it may well be that one of these two components is itself a fusion. However we are able to compute by means of Hill's intensity rules the relative intensities of the component lines. Upon doing so we find that for $I=\frac{1}{2}$ the weaker line should contain 33.3 percent of the total intensity which is close to our observed value. For $I=1$ we find, taking account of the interval rule, that 44.4 percent of the total intensity would be in the weaker component. Making similar calculations for other values of I we see that the intensity varies with I in such a manner that I would have to be considerably larger before we reach a point at which one component will possess such a percentage of the total intensity. Furthermore, I values of that size would give nearly equal separations of the components and nearly equal intensity. Therefore it seems reasonable to conclude that the nuclear spin of Be^9 is $\frac{1}{2}$.

A determination of the separation of the two components shows it to be 0.033 ± 0.005 cm^{-1}. The levels are normal since the weaker component is on the violet side of the fringe.

We have been assuming above that the asymmetry of the line is due to hyperfine structure. It seems advisable, in fact necessary, to consider other possible causes of this asymmetry.

Firstly, there is the question of an overlapping line. However, upon photographing the discharge at high dispersion, 2 to 3A/mm, this region of the spectrum was found to be free of adjacent lines.

Secondly, there is the possibility of the asymmetry being due to some quality of the apparatus. The examination of the fringes of singlet lines of helium shows that there is no asymmetry present in them and hence this objection is removed.

Thirdly, there is the question of pressure broadening. Pressure broadening is usually to the red while this asymmetry is to the violet. It is of course true that certain lines are broadened to the violet by pressure but it seems doubtful if the effect can be present here to an appreciable extent. However no accurate calculations can be made of the effects to be expected from either of these causes. The probability of a fortuitous pressure broadening of an amount to give an apparent relative intensity value so close to the theoretically expected value seems rather small. Therefore we may conclude that the broadening observed is due to hyperfine structure though the effect of pressure broadening is not completely barred as the source of the asymmetry.

An attempt to detect hyperfine structure in the λ3321

---

[1] R. Ritschl, Zeits. f. Physik **79**, 1 (1932).

* Tests on helium lines known to be single show that a trace of a fringe of a spectral line is sufficiently close to an error function to permit of using this method of analysis.

[2] F. N. D. Kurie, Phys. Rev. **41**, 701 (1932).

triplet of beryllium with the same apparatus has been unsuccessful. The narrowness of the triplet does not permit an analysis of the photometer curves of the individual members of the triplet even though they are resolved.

In as much as Chadwick[3] has found that neutrons are emitted from beryllium atoms which have been struck by α-particles, we know that the neutron is one of the components of the beryllium nucleus. He gives the equation of the reaction as

$$\alpha+Be^{9}\rightarrow C^{12}+\nu.$$

Now as the α-particle and the C^{12} atom possess no angular momentum the neutron must have an angular momentum equal to that of the Be^{9} nucleus, provided we assume that there is conservation of angular momentum. Therefore we reach the conclusion that the spin of the neutron is $\frac{1}{2}$ which fixes its statistics as Fermi-Dirac.

The author wishes to thank Professor W. W. Watson and Dr. F. N. D. Kurie for their interest and welcome suggestions.

ALLAN E. PARKER

Sloane Physics Laboratory,
Yale University,
May 29, 1933.

[3] J. Chadwick, Nature **129**, 312 (1932).

Weak X-Ray Lines of Columbium and Antimony

Emission lines of the K-series of 41 Cb and 51 Sb, including some lines not hitherto observed in these elements, have been studied with a two-crystal x-ray spectrometer, with results as given in Table I.

TABLE I.

	δ		β_4		β_5		β_1	β_2
	λ	ν/R	λ	ν/R	λ	ν/R	λ	λ
41 Cb Obs.	651.84	1397.7	658.67	1383.20	662.89	1374.4		
Calc.		1398.13		1383.30		1377.9		
51 Sb Obs.	406.37	2242.0	412.89	2206.6	414.73	2196.8	416.23	416.88
Calc.		2243.8		2207.2		2199.3		

The line here denoted by δ is a faint one quite near the K-absorption limit, homologous with a line first observed by Leide[1] in molybdenum. The line referred to here as β_4 corresponds to the lines listed by Siegbahn as β_5 but not found heretofore above 47 Ag. β_5 listed above corresponds to lines recently found by Ross[2] in other elements and similarly designated. All of these lines are extremely faint and may be accounted for only by forbidden transitions.

In addition to data upon these weak lines, thought to be new in these elements, we have listed for antimony the wave-lengths of $K\beta_1$ and its companion, known both as β_2 and β_3. This pair has perhaps not previously been resolved in antimony.

The wave-lengths and frequencies given here were obtained by measuring the separations of the new lines from prominent K lines of known wave-length. In Cb the lines taken as standards were γ, β_1 and β_2, and the standard values adopted were the values of ν/R given by Siegbahn.[3] In Siegbahn's notation these lines are respectively β_2, β_1 and β_3. From the values of ν/R thus obtained the wave-length values have been calculated.

For Sb the wave-lengths and frequencies are referred to the γ-line only (Siegbahn's β_2). δ, β_4 and β_5 values are based upon $\nu/R=2238.4$ for this line. In computing the wave-lengths of β_1 and β_2 in Sb the wave-length of γ has been taken to be 407.10 x.u., after Siegbahn.

The table also presents calculated frequencies for δ, β_4 and β_5. These were calculated from formulas which have been suggested by Carlsson[4] for δ, by Idei for β_4 and by Ross for β_5. Carlsson supposes δ to originate in a transition $N_{32,\ 33}-K$, which would require the frequency of δ to equal the sum of the frequencies of $K\alpha_1$ and $L\beta_2$. The agreement between the observed frequencies and those calculated in this way lends support to this hypothesis.

Idei[5] suggested that β_4 originates in a transition $M_{32,\ 33}-K$. The calculated frequencies in the table, derived on this basis, are in good agreement with the observed results. For β_5 Ross proposed the double transition $(M_{21}-K)+(M_{22}-M_{11})$. In this case the agreement is less good, the calculated values being too high.

In both Cb and Sb β_4 is unsymmetrical in a manner indicating the existence of a weaker component on the long wave-length side.

P. A. ROSS
PAUL KIRKPATRICK

Department of Physics,
Stanford University,
June 1, 1933.

[1] Leide, Dissertation Lund, 1925.

[2] Ross, Phys. Rev. **39**, 748 (1932).

[3] Siegbahn, Spektroskopie der Röntgenstrahlen, 1931, p. 188.

[4] Carlsson, Zeits. f. Physik **80**, 604 (1933).

[5] Idei, Sc. Reports Tôhoku Imp. Univ. **19**, 641 (1930).

Lateral Distribution of Photoelectrons and the Measurement of X-Ray Polarization

The first measurements of x-ray polarization, made by Barkla in 1906, consisted of observations of the lateral asymmetry which characterizes the distribution of scattered x-rays when the incident rays possess polarization. All subsequent investigators of polarized x-rays have used the same effect as the measure of polarization, though in a few cases it has been the coherent scattering which constitutes crystal reflection that has been utilized, rather than the diffuse scattering from amorphous bodies. It in no way reflects upon the excellence of past work in this field to remark that in spite of a lapse of nearly three decades we have not yet learned anything about x-ray polarization by these methods sufficiently fundamental and precise to serve as a check on the theories of continuous spectral emission. Some modification of technique is indicated.

Accordingly we are making polarization measurements by observing the lateral *photoelectron* distribution about the beam rather than the scattered x-ray distribution. A method of this kind, using the Wilson condensation method, has been proposed heretofore, but in the present case the photoelectron distribution is more expeditiously obtained by means of a subdivided ionization chamber of special design.

The collecting compartments in this chamber were designed to collect only ions formed by photoelectrons ejected in directions lying approximately in a single plane through the incident x-ray beam. The construction is such that any such plane may be chosen at will, that is, the chamber and its collecting compartments may be rotated about an axis coincident with the beam of x-rays under investigation.

With polarized radiation the ionization current varies with the angle at which the chamber is placed, being a maximum when the effective plane of photoelectron ejection contains the predominant electric vector of the beam, and being a minimum when the orientation differs from this adjustment by ninety degrees. This, of course, is to be expected on the basis of the recognized asymmetry of photoelectron ejection under the action of polarized x-radiation. In reality the ionization current also includes ions resulting from the absorption in the collecting chambers of fluorescent x-rays emitted by the atoms of the vapor excited by the incident beam. The proportion of such ions to the total number collected is known.

In order to obtain maximum x-ray absorption and still retain long photoelectron range a gas or vapor of high atomic number must be used in the chamber and for this purpose the vapor of methyl iodide is employed. This vapor is often avoided because of its tendency to impair amber insulators so it seems worth while to report that this trouble may be entirely obviated (at least when the vapor is unsaturated) by painting a ring of melted ceresin wax around each insulator so that all surface leakage must cross the wax.

The quantitative determination of polarization as described here rests upon a knowledge of the lateral distribution function for photoelectrons ejected by completely polarized x-rays. Since this distribution has been examined experimentally at least four times, and since the results of the more recent investigations are in satisfactory agreement with theory it cannot be claimed that an additional verification is a necessity at the present time. Nevertheless we have felt it worth while to reinvestigate this distribution by the entirely unique method made possible by the present apparatus. The radiation used was produced by a standard Coolidge tube and filtered through 4.8 mm of aluminum. Ion currents were measured for a number of orientations of the chamber and maxima were obtained in the expected positions. The electron distribution so deduced was decomposed into isotropic and anisotropic portions, the former representing the unpolarized radiation and, according to theory, a small number of L electrons ejected by polarized radiation. The anisotropic portion is plotted against direction of ejection in Fig. 1, the

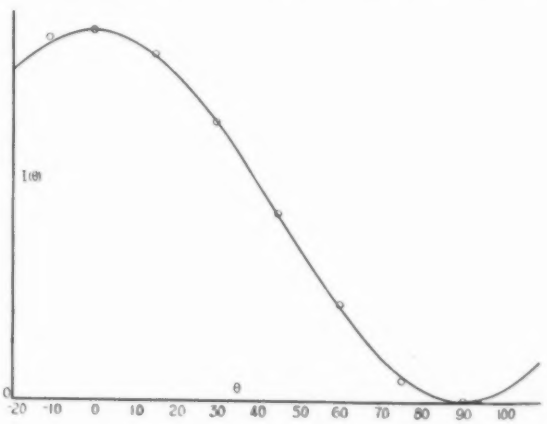


FIG. 1.

circled points representing the observations. In this figure, θ is the angle between the plane of photoelectron ejection and a plane containing the entering beam and the cathode stream of the x-ray tube. The solid curve is a graph of $\cos^2\theta$ against θ, a distribution supported by current theory and the consensus of prior experiments. The ordinate scale of this curve has been arbitrarily adjusted to agree with the experimental point at $\theta =$ zero. The agreement between the data and the curve is very close.

Since the method of polarization measurement outlined here is based upon a measurement of the ionization produced by the primary beam rather than by the minute fraction which is available after scattering through ninety degrees it is expected that this method will be found applicable to the measurement of polarization of x-rays from thin films. This work was started by one of the present writers (P. K.) at Cornell University and it is a pleasure to acknowledge the help and facilities provided by members of the Cornell Physics Department.

DONALD G. HARE
PAUL KIRKPATRICK

Department of Physics,
Stanford University,
June 6, 1933.

Angular Distribution of Cosmic-Ray Particles

The recent cosmic ray[1] survey indicates that at least a part of the cosmic radiation comes to the earth in the form of charged particles, whose paths are influenced by the earth's magnetic field. This suggests an experiment to determine the predominating direction of these particles at several altitudes. To do this the directional intensity of these rays at different angles of elevation was measured in two planes perpendicular to each other. It was hoped to detect any possible shift of the maximum from the vertical which would indicate a predominance of charged particles of one sign. Such a survey was begun at Denver last October.

The apparatus, which was the familiar double Geiger counter arrangement,[2] had been developed for another purpose, but was readily adapted to this work. The counters were 1.5 cm inside diameter, 9 cm in length, and spaced 17 cm between centers. The experiment was performed in a thin roofed frame building whose absorption was negligible. The horizon was clear except for a brick building on the east, whose top subtended an angle of elevation of 40°. The results indicate that the shielding or scattering effect of this building was not significant.

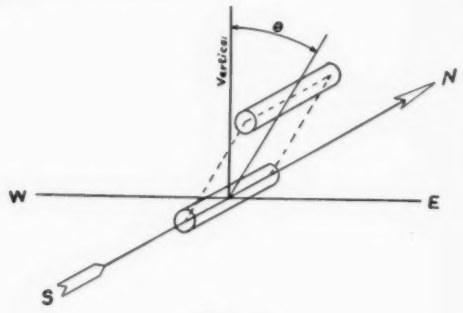


FIG. 1.

The counters were arranged as in Fig. 1 to obtain the data of Fig. 2a, while the data in 2b were determined by rotating the whole structure through 90° about the vertical axis. The points were taken alternately on opposite sides of the vertical, starting with the vertical, to reduce any effects due to slow changes in the sensitivity of the apparatus. Each point represents a run of 24 hours. The constancy of the apparatus was checked each ten days by determining the number of coincidences for a 24 hour period with the counters in a vertical direction. A record of the barometric pressure was kept, but its small variation was not considered significant on the final result.

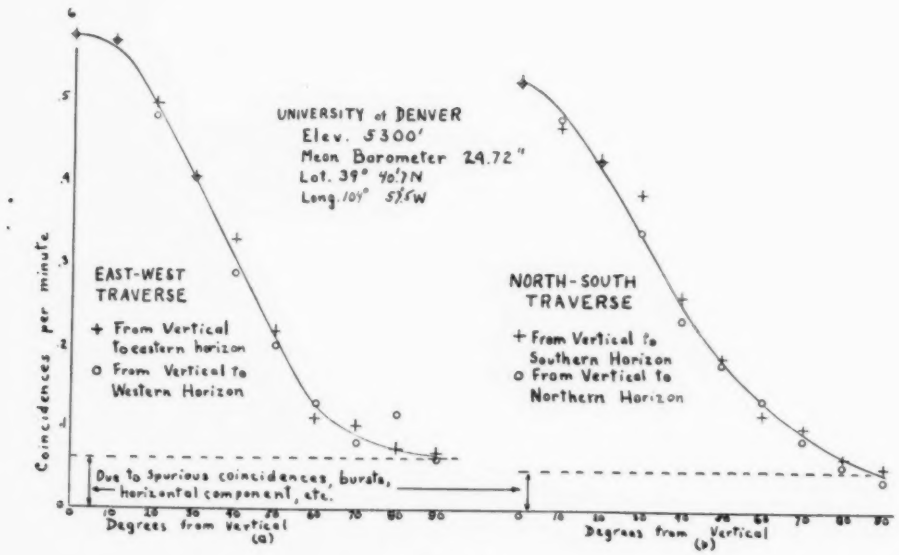


FIG. 2.

During the progress of the experiment the Vallarta-Lemaitre[3] theory of paths of charged particles in the earth's magnetic field appeared. This theory predicts no appreciable asymmetry of the radiation at the latitude of Denver.

The deviations of the points of Fig. 2 from the smooth symmetrical curve are within the range to be expected from a statistical study of the data. This would indicate a maximum intensity in the vertical direction. After the two curves in Fig. 2 had been obtained, it was decided to investigate the intensity in the region of the vertical more

[1] A. H. Compton, Phys. Rev. **43**, 387 (1933).
[2] Bennett, Stearns and Overbeck (in publication).
[3] Lemaitre and Vallarta, Phys. Rev. **43**, 87 (1933).

carefully. Alternate readings of 24 hours' duration were taken 2.5° north and south of the vertical, respectively. Because of an accident, these readings extended over only an eight day period. The results of these observations were 0.540 ± 0.020 and 0.587 ± 0.040 for directions 2.5° north and south of the vertical, respectively. This work is being repeated at the present time.

This result, like the preliminary results recently reported by Johnson[4] is in accord with the theory of Vallarta and Lemaitre.

The equipment was purchased with funds provided by the Rumford Committee.

J. C. Stearns
University of Denver

R. D. Bennett
Massachusetts Institute of Technology

June 7, 1933.

[4] T. H. Johnson, Phys. Rev. **43**, 1059(A) (1933).

Hyperfine Structure and Isotopic Constitution of Tungsten

The arc spectrum of tungsten was excited in a Schüler tube cooled in liquid air, the tungsten lines being very much stronger than those due to the inert gas (argon) used as an activator. Lines in the visible were photographed with a three prism glass spectroscope using silvered Fabry Perot etalons placed in the parallel beam behind the collimator.

Aston[1] reports four isotopes of tungsten of mass and abundance 182, 22.6 percent; 183, 17.2 percent; 184, 30.1 percent and 186, 30.0 percent. Our patterns show but three components which, since they have nearly equal separations, are attributed to the three isotopes of even atomic weight. Microphotometer traces taken on a number of lines show that the intensity relations between the three components differ materially from line to line. This indicates that a fourth isotope is present and shows that the component (or components) due to the odd isotope is differently placed with respect to the three even isotopic components in various lines. This is just what one would expect if the fourth isotope has both a magnetic and a mechanical nuclear moment.

Several lines, arising from transitions into levels involving a single $6s$ electron, are not split even with the largest separators employed, i.e., the isotope shift is practically zero. A preliminary investigation of such lines with small and large separators, having revealed no h.f.s. for isotope 183, indicates that the g-factor is quite small. Although the electron configurations for most of the levels in tungsten have not yet been published, the ground state[2] is reported to be $5d^4 6s^2$, ^5D and the first excited state to be $5d^5 6s$, ^7S. The theory of isotope shift in spectrum lines, as developed by Breit[3] indicates, in general, that the greatest isotope shift is to be expected in levels involving two tightly bound s electrons like $5d^4 6s^2$. Assuming the resolved lines are due to transitions into the ground states, and that the ground states are the levels widely split, the isotope intensities indicate that the levels of the heavy isotope lie deepest. This is in contradiction to the isotope shifts in Pb, Tl and Hg, treated by Breit but in agreement with the shifts in Zn observed by Schüler and Westmeyer.[4]

Norman S. Grace*
H. E. White

University of California,
June 7, 1933.

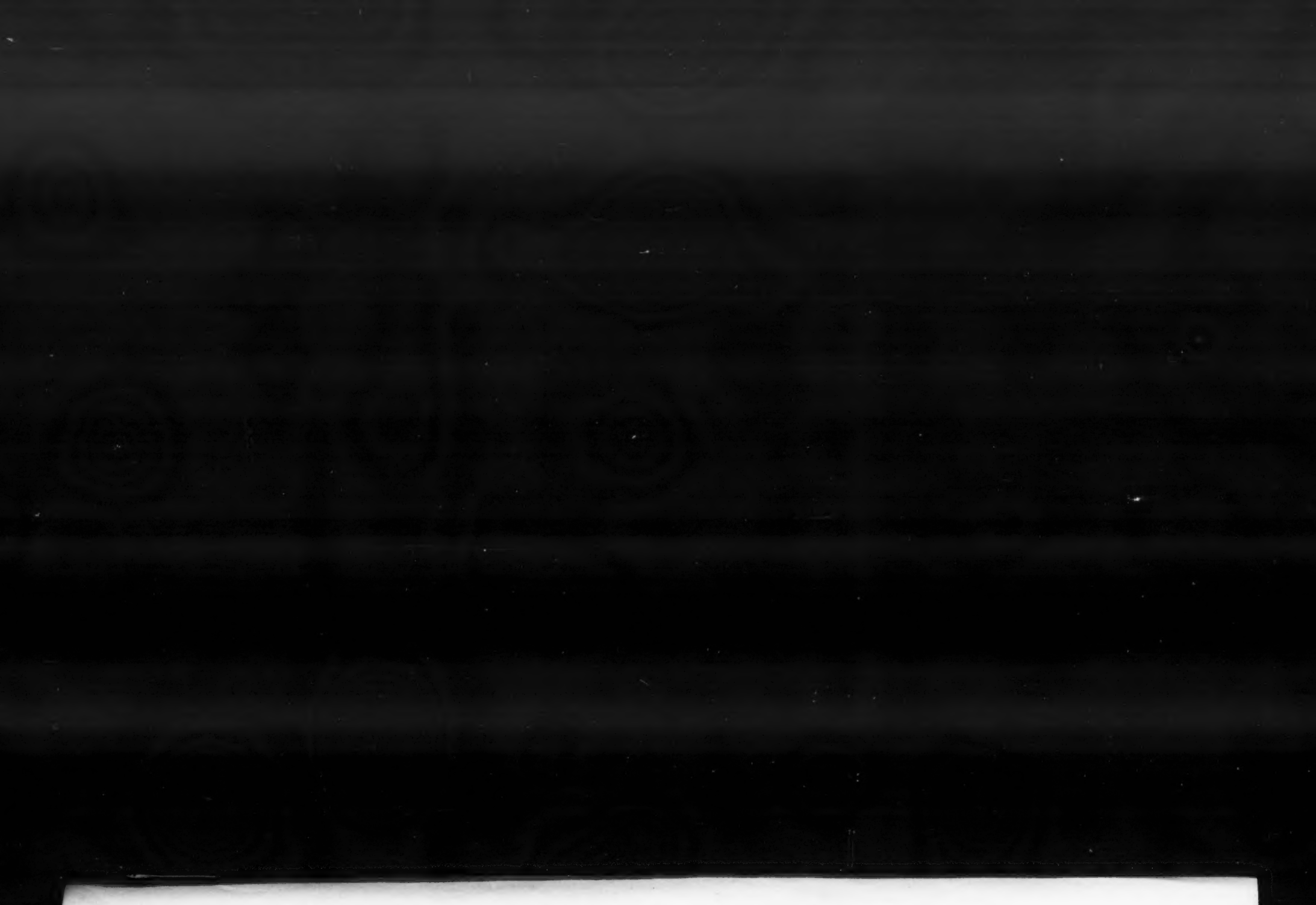
* Commonwealth Fellow.

[1] Aston, Nature **126**, 913 (1930).

[2] Bacher and Goudsmit, *Atomic Energy States*, p. 500.

[3] Breit, Phys. Rev. **42**, 384 (1932).

[4] Schüler and Westmeyer, Zeits. f. Physik **81**, 565 (1933).



Proceedings of the American Physical Society

MINUTES OF THE WASHINGTON MEETING APRIL 27–29, 1933

THE 184th regular meeting of the American Physical Society was held in Washington, D. C. at the Bureau of Standards on Thursday and Friday, April 27–28, and at the National Academy of Sciences on Saturday, April 29, 1933. There were two parallel sessions for the reading of ten minute contributed papers on Thursday morning. On Thursday afternoon there was a session of contributed papers and also a session of invited papers for a special Hydrodynamics Program. Lyman J. Briggs presided at the Hydrodynamics Session and after the program there was a tour of inspection of the Hydraulics Laboratory. The abstracts of these invited papers appear as numbers twenty-seven to thirty-three inclusive in the following pages.

Orchestral Concert in Constitution Hall. Under the auspices of the National Academy of Sciences, and through the cooperation of Music Director Leopold Stokowski, the Philadelphia Symphony Orchestra, and the American Telephone and Telegraph Company, an orchestral concert was given in Constitution Hall, Washington, on the evening of Thursday, April 27th, in demonstration of notable advances recently achieved by the Research Division of the Company in the telephonic transmission of music and its reproduction before a distant audience. The concert program was rendered by the Orchestra in Philadelphia, and the music was transmitted to Washington over three telephone lines, where it was received and its qualities electrically controlled by Doctor Stokowski in Constitution Hall. The National Academy cordially invited the members of the American Physical Society to attend the concert.

There was also a demonstration of interesting phenomena of acoustical perspective and loud speaker characteristics, with various portions of the frequency scale removed, presented by Harvey Fletcher of the Bell Telephone Laboratories, Inc.

On Friday morning the Society met in three parallel sessions for the reading of contributed papers, two on Friday afternoon and one session each on Saturday morning and Saturday afternoon. The presiding officers were Paul D. Foote, President of the Society, A. H. Compton, Vice-President, G. Breit, L. J. Briggs, H. L. Curtis, K. K. Darrow and H. E. Ives.

The Society held a dinner at the Hotel Washington on Friday evening with one hundred and forty-five present. The President presided and the after dinner speakers were L. J. Briggs, A. H. Compton, Th. von Karman, Professor L. von Mintrop from the University of Breslau, Germany, W. F. G. Swann and Professor L. Vegard from the Physical Institute, Oslo, Norway.

Meeting of the Council. At its meeting held on Thursday, April 27, 1933, the deaths of two fellows, Frederick E. Beach and Frederick W. Meyer, and two members, Tillman G. Titus and Albert F. Wagner, were reported. Three candidates were transferred from membership to fellowship and thirty-two were elected to membership. *Transferred from membership to fellowship:* Gerald M. Almy, R. T. Dufford and P. Gerald Kruger. *Elected to membership:* Leonard H. Abbot, Richard P. Bien, Cornelius G. Brennecke, Stephen L. Burgwin, Emma P. Carr, James A. Chiles, Joseph H. Cope, Arnold Euken, Roger S. Firestone, Frank G. Foote, Marvin Fox, A. Pharo Gagge, R. O. Grisdale, Cyril N. Hoyler, Paul H. Keister, Irving S. Lowen, John H. Marchant, George T. Merideth, Madeline M. Mitchell, Richard Proskauer, Gilford G. Quarles, D. H. Rank, Harry Schecter, D. Gordon Sharp, Nathaniel Shear, George A. Smith, Lewis R.

Smith, David R. Tashjian, Takehiko Tesima, Herbert Trotter, Jr., Dana T. Warren and Alfred H. Weber.

The regular scientific program of the Society consisted of one hundred and two papers of which numbers, 55, 63, 67, 72, 88, and 97, were read by title. The abstracts of these papers are given in the following pages. An *Author Index* will be found at the end.

W. L. SEVERINGHAUS, *Secretary*

ABSTRACTS

1. Some Micro-Moving-Pictures Showing the Lethal Effects of Ultraviolet Radiation on Certain Living Protozoa. RACHEL FRANKLIN, A. J. ALLEN AND ELLICE MCDONALD, *Cancer Research Laboratory, Univ. of Penn.*—A series of micro-moving-pictures showing the effects of ultraviolet radiation on certain living protozoa (entosiphon, euglena, colpoda and paramecia) was taken. Ultraviolet radiation below 2900A causes immediate cessation of all motion of the organisms, followed by marked internal changes and in some cases a complete breakdown of the cellular structure. These pictures were taken with a microscope having a quartz optical system for the condenser. A small constricted mercury arc, placed directly under the condenser, served as a source of ultraviolet radiation as well as a source of illumination. The organisms to be studied were placed in a quartz dark field cell, and for observation under normal conditions a glass filter was interposed between the light source and the condenser. When this was removed the organisms received the ultraviolet radiation and the specific effects for each organism were shown in a striking manner. Some ideas as to the mechanism of the killing caused by the photochemical changes which take place can be obtained.

2. Microphotography and Some Radiation Effects on Living Organisms with Various Wave-Lengths of Monochromatic Ultraviolet Light. A. J. ALLEN, RACHEL FRANKLIN AND ELLICE MCDONALD, *Cancer Research Laboratory, University of Pennsylvania.*—A microscope with an all quartz optical system was originally designed to make use of the higher resolving power which could be obtained by using the short ultraviolet wave-lengths to which the glass systems were opaque. Such a microscope equipped with a quartz monochromator and a small constricted mercury vapor arc enables one not only to take microphotographs of high resolving power but also to study the absorption of various parts of micro organisms to the various wave-lengths. It can also be used to study the effects of these radiations on living organisms and to correlate the effects with the absorption. This has been done for some protozoa, microphotographs having been taken with fifteen different lines in the ultraviolet spectrum of the mercury arc. Similar photographs have been taken of sections of various kinds of tissue. Absorption of wave-lengths below 2900A by the nuclei of the cells produces an effect similar to staining. Microphotographs of cancer tissue taken five minutes after an animal has been killed and those of the same tissue taken three hours later show a marked change in the micro absorption spectra of the cells.

3. Diffraction Spectra from Striated Muscle and the Mechanism of Contraction. A. SANDOW, *Biology Department, Washington Square College, New York University.*—Striated muscle consists of long cylindrical cells (fibers) crossed by alternate dark and light bands with a separation between two similar successive bands, in the frog, of about 2.5μ. The frog sartorius, a thin, flat muscle, thus constitutes a natural diffraction grating, better, a set of closely layered gratings, whose lines number about 10,000 to the inch. The entire diffracted spectrum of a beam of white light obtained by using this muscle as a transmission grating has been photographed and in elaboration of previous citations of this effect (Ranvier-1874, Renault-1884, and others) 0, 1st, 2nd, 3rd, and sometimes 4th order spectra appear on the plates. The width of the muscle "grating element" calculated from the angular separation of the various order spectra from filtered blue light agrees with that found by direct measurement, 2.5μ. When the muscle is lengthened by stretching, or shortened by stimulating it to contract, the percentage change in the grating element width equals the percentage change in the muscle length. These results are in agreement with the supposition that the muscle behaves as if it were an elastic ideal grating. The intensity distribution of light among the various spectra formed by a resting muscle and a muscle excited to develop tension but not permitted to contract has been studied in the attempt to follow the change due to excitation in the ratio: width of dark band to width of light band. The results show that this ratio decreases in the excited muscle thus indicating, in agreement with Meyer's theory of muscle contraction, that the contractile elements are localized in the dark bands.

4. An Absolute Determination of the Ohm. HARVEY L. CURTIS, CHARLES MOON AND C. MATILDA SPARKS, *Bureau of Standards.*—A determination of the ohm in terms of the units of length and time has been made. The method depends on the measurement of a self-inductance in terms of time and the present unit of resistance, and on the computation of the inductance from measurements of the dimensions of the inductor. Three single layer solenoids have been measured. They have been constructed with such care that the inductance can be computed from the measured dimensions with an error of only a few parts in a million. The measured inductance can be measured in terms of resistance and time with about the same accuracy. The result is expressed as 1 B.S. International Ohm $= 1.00045_2$ Absolute Ohms.

5. The Resistivity of Expanded Conductors. P. I. WOLD, *Union College.*—At the meeting of the Physical Society in Atlantic City a report was given under the title above, based on experimental data and on theoretical work of Lord Rayleigh. In that report an extension of Rayleigh's simple formula was given for obtaining the fractional volume p occupied by spherical insulators in a conducting matrix, in terms of σ, the relative resistivity with and without insulator. Rayleigh's simple formula is satisfactory to about 30 percent of insulator. The one mentioned above is satisfactory to about 60 percent. A further extension has now been obtained in a form of increasing powers of p, to p^{13}, the coefficients of p each involving products of certain surface spherical harmonics. By a method of approximations, values of p, satisfactory to over 85 percent, can be calculated and a curve has been plotted between p and σ. This curve takes on a universal character, since it is independent of the nature of the conductor. The results are also applicable to many problems similar to flow of electricity, such as a fluid through a barrier, as of oil or water through sand.

6. A Sensitive Ballistic Galvanometer Operating in High Vacuum. W. B. ELLWOOD, *Bell Telephone Laboratories.*—A resonant ballistic galvanometer has been developed especially for precise magnetic flux measurements. The sensitivity is high, about 4 flux turns per mm deflection at one meter scale distance compared with 500 flux turns/mm/m attained by commercial galvanometers. The galvanometer is based upon the principle stated by Maxwell that, if the charge to be measured by a ballistic galvanometer is sent repeatedly through the galvanometer in proper phase, the amplitude of the swings will increase to a steady value which is proportional to the charge and which is greater the smaller the damping. Operating the moving coil in high vacuum eliminates air damping. A flat tape suspension, thin enough to act substantially as a bifilar suspension, reduces elastic hysteresis. Light from the galvanometer mirror acts on a photo-cell controlling the switching mechanism, to ensure the correct synchronization of the repeated impulses. A high accuracy of integration is attained by reducing to a minimum the eddy-currents and hysteresis losses in the galvanometer itself. This is done by avoiding the use of metal in the neighborhood of the coil.

7. Luminous Discharge Between Lecher Wires in a Partially Evacuated Tube. W. D. HERSHBERGER, H. A. ZAHL AND M. J. E. GOLAY, *Signal Corps Laboratories, Fort Monmouth, New Jersey.*—In experiments with a Barkhausen-Kurz oscillator generating undamped 65 cm waves a study was made of the potential difference between the Lecher wires coupled to the oscillator by enclosing these wires in an evacuated glass tube several wave-lengths long. The oscillator itself employed two type −52 tubes connected in push-pull and set up radiofrequency currents of one ampere in the Lecher wires and potential differences in excess of 100 volts between the wires at positions separated by half wave-length intervals. At these voltage loops, for certain critical pressures, a discharge took place between the wires which was photographed as well as employed in a direct study of the standing waves in the wires. The gases used in the tube were air, helium, mercury vapor, and mixtures of the latter two with air. The use of this tube made possible a visual study of the changes in potential distribution as various circuit adjustments were made. A series of double maxima was observed when the wires were mistuned. At certain pressures the losses introduced in the system by the discharge were so great as to cause the oscillations to diminish in amplitude giving rise only to a brilliant but intermittent discharge, playing back and forth from one nodal position to another.

8. The Principle of Dimensional Homogeneity and Aerodynamical Resistance, with Application to the Cup Anemometer. G. GRIMMINGER, *U. S. Weather Bureau, Wash., D. C.* (*Introduced by* W. J. HUMPHREYS.)—There is no adequate theory of the motion of the anemometer cup wheel because the forces involved are too complicated to be treated by existing aerodynamical theory. One must therefore resort to experiment, in which case, the principle of dimensional homogeneity can be used advantageously. This principle is discussed, and it is shown that its application to the thrust and torque exerted on the cup wheel has the effect of reducing the number of independent variables, thus reducing the number of experiments necessary to determine the unknown functions. Its application also shows that the forces exerted on the instrument may be expected to vary with Reynold's number, the effect of which has hardly been recognized heretofore. Experiments are in progress for the purpose of determining this effect as well as the effect of variations in the size of the cup wheel and in the shape of the cups.

9. The Effect of Turbulence on the Rating of Cup Anemometers. HUGH L. DRYDEN, W. C. MOCK, JR., AND G. B. SCHUBAUER, *U. S. Bureau of Standards.*—In recent attempts to determine accurately the rates of the standard three-cup and four-cup Robinson anemometers used by the Weather Bureau for the measurement of wind speed, it was discovered that the rate of rotation depended on the small fine-grained turbulence of the wind stream as well as on the mean speed. When the turbulence was increased from 0.7 percent to 5 percent, the rate of the four-cup standard at a mean speed of 30 miles per hour increased by 8 percent, that of the three-cup by 3½ percent. Measurements of the fine grained turbulence in the natural wind showed that calibration should probably be made with large turbulence.

10. A Review of Observation and Theory of Turbulence in the Atmosphere. LOUIS P. HARRISON, *U. S. Weather Bureau.* (*Introduced by* W. J. HUMPHREYS.)—The various methods of observing atmospheric turbulence are summarized and discussed with regard to the particular aspects of turbulence which they each show most clearly. The results of the direct observations disclosed by these methods are reviewed. From these results an attempt is made to describe atmospheric turbulence in terms of physical models. The various theories which have been proposed to explain and describe turbulence in the atmosphere are next reviewed and their agreement with observation discussed.

Some of the difficulties inherent in these theories are pointed out. Finally, certain desiderata which must be fulfilled by more satisfactory theories are presented and discussed.

11. Magnetic Rotation Spectrum of the Red Bands of Sodium. W. R. FREDRICKSON AND CARL R. STANNARD, *Syracuse University.*—The magnetic rotation spectrum falls into two distinct sections; the first, from the *D* lines to about λ6800A, in which the lines appear more or less as individual lines, and the second, to the red of λ6800A, where the lines appear in groups. When the vapor pressure of sodium in the absorption tube is relatively low, the first region shows only two strong series of doublets, the doublet spacing increasing very rapidly as a series progresses to lower frequency. The low-frequency component of a doublet has positive rotation and the other, negative. The doublet is interpreted as a *P* and *R*-branch line having a common K'. The two series turn out to be a v' progression ($v''=0$) arising from different K' levels in the upper vibrational states,—the K' values in the states 1 to 9 being 60, 52, 43, 31, 16, 58, 51, 40 and 28, respectively. Corresponding doublets for other v'' values are found on the high vapor density plates and are arranged in a wide Condon parabola. In trying to extend the doublets to the far red region where the groups of lines occur, one has difficulty because of the great number of lines and the uncertainty of the B'' values for large v''. The appearance of the doublets and groups strongly suggests the presence of another state, probably $^3\Pi$, closely parallelling the $^1\Sigma'$ and causing perturbations.

12. Pure Rotation Absorption Spectra of NH_3 and PH_3 Gases Under High Resolving Power. H. M. RANDALL AND NORMAN WRIGHT, *University of Michigan.*—A large aperture spectrometer employing a reflection grating of the echelette type ruled to concentrate radiation in the region of the first order of 90μ has yielded a resolving power several times that hitherto obtained in the region between 50 and 125μ. The pure rotation absorption spectra of the two gases ammonia and phosphine have been investigated. In the case of NH_3 the absorption lines shown as single in the work of Badger and Cartwright have been shown to be doublets with a separation of 1.33 cm^{-1} under the higher resolving power obtained in the present work. Despite the similarity of the PH_3 molecule to that of NH_3 the PH_3 pure rotation lines have been found to be single and quite sharp. Determinations of the A and C moments of inertia of the NH_3 molecule and of the A moment of inertia of the PH_3 molecule have been made.

13. The Infrared Spectrum of CO_2. ARTHUR ADEL AND DAVID M. DENNISON, *University of Michigan.*—About thirty infrared bands of CO_2 are now known with great precision, thirteen having been mapped during the past year. The qualitative features of the spectrum are well explained by the theoretical work of Fermi and Dennison which, however, fails to predict the exact positions of the bands. A second order perturbation involving eleven anharmonic coefficients and featuring the resonance degeneracy of the system accounts for the entire infrared spectrum to within the experimental error. A second mode of attack on the anharmonic constants which define the potential is through an analysis of the interaction between rotation and vibration. The convergence of the fine structure lines is due to the displacement of the atomic equilibria positions in the anharmonic force field by the centrifugal forces of rotation. The slight asymmetry of the excited molecule is included in the analysis. An attempt has been made to utilize these anharmonic constants in establishing a closed analytical potential function for the system. It resembles the Morse function for diatomic molecules and possesses but four constants. The solution to the entire problem is characterized by an excellent agreement with experiment.

14. The Absorption Spectrum of Diatomic Bismuth. G. M. ALMY AND F. M. SPARKS, *University of Illinois.*—The absorption spectrum of bismuth, previously observed but not analyzed, has been obtained by heating bismuth in an atmosphere of nitrogen in a carbon tube furnace at temperatures from 800°C to 1500°C, and also by heating in an evacuated quartz tube, up to 1100°C. Vibrational analyses have been made of four systems of bands: (1) a visible system, 4500A to 8000A, consisting of some 300 bands which can be arranged in v', v'' array extending to $v'=59$, $v''=55$. $\omega_e''=173.4$, $x''\omega_e''=0.41$ (normal state of Bi_2); $\omega_e'=132.5$, $x'\omega_e'=0.33$; (2) an ultraviolet system, 2600A to 2900A, of about 40 bands having the same lower state as (1) and $\omega_e'=156$, $x'\omega_e'=0.7$; (3) a far ultraviolet system, from 2250A down, having the same lower state as (1); (4) a violet system, 4050A to 4200A, of 15 bands, appearing only above 1000°C. ((1), (2), (3) are well developed at 900°C.) The lower state of this system is probably the upper state of (1) while $\omega_e'=130$, $x'\omega_e'=9.7$. Assuming the lowest state dissociates into two normal $^4S_{3/2}$ Bi atoms, satisfactory extrapolations to atomic states can be made in all cases, including three observed regions of continuous absorption. The heat of dissociation of the lowest state is 1.8 volts.

15. Vibrational Analysis of the GeO Emission Band Spectrum. R. WILLIAM SHAW, *Cornell University.*—Fifteen bands in the region λ2990–λ2500 have been observed in an arc with electrodes composed of graphite rods cored with fused germanium dioxide. Measurements on the heads of the bands have been made visually and from microphotometer records. A vibrational analysis based on these data yields the following equation:

$$\nu=37{,}763.97+6[47.9(\nu'+\tfrac{1}{2})-3.4(\nu'+\tfrac{1}{2})^2]$$
$$-[984.0(\nu''+\tfrac{1}{2})-4.1(\nu''+\tfrac{1}{2})^2].$$

This analysis is substantially the same as one communicated to the author by W. F. C. Ferguson of New York University. The rotational structure near the band heads is completely unresolved with the instruments used thus far. Considerable overlapping of the bands is apparent, and moreover the vibrational analysis indicates that a number of faint bands are obscured by the strong ones observed.

Thus progress in rotational analysis and in the detection of isotope effects has been hampered.

16. Quenching of Iodine Fluorescence by Iodine and Argon. JAMES F. KOEHLER, *Smith College.*—In experiments conducted at Princeton University the fluorescence intensity of iodine between 0.03 mm and 0.8 mm when excited monochromatically by the mercury arc, has been measured by photographic photometry with a spectrograph. Refinements of the work of Wood and Speas (Phil. Mag. [6] **27**, 531 (1914)) consist in monochromatic excitation and corrections for absorption changes of exciting light and reabsorption of fluorescence. Absorption measurements under similar conditions have been made with two compensating photoelectric cells. The fluorescence and absorption data are combined to evaluate the ratio of the rate of dissociation of excited molecules to the rate of radiation at given concentrations. At 0.03 mm this ratio is 0.42 for the green line and 0.55 for the yellow; at 0.2 mm these are 2.8 and 3.6. This radiation includes energy transferred and radiated from nearby states. Yellow fluorescence is quenched by iodine more than green, indicating that collision quenching is not due to enhancement of predissociation. Similar measurements of fluorescence and absorption were made with varying amounts of argon up to 16 mm. The quenching efficiency of argon is only one-twentieth of iodine.

17. Electro-Optical Kerr Effect in Gases. C. W. BRUCE, *University of Virginia.* (*Introduced by* J. W. BEAMS.)—The variation of the Kerr effect with density in carbon dioxide has been measured with increased precision for densities up to 0.32 g/cc. The temperature variation in carbon dioxide has been measured for temperatures 7°C and 35°C. The variation with density and temperature is about that expected from the Langevin-Born theory. A method is given for obtaining the absolute value of the Kerr constant in gases at high pressures. Preliminary values have been obtained for carbon dioxide and nitrogen.

18. Kerr's Law at High Field Strengths. JAMES A. HOOTMAN, *University of Virginia.*—Deviations from Kerr's law for liquids subjected to intense electric fields have been quantitatively investigated by the use of improved compensation methods similar to that of Beams (Phys. Rev. **37**, 781 (1931)). The law was found to hold, within the limits of precision of one-eighth of one percent, for pure nonpolar carbon disulphide up to the highest field intensity the liquid could sustain. In the case of polar liquids ethyl ether and chloroform, on the other hand, easily measurable deviations were observed. Curves showing the departure from Kerr's law at high field strengths are given for ethyl ether at −78.5°C and for chloroform at −22.5°C.

19. An Improved Magnetic Velocity Selector for Molecular Beams. I. I. RABI AND V. W. COHEN, *Columbia University.*—The magnetic velocity selector previously described (Phys. Rev. **43**, 377 (1933)) has been improved by the addition of a magnetic field at the end of the path of the beam. The analyzing field is between the selector slit and this last field. The field is so arranged as to produce a deflection opposite to that of the velocity selector. The advantages are twofold. The selected beam is considerably sharpened allowing better spatial resolution. Secondly the selected beam is brought away from the region where scattered atoms from the small tail of the beam would interfere with the deflection pattern. When one selects small velocities ($\bar{c}/2$ and $dv/v=1/10$) the intensity is less than one-thousandth of the original beam intensity. This amount is scattered into the same region from the tail of the beam. This form of the magnetic velocity selector has been applied to the measurement of the nuclear spin of sodium.

20. The Spin of Li^7 from Hyperfine Structure Data. NEWTON M. GRAY, *New York University, University Heights.*—It has been shown by Güttinger and Pauli and by Goudsmit and Inglis that the *hfs* of the 5485A ($1s\ 2s\ ^3S - 1s\ 2p\ ^3P$) line of Li II as observed by Schüler is in fair agreement with a spin $i=3/2$. According to Granath's newer measurements on components (1) (2) (3) the interval ratio of the $1s\ 2s\ ^3S - 1s\ 2p\ ^3P_0$ part of the pattern agrees closely with the theoretically expected value for this i. The calculations of Güttinger and Pauli have been corrected for the effect of the $2s$ electron and carried through for $i=3/2$, 2, 5/2. The part of the pattern due to $^3P_{1,2}$ is not very sensitive to the spin. Some observable differences were found. Thus the distance between components (3) and (5) should differ by about 0.1 cm^{-1} for $i=3/2$ and 5/2 and Schüler's observations are in agreement with 3/2. Unpublished photographs and measurements of Granath were used to redetermine this and other distances and were found to be in very close agreement with Schüler's. The observed pattern agrees best with $i=3/2$, $i=2$ is barely possible while $i=1$, 5/2 are definitely excluded.

21. Hyperfine Structure in Intermediate Coupling and Nuclear Magnetic Moments. G. BREIT AND LAWRENCE A. WILLS, *New York University, University Heights.*—The theory is extended to intermediate coupling and to cosine laws of force between nucleus and electron varying with different powers of their distance apart. In addition to Goudsmit's constants a', a'' there appears a third coupling constant a''' for each electron, which gives energy matrix elements for the same l and different j. The theory is in approximate agreement with observation. The gross structure is used to determine the parameters of the theory of intermediate coupling and hence $\overline{1/r^3}$. This together with a'' gives nuclear magnetic moments having roughly the same value for different configurations and stages of ionization. Approximately g is ($\mu=gi\mu_0/1840$) for Tl 3, Bi 1.1, Pb^{207} 1.5, Hg^{199} 0.8, Sb^{121} 1.3. The variation of $1/r^3$ as determined from gross structure shows changes in screening agreeing with those supposed to exist for $(6s)^2$ of Pb I, Tl I in order to explain the isotope shift as due to differences in nuclear radii. The discrepancies between gross structure and theory are shown to be connected with similar discrepancies in *hfs*.

22. On the Magnetic Moment of the Nitrogen Nucleus. R. F. BACHER, *University of Michigan.*—The lines λ5329

($2s\,2p^4\,{}^4P_{2\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{2\frac{1}{2}}$), λ5357 ($2s\,2p^4\,{}^4P_{1\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{2\frac{1}{2}}$), and λ5373 ($2s\,2p^4\,{}^4P_{\frac{1}{2}}-2s^2\,2p^2\,4p\,{}^4D_{1\frac{1}{2}}$) of N I have been examined for hyperfine structure with an interferometer, using various plate separations from 5 mm to 29.5 mm. Although the levels of the $2s\,2p^4$ configuration are expected to show hyperfine structures roughly twenty-five times as large as any of the usual levels of N I, the lines investigated always appeared single. Since the nitrogen nucleus is known from band spectra to have a *mechanical* moment of $1\ h/2\pi$, the absence of hyperfine structure must be due to the small size of the nuclear magnetic moment. With the expressions recently applied by Goudsmit, the measured line width of λ5329 indicates that the nuclear magnetic moment is $\leqq 0.2$ "proton magneton." ($eh/4\pi Mc$.) If the nuclear spin is due to the spins of component particles and if, as has been supposed, the nitrogen nucleus consists of a closed group plus one proton and one neutron, then this low value for the nuclear magnetic moment means that the neutron must have a magnetic moment in the opposite direction from its mechanical moment and of the order of one proton magneton.

23. Anomalies in Hyperfine Structure. S. GOUDSMIT AND R. F. BACHER, *University of Michigan.*—Deviations from the interval rule have been found by Schüler and Jones (Zeits. f. Physik **77**, 801 (1932)) in two adjacent levels of the $6s\,6d$ configuration of Hg I. This anomaly can be explained satisfactorily by the fact that in this case the hyperfine structure is of the same order of magnitude as the separation of the two levels, causing a distortion similar to the Paschen-Back effect. The method used is the same as that for change in coupling (Phys. Rev. **35**, 1325 (1930)). The hyperfine structure separations for the $3s\,4f\ {}^3F$ and ${}^1F$ levels of Al II found by Sawyer and Paschen are unusual in that they do not agree with the theoretical formulas and cannot be explained as due to perturbations by other levels. It can be shown that this hyperfine structure, which is observed only in transitions from the higher members of the $3s$ ng series to the $3s\,4f$ levels, is partially that of the $3s$ ng states. The latter all have the same hyperfine structure which is larger than their multiplet structure and singlet triplet distance.

24. Paschen-Back Effect and the Polarization of Resonance Radiation. Sodium ($5^2P_{\frac{1}{2},\frac{3}{2}}-3^2S_{\frac{1}{2}}$). N. P. HEYDENBURG, *State University of Iowa.*—Preliminary data on the polarization of the resonance radiation of the second doublet of sodium when excited by nonpolarized light with a magnetic field applied parallel to the direction of the incident beam shows that the polarization increases with increasing field strength reaching a maximum of slightly over 33 percent in fields of less than 100 gauss. In order that the Paschen-Back effect of the hyperfine levels be completed in fields of this order of magnitude the separation constant of the upper hyperfine levels must be of the order $5\times10^{-4}\,\mathrm{cm}^{-1}$.

25. Absorption and Emission of Hyperfine Structure in Optically Excited Mercury Vapor. M. L. POOL AND S. J. SIMMONS, *Ohio State University.*—Radiation from a mercury arc was passed through a resonance tube containing various lengths of optically excited mercury vapor and pressures of admixed nitrogen. Both components of 2967 showed strong absorption. None of the components of 4358 and 5461 showed any measurable absorption. The absorption of the five components of 4047 was strong and increased with increase of admixed nitrogen. The strongest component showed the strongest absorption and the amount of absorption decreased with decrease in intensity of the components. The intensity of each component decreased nearly exponentially with increase in length of excited mercury vapor. The linear absorption coefficient for each component increased with increase of nitrogen pressure. In fluorescent emission the four strongest components of 4047 were observed. However, in fluorescence the weaker components were much weaker in comparison to the central component than in an arc.

26. The Arc Spectrum of Sulfur in the Ultraviolet. J. E. RUEDY, *Cornell University.*—A Schüler tube with a water-cooled aluminum cathode, with He gas for maintaining the discharge, was attached to a vacuum spectrograph, employing a 1.5 meter grating. The sulfur spectrum was excited by vaporizing some sulfur in the tube while the discharge was in operation. The maximum potential used was 2000 volts d.c., and currents up to 2 amp. could be maintained without any injury to the tube. The wavelength range photographed was from 0 to 2600A. The spectra of both S I and S II were strongly excited, and no certain means of distinguishing them was found, although varying the discharge conditions gave some indication. Under proper conditions, strong reversal of lines due to transitions into the low or metastable states was obtained, and this aided greatly in classifying some of the singlet lines. All the ultraviolet lines reported at various times by Hopfield, and the revised classification of them given by Frerichs (Zeits. f. Physik **80**, 150 (1933)) (except that the total quantum numbers of the d electrons, as he gives them, appear to be too large by one) are confirmed. In addition, transitions have been found which permit the following new term values to be added to Frerichs' table.

$$3p^4\begin{cases}(^2D)^1D_2-74{,}315\\(^2P)^1S_0-64{,}522\end{cases}\qquad 3p^34s\begin{cases}(^2P)^3P_0-6418\\(^2P)^3P_1-6403\\(^2P)^3P_2-6373\end{cases}$$

$$3p^34s\begin{cases}(^2D)^1D_2-14{,}315\\(^2P)^1P_1-\ 5264\end{cases}$$

Several higher members of some of these and other series have also been observed. A complete report of this work will be published later.

27. Theory and Experiment in Fluid Dynamics. L. B. TUCKERMAN, *Bureau of Standards.*—Until recently fluid dynamics offered a most striking example of well-founded theory apparently contradicting well-established experimental results. As a consequence practical hydrodynamics in river and flood control, in water works, in water power utilization, and in ship design developed on a basis largely empirical. Within the last few decades, chiefly under the spur of the demands of aviation, a reconciliation of theory

and experiment has been achieved. The contradictions are now mere paradoxes. However, this reconciliation has made even more clear that, for a long time to come, direct experiment must form the basis of practical hydro- and aerodynamics. The general theory of fluid dynamics can, at present, furnish only the framework into which the experiments may be fitted. (10 minutes, by invitation.)

28. Our Present Knowledge of Turbulent Flow. TH. VON KARMAN, *California Institute of Technology.* (30 minutes, by invitation.)

29. Studies of Boundary Layer Flow at the Bureau of Standards. HUGH L. DRYDEN, *U. S. Bureau of Standards.*—The flow of air and water about obstacles has been found to depend on the state of motion in the "boundary layer," that thin region near the surface of the obstacle in which the frictional forces are comparable with the inertial forces. The simplest case of boundary layer flow is that near a thin flat plate of great length and width placed parallel to the flow. The distribution of mean speed near a plate has been measured at the Bureau of Standards in a wind tunnel in which the turbulence was smaller than that in the experiments of Zijnen and of Hansen and quantitatively known. The transition from laminar to turbulent flow occurred at a greater distance from the front of the plate than in their experiments. The amplitude of the fluctuation of speed in the boundary layer was also measured. The surprising result was obtained that in the laminar flow near the front of the plate, the fluctuations were nearly as large as in the turbulent flow. Computations were made, by a modification of Pohlhausen's method, of the flow in a boundary layer subjected to pressure gradients produced by a small sinusoidal speed fluctuation superposed on the steady flow outside the layer (simplified turbulence in the external flow). The computations indicate that the fluctuations in the external flow are amplified within the layer. Separation finally occurs, a probable preliminary to the beginning of turbulent flow. The computed distribution of the amplitude of the fluctuation corresponds qualitatively to that measured experimentally.

30. Tides. P. SCHUREMAN, *Coast and Geodetic Survey.* (*Introduced by* F. L. MOHLER.)—The tides of the ocean are primarily the result of a system of oscillations known as stationary waves, these oscillations being maintained by periodically applied forces exerted by the moon and the sun. The principal periods of these forces correspond approximately to the day and the half day, the effect of the latter usually predominating. The daily forces depend upon the declination of the moon and sun. The free period of a stationary wave depends upon the depth and length of the oscillating body of water and when these are such as to provide for a free period corresponding to one of the periodic tidal forces a large amplitude of oscillation can be sustained. While in all oceans areas can be delineated which will favor semidiurnal oscillations, we must look to the larger oceans for conditions most favorable to the diurnal oscillations. Thus the diurnal oscillations in the Pacific Ocean are much larger than in the Atlantic Ocean. Disregarding the peculiarities in the tide which may be occasioned by shallow water conditions, the type of tide in any locality depends upon the amplitudes and phase relation of the diurnal and semidiurnal waves in that locality.

31. The Present Status of Knowledge of Pressure Loss for Flow in Curved Pipes. G. H. KEULEGAN, *U. S. Bureau of Standards.* (*Introduced by* L. J. BRIGGS.)—Although the problem of pressure loss for flow in curved pipes has been attacked by many investigators during the last fifty years, our knowledge regarding this phenomenon is far from being definite. This is true particularly for the regime of turbulent flow. The results of various experiments on the coefficient of loss are not in accord, except perhaps for the qualitative result that the loss is a minimum for bends for which the ratio of the radius of bend to radius of pipe falls within certain limits. The many attempts to analyze the different factors contributing to the loss have established the existence of transverse currents and regions of separation of flow. Unfortunately, these phenomena cannot as yet be treated theoretically. On the contrary, theory has been of considerable assistance when laminar flow exists. (10 minutes.)

32. Capacity of Overflow Troughs. K. H. BEIJ, *U. S. Bureau of Standards.* (*Introduced by* L. J. BRIGGS.)—Overflow troughs are channels having uniform inflow along their entire length. Familiar examples are roof gutters, constant-level tank troughs and wash water troughs. Side channel spillways present the same problem on large scale. The discussion applies to rectangular troughs discharging freely at one end, assuming the inflow imparts no momentum in direction of flow, vertical velocities are negligible and velocity is constant over all parts of a cross section. Then, $d/dt(MV)=G+F$, where M = mass passing a section in unit time, V = velocity, G = gravitational force in direction of flow, F = frictional forces. If friction is small and troughs are level, $q^2x^2/gb^2=-h^3/2+hC$, where q = quantity of inflow per unit length, x = distance from upper end, b = width of trough, h = depth. The depth at the outflow end approximately equals the critical depth: $H=(q^2l^2/gb^2)^{\frac{1}{3}}$ where l = length of trough. This is confirmed by experiment. Using this relation to evaluate C, we find: $(h/H)^3-3(h/H)+2(x/l)^2=0$. The maximum depth, which occurs at upper end, is $h_m=3^{\frac{1}{2}}H$. Comparison with observations shows that, when the maximum depth is about the same as the trough width, the discrepancy is of the order of 5 percent. (10 minutes.)

33. The National Hydraulic Laboratory. H. N. EATON, *U. S. Bureau of Standards.* (*Introduced by* L. J. BRIGGS.)—The greatest advances in science of hydraulics have been made through controlled experiments in the laboratory, frequently, though not always, guided by theory. Until the beginning of the present century, however, hydraulic laboratories were relatively few and progress was correspondingly slow. About thirty-five years ago, what has been called aptly "the new awakening in hydraulic research" began to spread through Europe as a result of

model experiments on river problems in the laboratory of Engels in Dresden, and during the next 20 years laboratories were built in all parts of Europe. When the awakening reached the United States about 15 years ago, we had no hydraulic laboratories comparable with the best in Europe. The movement to establish a National Hydraulic Laboratory was a concerted effort on the part of the engineering profession to remedy this situation and to have a large adequately-equipped laboratory established in this country for the purpose of carrying out hydraulic investigations which require continuous work over relatively long periods of time and which are often too expensive to be conducted by most engineering colleges. (By invitation, 15 minutes.)

34. Spectra of Cb V and Mo VI. M. W. TRAWICK, *Cornell University.*—The rubidium-like spectra of columbium and molybdenum have been excited in a vacuum spark and measured by means of a vacuum spectrograph. Identifications of lines corresponding to the transitions $5s\ ^2S-5p\ ^2P$, $5p\ ^2P-5d\ ^2D$, $5p\ ^2P-6s\ ^2S$, $4d\ ^2D-5p\ ^2P$, and $4d\ ^2D-4f\ ^2F$ for Cb V and Mo VI have been facilitated by extrapolation of the frequencies of the corresponding lines in the spectra of the four preceding elements in the Rb I isoelectronic sequence. The relative term values $4d\ ^2D$, $5s\ ^2S$, $5p\ ^2P$, $4f\ ^2F$, $5d\ ^2D$, and $6s\ ^2S$ have thus been determined. The Rydberg formula applied to the $4d\ ^2D-5p\ ^2P$, $5p\ ^2P-5d\ ^2D$ and $5s\ ^2S-5p\ ^2P$, $5p\ ^2P-6s\ ^2S$ transitions gives results for the absolute term values, which, after correction by the method of Russell, are in substantial agreement with the values found, when $\Delta n=0$, by extrapolating $\nu^{\frac{1}{2}}$ for the terms of the sequence in such a way as to maintain the validity of the irregular doublet law.

35. The Configuration p^5p of the Rare Gases. GEORGE H. SHORTLEY, *Princeton University.*—Quantum mechanically, the ten energy levels of the configuration p^5p are given as the solutions of a quartic, a cubic, a quadratic, and a linear equation involving six parameters (radial integrals): three electrostatic, two spin-orbit, and one giving the absolute height of the configuration. These equations are difficult to handle and to visualize. But by neglecting the interaction between states which have different levels of the ^2P of the p^5 ion as parents in the *jj*-coupling scheme, one obtains six linear and two quadratic equations which are very easy to handle. This approximation is quite valid for the higher series members of neon and argon ($6p$, $7p$, $8p$) and for all members of p^5p for krypton and xenon. One of the parameters is given by the splitting of the parent doublet, and if the others are determined from some of the levels the calculated positions of the rest agree better with the observed data than has been the case for any configuration except p^5s and d^9s. This gives the states exact specification in the *jj*-coupling scheme, which permits calculation of intensities, Zeeman effect and Stark effect for these configurations. A similar reduction has been made for p^5d, with corresponding agreement.

36. Configuration Interaction in Complex Spectra. C. W. UFFORD, *Princeton University.*—The electrostatic interaction, $\Sigma(e^2/r_{12})$, between the multiplets in Russell-Saunders coupling of the electron configurations d^3ns, ds^2, d^3 and $d^2(n+1)s$ has been calculated and compared with experiment for Ti II and Zr II. The matrix elements between the quartets of d^2s and d^3 vanish. Since there is no electrostatic interaction between different kinds of multiplets, interactions exist between all the doublets and between the quartets of d^2ns and $d^2(n+1)s$. This means that these multiplets cannot be assigned to any single electron configuration, but have eigenfunctions which are linear combinations of those of the multiplets belonging to each configuration separately. Secular equations are obtained for the energy levels, the order of each equation being equal to the number of multiplets of the same kind which occur in all the electron configurations. The energy levels obtained as solutions of these equations still contain integrals over the radial parts of the central field eigenfunctions as parameters. Only 7 parameters are used to determine the 16 energy levels to be compared with experiment. The energy levels agree distinctly better with the observed values than do those calculated neglecting configuration interaction.

37. The Interaction of a Valence Electron with a Closed Shell. M. H. JOHNSON, JR. (*National Research Fellow*) AND G. BREIT, *New York University, University Heights.*—The magnetic interaction terms between two electrons are well known (Heisenberg, Zeits. f. Physik **39**, 499 (1926); Breit, Phys. Rev. **34**, 553 (1929)). By using antisymmetric wave functions the diagonal terms of the energy matrix for this interaction are calculated for an electronic configuration consisting of a valence electron and a closed shell. From the diagonal elements the doublet separation is easily obtained by applying the energy sum rule. The direct integrals, that is terms which would occur even though the wave functions were not symmetrized for the Pauli principle, yield the expected contribution to the doublet separation, namely the energy due to the interaction of the spin of the valence electron with the electric field produced by the closed shell. The exchange integrals give a contribution which decreases the doublet separation in the special cases examined. An attempt has been made to evaluate them for the *f* terms of the Cs I by using approximations to Hartree functions for inner shells and hydrogenic functions for the valence electron. The effect apparently is too small to account for the inversion of the doublets but because of the sensitivity to the type of wave functions used this result is not certain.

38. Atomic Wave Functions. F. W. BROWN AND C. G. DUNN, *University of Illinois.* (*Introduced by* J. H. BARTLETT.)—A Hartree field for normal boron has been found, and the radial electron functions represented by analytic expression of the form devised by Slater. The energy parameters for the $1S$, $2S$, and $2P$ electrons are found to be approximately 15.50, 0.915, and 0.437, respectively, in units of the ionization potential of hydrogen.

39. Perturbations Due to Spin-Orbit Forces in CO and Other Band Spectra. J. H. VAN VLECK, *University of Wisconsin.*—The perturbations encountered by Rosenthal and Jenkins (Proc. Nat. Acad. **15**, 381 and 896 (1929)) in

the CO Angstrom bands were shown by them to be due to disturbance of a $^1\Pi$ by a $^3\Pi$ state. This contradicted Kronig's rule that perturbations exist only between states of like multiplicity. Kronig, however, considered perturbations which are due to rotational distortion. It is shown that magnetic "spin-orbit" forces cause perturbations between $^1\Pi$ and $^3\Pi$, as well as between the following other pairs ($^1\Sigma$, $^3\Pi$), ($^1\Pi$, $^3\Sigma$), ($^3\Sigma$, $^3\Pi$). If the $^3\Pi$ is nearly case *a*, then $^1\Pi$ is perturbed strongly by $^3\Pi_1$ and weakly by $^3\Pi_0$, $^3\Pi_2$, while just the reverse is true in case *b*. In intermediate cases, all three components of $^3\Pi$ are effective perturbers of $^1\Pi$ and cause distortions comparable with those due to the Kronig rotational effect, except that the latter increase more rapidly with J. Allowance for Λ-doubling increases the number of perturbing components from three to six, about the number found by Rosenthal and Jenkins. The Zeeman behavior agrees with Watson's observations if the $^3\Pi$ perturber is more nearly case *b* than *a*.

40. The Structure of Ethylene and the Theory of the Double Bond. W. G. PENNEY, *University of Wisconsin.* (*Introduced by* J. H. VAN VLECK.)—Calculations have been made by the Heitler-London method to determine which of several plausible arrangements of the ethylene molecule is the most stable. The wave functions for the C atoms are based on the aggregate configuration sp^3; consequently the Coulomb energies are the same for all models. The signs and magnitudes of the various exchange integrals can be determined from the work of Bartlett, Mulliken and others. It appears that the model involving strictly tetrahedral wave functions, suggested by Pauling and Slater, is not as stable as another type of plane model which utilizes a somewhat different choice of directed wave functions. The electrons constituting the double bond in the tetrahedral configuration are not as favorable an arrangement as that involving one $\pi-\pi$ bond and another $(\sigma, s)-(\sigma, s)$ directed along the C−C axis. The hybridization terms are of paramount importance, and to get the full benefit of them, the HCH angle must be large, perhaps about 140°. Another possibility, more difficult to eliminate, is that where the HCH angle is a right angle, and the HCH planes are parallel to each other, an arrangement which possesses the curious property of free rotation around the C−C joint. Again, it is the hybridization terms which lead to the rejection of this model.

41. Note on Contact Potential Difference. A. T. WATERMAN, *Yale University.*—Assuming Fermi distribution among the conduction electrons in a metal the condition for equilibrium between two metals in contact at the same temperature is: $A_1=A_2 \exp (V_1-V_2)\epsilon/kT$, where A, the Fermi constant, is given here by $\log A=\zeta/kT$, ζ being the thermodynamic potential (Sommerfeld's W_i), and V is the average potential within the metal. Thus $\zeta_1-\zeta_2=(V_1-V_2)\epsilon$, (the Volta or junction P.D.). If the work functions of the metals are different electrons will be displaced across the boundary. Let $V=w+v\epsilon$, where $w/\epsilon=$intrinsic potential (Sommerfeld's W_a), and $v=$potential due to loss or gain of electrons. Then $(v_1-v_2)\epsilon=\phi_2-\phi_1$, where $\phi=w-\zeta$, the thermionic work function. If v_1-v_2 is taken to be the contact P.D. it has the correct value and sign, and is interpreted as the P.D. *between the metals themselves* (in the ordinary sense), in a manner similar to the P.D. of a charged condenser. On examination this interpretation appears to be consistent with the facts. This P.D. is shown to exist between any metals whose thermionic atmospheres are in communication, is strictly independent of the distance between them and independent of temperature insofar as are ζ and w.

42. The Constitution of Metallic Sodium. E. WIGNER AND F. SEITZ, *Princeton University.*—The free-electron picture of metals is employed in an investigation of the binding properties of metallic Na, such as heat of evaporization, lattice constant, and compressibility, under the assumption that the forms of the closed shells are not altered by metallic binding. The boundary conditions to be satisfied by the wave functions of the outer electrons are derived from consideration of crystal symmetry and the work of Bloch. The Schroedinger equation for each electron is approximated by a spherically symmetric one about each atom which is solved using the field of Prokofjew. The extent to which interactions of electrons are contained in the picture is discussed. It is found that to each value of the lattice constant, d, there corresponds a definite energy that is to be associated with the electron in the lowest state. This energy is determined as a function of d. Because of the Pauli principle, only two electrons will possess this energy and the mean energy of all of the electrons is obtained by adding an appropriate correction. The resulting curve possesses a minimum below the energy of the free atom, and from its characteristics the binding properties are determined with favorable agreement.

43. Crystal Structure of Sodium Uranyl Acetate. I. FANKUCHEN, *Cornell University.*—Powder, rotation, and oscillation photographs of sodium uranyl acetate were made. The powder photographs showed that it is cubic, a = 10.69A and there are four molecules to the unit cell. Independently, De Jong, in Physica **10**, 101 (1930) on the basis of powdered crystal measurements reported a = 10.690A, and assigned it to space group T^4. Rotation and oscillation studies confirmed this space group assignment but necessitated a revaluation of parameters. The structure is like that of sodium chlorate with the UO_2 group corresponding to the chlorine and the acetate groups to the oxygen of the sodium chlorate. The parameter for uranium is 0.42, that for sodium is 0.06. The location of the acetate groups has not been determined.

44. X-Ray Diffraction Study of Vitreous Silica and Pyrex. B. E. WARREN, *Massachusetts Institute of Technology.*—X-ray diffraction patterns of vitreous silica and Pyrex glass have been made with radiation Cu $K\alpha$. The silica pattern shows a broad but rather strong peak at a scattering angle corresponding to d = 4.32A in the Bragg law. The pattern from Pyrex is very similar, the peak occurring at an angle corresponding to d = 4.26A. The atomic grouping in vitreous silica must be essentially the same as found in the silicates, each silicon tetrahedrally surrounded by four

oxygens and each oxygen shared between two-tetrahedral groups. These conditions are so flexible that a number of crystalline arrangements and also a non-crystalline random network would be possible. A systematic attempt has been made to interpret the pattern in terms of non-crystalline diffraction. It appears to be impossible. The pattern can be interpreted as due to very small cristobalite crystals, the observed ring being the strong 111 reflexion of cristobalite. The spacing d=4.32A is somewhat larger than that obtained from large cristobalite crystals (4.11A), but it can be shown that direct application of the Bragg law to tiny cristobalite crystals will give too large a spacing due to (a) rapidly falling background, (b) effect of structure factor in determining the intensity maximum.

45. A Method of Doubling the Energy of Electrons Obtainable in a Sloan Type Tube and of Measuring e/m at a Million Volts. JESSE W. M. DUMOND, *California Institute of Technology.*—The freely suspended Tesla coil developed at Berkeley by Sloan under the leadership of E. O. Lawrence permits the generation of H.F. voltages of the order of 10^6 with wave-lengths of 20 to 40 meters. The author proposes and describes in some detail a technique in which electrons emitted from the high voltage terminal of the Tesla coil are accelerated into a field-free space, focussed by a magnetic "lens" on a magnetic "mirror," then reflected back and refocussed by the same magnetic lens on a target adjacent to the cathode and supported by the free end of the Tesla coil. The distance from target-cathode to mirror is about one-fourth the wave-length used to excite the Tesla coil and since the electrons have nearly light velocity they will receive two accelerations of a million volts each, impinging on the target with an energy of two million volts. The device permits of obtaining not only very hard x-rays with which the author proposes to perform fundamental investigations in scattering similar to his own in the field of softer radiation and those of Gray and Tarrant with gamma-rays, but also offers excellent promise as a method of studying e/m (both "transverse" and "longitudinal") for these high energy electrons with considerable precision.

46. The *K*-Series Spectrum of Tungsten. J. C. HUDSON AND H. G. VOGT, *Harvard University.*—The components of the $K\beta$ x-ray doublet have been completely separated and the $K\gamma$ doublet has been resolved for tungsten. A δ-line previously found by Armstrong and lying very near the absorption limit has also been recorded. The investigation was carried out by using the Bragg focussing arrangement with calcite as the diffracting crystal. The distance from the slit used as a source to the axis of the spectrometer was about six meters and a lever arm and cam served to oscillate the crystal through an arc of five minutes for each setting. On the photographs the α-lines are separated by more than 9 mm. The wave-lengths of the β, γ and δ-lines have been determined with reference to the α_2 and α_1-lines, whose wave-lengths have been taken as 0.21341A and 0.20860A respectively (d for first order reflection from calcite is taken as 3.028A). Measurements of the distances between the lines on the photographs give the following as the wave-lengths in angstroms: $\beta_2=0.18477$, $\beta_1=0.18397$, $\gamma_2=0.17917$, $\gamma_1=0.17899$, $\delta=0.17803$. The notation is an extension of that used by the Braggs, the subscript 1 denoting the shorter wave-length component in each case.

47. The Widths of the Lines of the *K*-Series of Tungsten. F. K. RICHTMYER AND S. W. BARNES, *Cornell University.*—With the direct-reading two-crystal spectrometer previously described (Phys. Rev. **35**, 1428 (1931)) measurements have been made of the widths of the several lines of the *K*-series of W(74). After making corrections for the rocking curve widths by the method described in the preceding paper, the half-widths at half-maximum are:

$K\alpha_1$: 0.075 X.U;	21 volts	$K\gamma$: 0.12 X.U;	45 volts
$K\beta_1$: 0.07	25	$K\delta'$: 0.05	19
$K\beta_2$: 0.082	29		

Since $K\gamma$ is an unresolved doublet, its greater width is to be expected. The line designated $K\delta'$ is apparently a line not previously reported. Its wave-length is about 178.00 X.U. It lies very close to the *K*-limit, and it has an intensity of about one-fifth that of the $K\gamma$ line. $K\delta'$ may tentatively be ascribed to the transition $K\rightarrow 0$. The *K*-limit, uncorrected for rocking curve width, has an observed width of about 0.46 X.U. (170 volts) and extends from about 177.8 X.U. to 178.2 X.U., taking the center of the $K\gamma$ line at 178.98 X.U.

48. The Measurement of the Relative Intensity of X-Ray Satellites in Cathode-Ray and in Fluorescence Excitation. F. R. HIRSH, JR., AND F. K. RICHTMYER, *Cornell University.*—A knowledge of the relative intensities of x-ray satellites is very important in deciding among the several theories proposed for the origin of these lines. Existing data are very fragmentary. Accordingly, spectrograms were taken of the $L\alpha$ doublet with its satellites for elements Mo(42) to Ag(47) with both cathode-ray and fluorescent excitation. The cathode-ray excitation was by 20 kv electrons; the fluorescence excitation by the radiation from a silver target bombarded by 20 kv electrons. The relative exposure-times were such as to produce spectrograms of approximately the same density. Microphotometric records, taken by the "oscillating" method (paper in press), were then analyzed to determine the ratio of the intensity of the total satellite structure to that of the parent line. Preliminary results, roughly quantitative, suggest that in cathode-ray excitation, the satellite intensity is a maximum at Rh(45) and is of the order of 80 percent of that of the parent line. For fluorescence excitation the corresponding ratio lies between 4 percent and 19 percent for the elements studied, probably depending on the "line" nature of the exciting radiation. The results are rendered somewhat uncertain because the satellites lie on the foot of $L\alpha_1$, the estimated height of this foot being a matter of judgment. The method, however, gives promise of yielding more precise results.

49. Relative Intensities of the *K*-Series Lines of the Elements 24 to 52 by the Ionization Chamber Method. JOHN H. WILLIAMS, *National Research Fellow, University of Chicago.*—The validity of the ionization chamber method of measuring relative intensities of x-ray beams of different wave-lengths has been extended to the region 1.5 to 2.3A by

the method of Allison (Phys. Rev. **38**, 441 (1931)). By using five gases and the $K\alpha$ lines of alloys of Cr, Fe and Ni it was found that the intensity measurements agreed to one percent. Measurements of thick target relative intensities of the K-series of twenty elements, corrected for absorption in the air, mica, and cellophane, for the partial reflection by the calcite crystal, and for the partial absorption of the beams in the ionization chamber, show variations of β_2/β_1 and β_1/α_1 as a function of atomic number. β_1/α_1 rises smoothly from a value of 0.165 at Co(27) to 0.282 at Cb(41) and then remains approximately constant at 0.29 for the higher elements. β_2/β_1 starts from a value of 0.018 at Zn(30) and rises throughout the range of elements, being 0.205 at Pd(46). The value of β_1/α_1 is increased slightly for Cr and Fe due to the presence of satellites. The value of α_2/α_1 is approximately 0.50 throughout this atomic range. Equations have been obtained for the absorption coefficients of mica, cellophane, CH_2Br and CH_3I in significant wave-length regions.

50. Wave-Lengths and Shapes of the K Series Lines of Cu, Ni, Co, Fe, and Cr. J. A. BEARDEN AND C. H. SHAW, *Johns Hopkins University.*—Previous work (Phys. Rev. **43**, 92 (1933)) has shown the usefulness of the double crystal spectrometer for determining the wave-length and shape of x-ray lines. This work has been extended with an improved vacuum tube method for measuring the ionization currents. The crystals used gave rocking curve widths within about 10 percent of the theoretical value for perfect crystals. The equation

$$y=a/[1+(x/b)^2]$$

has been used to plot symmetrical curves with which the experimental curves have been compared. In most of the lines the asymmetry suggests unresolved component lines. The wave-lengths as measured at the position of maximum intensity are in reasonable agreement with the values obtained with precision single crystal photographic spectrometers.

51. The Rocking Curve Correction in the Determination of the Widths of X-Ray Lines. S. W. BARNES AND L. D. PALMER, *Cornell University.*—The familiar relation between true line width, W_t, observed line width, W_0, and rocking curve width, W_r, on the assumption that these curves have the shape of the Gauss error curve, is

$$W_t=(W_0^2-W_r^2)^{\frac{1}{2}}. \quad (1)$$

A corresponding relation, on the assumption that x-ray lines and rocking curves have the shape given by the empirical relation

$$y=a/[1+(x/b)^2]$$

suggested by Hoyt, has been obtained and is

$$W_t=W_0-W_r \quad (2)$$

The width of $K\alpha_1$ of $W(74)$ was observed by a two crystal spectrometer in the first, second, third and fourth orders. The observed widths in first and second orders were corrected for the rocking curve widths by Eqs. (1) and (2) in turn. Corrections in third and fourth orders are relatively small. Eq. (2) gave nearly the same value of W_t, approximately 0.15 X.U., for all orders. Whereas, according to Eq. (1) W_t is 0.26 X.U. in first order and approaches 0.15 X.U. in higher orders. Since W_t should be independent of order, Eq. (2) seems preferable.

52. The Effect of Etching on the Rocking Curve Widths of Calcite Crystals. K. V. MANNING, *Cornell University.*—A pair of freshly polished calcite crystals in the $(1, -1)$ position gave for $\lambda=0.707$A, a rocking curve width (half width at half maximum) of 15″. After washing with a very dilute solution of HCl in water the width decreased to about 10″. Further washing with a somewhat stronger solution decreased the width to 6″. A pair of freshly cleaved calcite crystals gave a rocking curve width of 3″. These surfaces were subsequently, and inadvertently, maltreated. As a result the rocking curve width increased to 25″. But washing with dilute HCl brought the width back to 5″. It seems possible, therefore, by this method to produce polished crystals with reasonably narrow rocking curves.

53. A Direct-Current Amplifier Circuit. LOUIS A. TURNER, *Princeton University.*—A vacuum tube is used in a circuit which is essentially that of the bridge method of measuring the amplification factor, μ. Current from the battery flows through two resistances in series having a ratio of μ. These are connected to the vacuum tube in the obvious way so that any increase in the battery voltage, ΔV, will produce a rise in the plate voltage of amount $(\mu/\mu+1)\Delta V$ and a simultaneous decrease of the grid voltage of $(1/\mu+1)\Delta V$. The ratio of these changes being $-\mu$ the plate current remains constant. By using a Western Electric 262-A tube, it was found that the compensation was better for this circuit than with the same tube in a comparable ordinary single tube compensated circuit. In the latter the efficacy of the compensation depends upon the constancy of the plate resistance of the tube. Apparently, for the 262-A tube μ is more constant than R_p. Examination of the characteristics of other tubes indicates that this is a fairly general property.

54. Superconductivity of Tin at Radio Frequencies. F. B. SILSBEE, R. B. SCOTT, F. G. BRICKWEDDE AND J. W. COOK, *Bureau of Standards.*—Calorimetric experiments on extruded tin wire carrying radiofrequency currents showed that the effective resistance at temperatures slightly below the transition temperature (3.7°K) was less than one percent of that at temperatures slightly above. The transition temperatures corresponding to various definite values of current and of frequency were observed for (a) direct current, (b) for alternating currents of frequencies ranging from 200 to 1200 kilocycles per second, and (c) for combinations of direct and alternating currents. Changes in resistance were indicated (a) by the deflection of a galvanometer connected across the terminals of the specimen, (b) by the changes in the readings of a thermoammeter in series with the specimen, and (c) by the simultaneous use of both. For large currents the transition temperature decreased by substantially the same amount for equal incre-

ments of the crest value of the resultant current (d.c., a.c., or combined a.c. and d.c.). At low currents there appeared to be a small difference (about 0.01°C) in the transition temperature for direct and for radiofrequency currents. This is being further investigated. Except for this, all our results including those of combined alternating and direct currents can be explained on the basis of skin effect and the variation of the transition temperature with magnetic field, without additional hypotheses. A conductor can be normally resistant to an alternating current and at the same time superconducting to a direct current flowing through the core inside the skin. This was observed.

55. Filtration of Elastic Waves in Solid Rods. R. B. LINDSAY, *Brown University*.—The theoretical studies previously made and reported (Physical Review **40**, 125 (1932); J. Acous. Soc. Am. **4**, 155 (1932)) have been extended to the case of the filtration of torsional waves in a solid rod, and of longitudinal waves in rods having solid rods as side branches. If an infinite cylindrical metal rod is loaded at equal intervals of length $2l$ with heavy disks (with planes perpendicular to the rod) of mass m and moment of inertia I about the axis of the rod, the resulting structure acts as a low-pass filter for torsional waves. The transmission bands are given by $+1 > \cos W > -1$, where $\cos W = \cos 2kl - \omega I/\pi a^4 \rho_0 c \cdot \sin 2kl$; and $k = \omega/c = 2\pi\nu/c$, a = radius of rod, ρ_0 = density, $c = (\mu/\rho_0)^{\frac{1}{2}}$, where μ = rigidity modulus. Other things being equal, increase in I decreases the cut-off frequency. There is next considered the case of an infinite solid cylindrical rod loaded at equal intervals of length $2l$ with solid side rods (perpendicular to the main rod) of length l'. The problem is simplified by the approximate assumption that longitudinal waves in the main rod set up transverse waves in the side rods. Calculation indicates that if the side rod has a *free* end the structure acts as a low-pass filter for longitudinal waves in the main rod, while if the end of the side rod is rigidly clamped, a high-pass filter results. The transmission band characteristics are rather complicated. Details will be published shortly.

56. The Absorption of Sound in Air and Water Vapor. V. O. KNUDSEN, *University of California at Los Angeles*.—By measuring the rates of decay of pure tones in two steel chambers—one a six-foot cube and the other a two-foot cube—it is possible to determine the absorption coefficient of any gas within the chambers. The absorption coefficient m is defined by $I = I_0 e^{-mx}$, where I/I_0 gives the fractional reduction in the intensity of a plane wave after travelling a distance x. Measurements have been made in air for frequencies between 500 and 11,000 cycles, at relative humidities between 0 and 100 percent, and at temperatures between 18°C and 55°C. The coefficient m increases with an increase of either frequency or temperature; at any one frequency and temperature m has a maximum for a certain concentration of water vapor. Thus, at 3000 cycles and 20°C, m has a maximum (1.9×10^{-4} cm^{-1}) at a relative humidity of about 10 percent, and at 6000 cycles the maximum (4.0×10^{-4} cm^{-1}) occurs at a humidity of 14 percent. At lower humidities, and for all frequencies tested, m drops off rapidly, sinking to values below 1.0×10^{-4} for nearly dry air. At higher humidities the values of m agree satisfactorily with those published two years ago (V. O. Knudsen, J. Acous. Soc. Am. **III**, 1, 126–138 (July, 1931)). As the temperature increases from 18° to 55°C, m increases approximately 100 percent. A probable explanation of these results may be expected from a consideration of the transfer of energy at collisions. (See following abstract by H. O. Kneser.)

57. The Transfer of Vibrational Energy between Molecules. H. O. KNESER, *University of California, Berkeley*. (*Introduced by* V. O. KNUDSEN.)—Sound absorption in air is to a large extent due to the finite rate of adjustment of the thermal equilibrium between vibrational energy and rotational translation energies (H. O. Kneser, Ann. d. Physik **16**, 337 (1933)). The results of V. O. Knudsen (see preceding abstract) can be accounted for by assuming that the rate is proportional to the number N of H_2O molecules present. Assuming only one constant, the absorption coefficient m can be calculated as a function of N and ω (sound frequency) and is found to agree very satisfactorily with the observed $(m - m_0)$ values (m_0 being the absorption coefficient at $N = 0$) over the whole range of N and ω to which the experiments have extended. Also at higher temperature the agreement seems to be good. From this the conclusion can be drawn that the probability of the transition $v = 0 \rightleftarrows v = 1$ of the O_2 molecule at a collision is about 10^4 times greater if the colliding particle is an H_2O molecule than if it is an O_2 or N_2 molecule. Moreover the fact that the maximum absorption occurs at the same value of N at different temperatures indicates that the excitation of vibrational quanta does not require energy of activation.

58. The Entropies of Nitrogen, Carbon Monoxide, and Hydrogen from 1 to 1200 Atmospheres and from −70°C to 400°C. W. EDWARDS DEMING AND LOLA S. DEMING, *Bureau of Chemistry and Soils, Washington, D. C.*—With the authors' graphical method (Phys. Rev. **37**, 638 (1931); **38**, 2245 (1931); **40**, 848 (1932)) of evaluating derivatives from observational data, it is possible to use p, v, T data of a gas to get values for the expansion coefficients $(T/v) \times (dv/dT)_p$, $-(p/v)(dv/dp)_T$, and for $-T(d^2v/dT^2)_p$ at temperatures and pressures covered by the data. From these derivatives, various thermodynamic properties of the gas can be calculated and plotted along isotherms and isobars. Some of these properties have been tabulated in the references cited above. A graphical evaluation of the integral $-\int_1^p (dv/dT)_p dp$ along an isotherm gives the change in entropy brought about by increasing the pressure from 1 to p atmospheres at temperature T. When this integral is added to the entropy at 1 atmosphere, the result is the entropy at temperature T and pressure p. If the gas were ideal, the integral would be $-R \ln p$. Actually, for nitrogen, carbon monoxide, and hydrogen, the integral is always greater in absolute value than $R \ln p$. The difference is greater as T decreases and p increases, and is more for nitrogen and carbon monoxide than for hydrogen. Curves are plotted showing the entropy of the three gases along isotherms between −70°C and 400°C from 1 to 1200 atmospheres.

59. Thermal Conductivity of Metals in the Range 0° to 600°C. M. S. VAN DUSEN AND S. M. SHELTON, *U. S. Bureau of Standards.*—The thermal conductivities of a number of metals and alloys have been measured in the range 0° to 600°C. The method used gives directly the variation of conductivity with temperature, but absolute values are obtained by comparison with pure lead, the conductivity of which at ordinary temperature is fairly well established. If later considered necessary an absolute determination of the conductivity of the lead used as the standard can be made at one temperature under conditions most conducive to accuracy, and all results reduced to this basis. The change of conductivity with temperature is determined by measuring the temperature distribution along a cylindrical bar, one end of which is heated and the other cooled. Heat loss from the convex surface is minimized by a surrounding guard tube. For the comparative measurements, the bar is soldered end to end with a standard bar of the same diameter, and similar measurements made. Data have thus far been obtained on a number of technically important metals, principally ferrous alloys and nickel chromium alloys. Some investigation has been made of the use of thermal conductivity measurements as a tool in metallurgical research.

60. The Response of a Chromic Acid Desensitized Emulsion to Alpha-Rays. T. R. WILKINS AND R. N. WOLFE, *University of Rochester.*—It has been found that the luminosity produced in the air through which alpha-particles pass produces a considerable photographic density. In some of our experiments this amounts to as much as 50 percent of the densities measured. The difference in the penetrations of alpha-rays and light, the difference in the response of the halide grains to these two radiations as well as the usual difficulties in fog corrections (see R. B. Wilsey—The Photographic Journal **LXV**, 454–458 (1925)) make it impossible to separate these effects. This difficulty has been overcome by chromic acid desensitization of the plate to light. The response of the emulsion to alpha-rays is only moderately reduced. A family of curves representing the photographic densities due to alpha-rays of varying number with speeds from 0.42 to 0.76 V_0 Ra *C* fit the general equation $D = D_{\max}(1 - e^{-cN})$, where D is the diffuse density and N = number of alpha-particles/sq. cm.

61. The Influence of Frequency of Flash on the Photographic Intermittency Effect. BRIAN O'BRIEN AND VERNON L. PARKS, *Institute of Optics, University of Rochester, N. Y.*—The dependence of the photographic intermittency effect upon frequency of flash, intensity, and wave-length has been investigated below optimal intensity for three slow speed emulsions from λ3500 to λ6400A, using sectors with ratio of flash to total period as small as 1/366. Since both the character of the emulsions and the sector ratios chosen result in a large intermittency effect, the critical frequency above which no further change in density occurs can be determined with some precision. Under these conditions the intermittency and reciprocity defects become equal as already reported (Phys. Rev. **37**, 471 (1931); **41**, 387 (1932)). An optical sensitizer which decreased the speed of one emulsion at λ4500A by one-half did not alter gamma, the magnitude of the intermittency effect, or the critical frequency at this wave-length. If the effective receptor area of a halide grain is a function of the wave-length, the present observations are in approximate agreement with the recent suggestion by Webb (Bull. A. P. S. February, 1933) that the critical frequency coincides with one quantum per grain per flash.

62. Remarkable Optical Properties of the Alkali Metals. R. W. WOOD, *Johns Hopkins University.*—The optical properties of films of the alkali metals deposited at liquid air temperature, reported at the last meeting, have been more fully investigated. Potassium behaves as a metal in the visible spectrum and down to λ = 3000, below which point it behaves like a transparent substance giving plane polarization by reflection at an angle of about 35°. A film, so thick that the sun's disk is invisible through it, shows Newtonian interference bands from the 1st to the 5th order in the spectrum of the reflected light, between wave-lengths 3000 and 2000. The thickness of the film was determined by titration with phenolphthalein.

63. Photoelectric Effect of Caesium Vapor. JAKOB KUNZ, *University of Illinois.*—The ionization efficiency curve of caesium vapor has been determined by Mohler, Foote and Chenault, by my students Little and Cooke, and by Lawrence and Edlefsen. The results obtained are in agreement, except those of Lawrence and Edlefsen, showing a pronounced minimum between 2700 and 2800A. The direct method applied by Little and Cooke suffered from thermionic emission of caesium metal deposited on the electrodes and the walls of the Pyrex tube. The new method indicated in Phys. Rev. **39**, 866 (1932), has now been applied in two different ways and the minimum in the efficiency curve has again been established even in vapor at ordinary temperature.

64. Spontaneous Recombination and the Effect of Vapor Pressure in a Caesium Discharge. F. L. MOHLER, *Bureau of Standards.*—Measurements of the continuous recombination spectra emitted by the positive column (Phys. Rev. **43**, 374 (1933)) have been extended to lower pressures. Below 3μ there is little change with pressure and the intensity is proportional to the square of the electron concentration. The effective area q for recombination of 0.3 volt electrons into $2P$ at zero pressure is 1.7×10^{-21} cm². There is a nearly linear increase with pressure at a rate to double q at 9μ. Since qv^2 (v is electron velocity) is a transition probability, this means that the transition probability is a function of the pressure. There is a strict proportionality between line intensity and continuous intensity at higher pressures and one is led to the surprising conclusion that the transition probability for line emission depends on the pressure. Some support is found for this view in the explanation that has been given for photo-ionization by line absorption (Bur. Standards J. Research **5**, 51 (1930)). Results indicated that excited atoms either become ionized or revert to normal upon collision. If it is assumed that they

radiate when they do not become ions, then the radiation probability is a linear function of pressure.

65. Conductivity of a Rare Gas Irradiated with Its Own Resonance Radiation. O. S. DUFFENDACK AND R. W. SMITH, *University of Michigan.*—Previous experiments (Phys. Rev. **43**, 374 (1933)) on the effect of impurities on the conductivity produced in Ne and He irradiated with their own resonance radiations have been extended to investigate the processes contributing to the observed conductivity. It has been found that probe measurements made with unguarded collectors in the irradiated gas are liable to misinterpretation because of large edge effects. When guarded parallel plane collectors are used, no measurable current ($<10^{-16}$ amp.) is obtained unless the collectors are oriented so that the direct primary radiation passes between them. Glass shields prevent any appreciable direct or reflected light from striking the collector faces. These results indicate that propagation of resonance radiation by diffusion down the tube is negligible when impurities are present, though the relatively large conductivities observed in pure gases are difficult to explain on any other basis. Furthermore the diffusion of metastable atoms is an unimportant factor. The currents measured between the guarded plane collectors do not attain even approximate saturation values at the radiation intensities from arcs of 50–200 m.a. current 20 cm from the collectors. In agreement with the observations of Penning (Zeits. f. Physik **78**, 454 (1932)), experiments using a glow discharge of 1 to 5 m.a. as a source indicate that saturation currents may be obtained at these lower light intensities.

66. Pressure Effects of Foreign Gases on the Sodium Resonance Lines. HENRY MARGENAU AND WILLIAM W. WATSON, *Yale University.*—The effects of foreign gases on the *D*-lines of Na are being studied by means of a steel absorption chamber, in which the Na-vapor pressure is kept low while the foreign gas pressure varies between 0 0.144 cm^{-1} per atmosphere. (2) Study of the line contours can be photographed and photometrically evaluated.—Results for N_2 are: (1) The shift of the absorption maximum varies linearly with the relative density of N_2 and equals 0.144 cm^{-1} per atmosphere. (2) Study of the line contours reveals marked red asymmetries, most pronounced near the base. While the half-value widths vary approximately linearly with density (0.36 cm^{-1} per atmosphere) it is shown that "quarter-value widths" follow a somewhat different law. Hence the characterization of line contours by half-value widths means little. (3) Both *D*-lines show the same broadening and shift. (4) This pressure shift leads to a value between 9 and 10A for the distance of closest approach between the excited Na and the N_2.—Similar data for A and H_2 will be presented.

67. Light Excitation by Alkali Ions in the Noble Gases. ROBERT E. HOLZER, *National Research Fellow, University of Chicago.*—By a method similar to the one recently described (Phys. Rev. **43**, 365 (1933)) measurements were made on the excitation of light by four alkali ions, Li^+, Na^+, K^+, and Cs^+ in three of the noble gases He, Ne, and A. It was found that the positive ion accelerating potential for which light excitation in the gas was first observed was lowest for the alkali ion closest to the particular gas in the periodic table. In He, Li^+ produced observable excitation at a potential lower than the other three ions studied. The same was true for Na^+ in Ne and K^+ in A. It is interesting to note that the conclusion drawn from the cases of light excitation studied corresponds to the generalization of Beeck and Mouzon who found that in a particular noble gas ionization was produced at the lowest potential by the alkali ion which was closest to that gas in the periodic table. Light excitation was observed at potentials considerably lower than the potentials at which ionization is produced. In those cases where adequate data on ionization by positive ions exist, excitation was observed at potentials approximately fifty percent below the reported ionization potential.

68. The Nature of Gas Ions. OVERTON LUHR, *Massachusetts Institute of Technology.*—Improved apparatus and technique have been employed to extend previous measurements of the mass of positive ions in a glow discharge (Phys. Rev. **38**, 1730 (1931)). Further progress has been made in determining the nature of normal gas ions, and it has been necessary to revise some of the conclusions drawn in the preliminary work. Nitrogen presents the most interesting results, with the progressive formation of N_3^+ and N_4^+ ions from the primary N^+ and N_2^+ by attachment to neutral molecules after about 10^7 or 10^8 collisions. O^+, O_2^+ and O_3^+ are found in oxygen, together with a strong peak at a point corresponding to a molecular weight of 30, which may be NO^+, resulting from nitrogen occluded in the walls. H^+, H_2^+ and H_3^+ are found in hydrogen. In dry air the ions consist principally of nitrogen and oxygen ions, with the probable addition of NO^+ and NO_2^+. Water vapor plays an important rôle unless carefully removed. Negative ions have not been observed even in oxygen where conditions were such that considerable electron attachment would be expected.

69. Ions in Oxygen and Hydrogen. CHARLES D. BOCK, *Yale University.*—The magnetic spectrometer with high order focussing, described last year at Cambridge, is being used to obtain increased intensity in the study of aged ions in oxygen and hydrogen. High intensity is obtained by reducing the number of slits to two and by concentrating the ions at the slits in convergent electric fields. The ions are generated in a glow discharge. They pass through a gauze cathode to drift along the gas stream under their own space charge field. In oxygen, at early stages the ions are O_2^+ and O_4^+, the latter, although a transition product, predominating. Its disappearance is accompanied by an instability, shown as a change to O^+ after acceleration for analysis. O^+ appears to carry most of the current at a later stage in pure oxygen. The presence of 0.5 percent nitrogen, however, makes this transition occur through the N^+ ion, which at one time carries about one-half the current. The ions in these measurements were generated in gas of the same pressure, the age being controlled through the conditions of drift to the first slit. Study is now being extended to

oxygen of much higher purity, in order to identify certain other rare products observed.

70. Electron Capture Gross Sections. L. A. YOUNG AND N. E. BRADBURY (*National Research Fellow*), *Massachusetts Institute of Technology.*—The process involved in the capture of electrons by neutral molecules and atoms is assumed to be a spontaneous transition from a continuous to a discrete state of the system molecule plus electron accompanied by the emission of radiation. Since the many electron problem involved for a real system is extremely complicated, an attempt has been made in which the molecule is replaced by an effective potential field. Such a potential field can be adjusted as to size and magnitude to simulate molecular conditions. Cross sections for capture are calculated for such a model and their variation with electron energy of the continuous state studied. These cross sections turn out to be of the order of magnitude observed experimentally for such electronegative gases as O_2 and CO_2 and the variation with electron energy is qualitatively correct.

71. A Mass Spectrum Analysis of the Products of Ionization by Electron Impact in Nitrogen and Acetylene. JOHN T. TATE, P. T. SMITH AND A. L. VAUGHAN, *University of Minnesota.*—A mass-spectrograph of high resolving power has been used to compare the ionization potentials of molecular nitrogen and argon. This comparison gives the ionization potential of N_2 as 15.65 ± 0.02 electron-volts or lower than that of argon. The probable error given is calculated from the variation of the data. A mass spectrum analysis of the products of ionization by electron impact in acetylene at low pressures has been made and their ionization potentials determined. Argon was left in the apparatus to calibrate the voltage scale. The ions formed with their ionization potentials and the estimated probable errors are given in the table. The efficiency of formation of the H^+ ion

ION	$C_2H_2^+$	C_2H^+	C_2^+	CH^+	C^+	H^+
I. P. (volts)	11.2	17.8	23.8	22.2	24.5	21.7 & 25.6
Error (volts)	±0.1	±0.2	±0.3	±0.5	±1.0	±1.0

increased sharply at 25.6 ± 1.0 electron-volts. The CH^+ and C^+ ions had as high as 1.5 volts kinetic energy while the H^+ ion had as much as 6 volts kinetic energy. No negative ions were found. The carbon isotope C^{13} was found forming an ion of mass 27. The ratio of C^{13} to C^{12} in this sample of C_2H_2 was 1 part in 100.

72. A Test of the "Momentum Transfer" Theory of Accommodation Coefficients of Ions at Cathodes. K. T. COMPTON AND E. S. LAMAR, *Massachusetts Institute of Technology.*—In a recent paper by one of us, read before the National Academy of Sciences, it was suggested that the accommodation coefficient for positive gas ions on a metal cathode should be less than unity only if the mass of the metal atom exceeds that of the ion. In order to test this hypothesis, experiments already reported for helium ions on molybdenum have been continued for argon ions on molybdenum and on aluminum. The cathode to be studied was the molybdenum or aluminum bob of a glass pendulum, whose deflection gave a measure of the pressure acting, and was immersed in the positive column of a low voltage argon arc. The pressures on the cathode were due to the recoil of those ions which retain some of their kinetic energy after neutralization and to radiometric effects resulting from heating of the cathode by ion bombardment. From an analysis of the data, it was possible to compute an accommodation coefficient for the positive ions. The results indicate an accommodation coefficient of about 0.79 for argon ions on molybdenum, and an accommodation coefficient of unity on aluminum.

73. The Absolute Motion of the Solar System and the Orbital Motion of the Earth Determined by the Ether-Drift Experiment. DAYTON C. MILLER, *Case School of Applied Science.*—Previous reports of the ether-drift observations made on Mount Wilson by the writer in 1921, 1924, 1925, and 1926 have given evidence of a cosmic motion of the solar system, but failed to show the earth's orbital motion. A complete reanalysis of the entire series of Mount Wilson observations has been made. The ether-drift effect in the interferometer, being a second-order effect, correctly defines the line in which an absolute motion takes place, but does not determine whether the motion in this line is positive or negative. The direction heretofore indicated was northward; by adopting the alternative direction, southward in the same line, a new remarkably consistent result has been obtained, which gives *for the first time* definite quantitative determinations of both the absolute motion of the solar system and the motion of the earth in its orbit. The motion of the solar system thus determined has a velocity of 208 kilometers per second, and is towards an apex having a right ascension of 4 h., 56 m., and a declination of $-70°\ 30'$. As before, the observed displacement of the interference fringes corresponds to about one-twentieth of the full velocity. This might be accounted for by an incomplete Lorentz contraction, or by a dragged ether. This reduced displacement does not invalidate the determination of the absolute motion.

74. Radiation Reaction Forces and The Expanding Universe. ROSS GUNN, *Naval Research Laboratory, Washington, D. C.*—An earlier suggestion of the author is examined. Without assigning special properties to space it is shown that great clouds of stars are non-conservative systems and that radiation reaction forces play a major rôle in guiding stellar motions. The forces are of importance in unsymmetrical stars and evidence is collected to show that perhaps all stars are possessed of deep seated thermal asymmetries. The forces continually act to increase the mean velocity of the stars and the galaxy is calculated to expand at a rate which closely approximates the observed rate of expansion. From a relation connecting the mean velocity of the stars and their age, we deduce from the observed motions that the ag. of the galaxy is about 10^{10} years. The deduced rate of expansion of the galaxy agrees with the observational data and is identical in form and numerical constant with Hubble's relation connecting the velocity of recession and distances of nebulae. The relation of radia-

tion reaction forces to the motions of nebulae is considered and Hubble's relation deduced. It seems probable that the nebulae were born from parent super nebulae by a process much like the stars themselves are thought to have been born. Initially the rate of expansion of the Universe was small but it has been increasing at a nearly uniform rate. Eventually, after the radiations of the nebulae decrease, a contraction may readily set in and the final state of the Universe will be a highly condensed system of cold stars and nebulae describing orbits about their common center of gravity. There is no evidence indicating that the expansion and contraction is periodic.

75. Production of Secondaries by Cosmic Rays. J. C. STREET, *Harvard University.*—Using an arrangement of three Geiger Mueller counters placed out of line (at the vertices of an isosceles triangle) a study has been made of the production of secondary rays in lead screens introduced between the upper and the two lower counters. Since no single ray travelling in a straight line can pass through the three counters a triple coincidence has the significance of two or more associated rays. An increase in the frequency of triple coincidences has been observed to result upon the introduction of the lead and curves of the variation have been obtained under various experimental conditions. The walls and roof of the building in which measurements are taken have a marked effect on the results obtained. A rough estimation has been made to determine the average rate of production of secondaries. This leads to a value 0.040 for the probability that a primary ray will in one centimeter path in lead produce a secondary travelling into unit solid angle at the average angle necessary for observation (about 35° with the original ray). If exponential absorption is assumed for primary and secondary rays it is possible to show that single scattering cannot account for the shape of curves obtained. This difficulty vanishes if multiple scattering is assumed but determination of the absorption coefficients is not possible from the present data.

76. Nuclear Physics Studies Using the Van de Graaff Electrostatic Generator. M. A. TUVE, L. R. HAFSTAD AND O. DAHL, *Carnegie Institution of Washington, Department of Terrestrial Magnetism.*—A 1-meter diameter air-insulated electrostatic generator of the type described by Van de Graaff (R. J. Van de Graaff, Phys. Rev. **38**, 1919-A (1931); R. J. Van de Graaff, K. T. Compton and L. C. Van Atta, Phys. Rev. **43**, 149–157 (1933)) has been used with simple cascade-type vacuum-tubes for disintegrating various elements. The positive ion-source is a hollow-anode low-voltage arc, with generators driven by the silk charging-belt 12 inches wide, 5000 feet per minute, 200μA charging-current. With a height of 12 feet from floor to rafters 8 feet apart a usual voltage of 600 kilovolts is attained, with a maximum of 750 kilovolts, apparently limited with sphere positive by its curvature rather than by head-room. Negative voltages are somewhat higher. Voltages are measured by a generating voltmeter (R. Gunn, Phys. Rev. **40**, 307–312 (1932); P. Kirkpatrick and I. Miyake, Rev. Sci. Inst. **3**, 1–8 (1932); P. Kirkpatrick, Rev. Sci. Inst. **3**, 430–438 (1932)), checked by sparks to a 2-meter sphere and roughly checked by magnetic deflection. Target-currents of 10μA and more are obtained. All disintegration-observations are made with a Wynn-Williams and Ward (C. E. Wynn-Williams and F. A. B. Ward, Proc. Roy. Soc. **A131**, 391–409 (1931)) pulse-amplifier. Bombarding with 600-kilovolt protons lithium gives two alpha-particle groups, the shorter 19 mm, in nearly equal numbers. The boron yield is about 20 times lithium, in two groups, the shorter 28-mm group greatly preponderating, with about 4,000,000 incident protons per alpha-particle (J. D. Cockcroft and E. T. S. Walton, Nature **131**, 23 (1933)). Search has been made for neutrons from Li, B, and Be bombarded by protons and by helium ions, with negative results. A target-shutter eliminates spurious effects. Calibrating the amplifier (neutron-recoil protons from paraffin) by neutrons from 3 millicuries Po on Be showed that 1/25 of Chadwick's neutron-intensity at the target would be detected. Observations are in progress on a series of heavier elements, with controls to evaluate surface and volume impurities of lighter elements.

77. The Radioactivity of Beryllium. R. M. LANGER AND R. W. RAITT, *California Institute of Technology.*—The decomposition products of Be disintegration have a range in air which, according to our measurements, is approximately 1 cm. Aluminum foil 1μ thick cuts out about 40 percent of the ionization. Such α-rays produce about 4×10^4 ions along their path. The ionization currents (7×10^3 ions per second from 100 cm^2 of Be surface) from Be samples corrected for radioactive contamination then give a means of calculating the decay constant of Be. This is $\lambda \doteq 10^{-21}$ sec.$^{-1}$. This may be compared with the helium content of old Beryls. The data from the analysis by Strutt of the Acworth Beryls lead to a decay constant of the same order of magnitude. There is no strong reason to expect that the Condon-Gurney-Gamow theory of radioactive disintegration will hold. Nevertheless it is of interest to note that even with appropriate modifications the ordinary formulae utterly fail to give the right order of magnitude for the decay rate. This is one more indication that the light nuclei, at least, are not composed of α-particles.

78. The Disintegration of Atomic Nuclei by Neutrons. WILLIAM D. HARKINS, DAVID M. GANS AND HENRY W. NEWSON, *University of Chicago.*—Many photographs have been taken to show the action of high-speed neutrons in different gases, such as nitrogen and neon. The former of these two exhibits nuclear disintegration much more often than the latter. A neutron of energy corresponding to fifteen

TABLE I. *Amount of energy (in 10^6 electron-volts) which disappears in the reaction and is presumably converted into γ-rays.*

	N^{14} + n^1 → N^{15} →	B^{11} + He^4
1.	1.4 (HA) 1.4 (FB) 1.3 (FC)	
2.	2.3 (HA) 2.1 (HA) 2.0 (FB)	2.0 (FB) 2.3 (FC)
3.	2.6 (HA) 2.8 (FA) 2.5 (FC)	
4.	3.7 (HA) 3.8 (FA) 3.8 (FA)	3.9 (FB)
5.	5.0 (FC) 5.8 (FC)	
6.	11.9 (HA)	

million electron-volts has been observed. Disintegrations by what has been called "non-capture" will be discussed. In disintegrations by capture energy always disappears in the process, and is presumably emitted as γ-rays. Table I, which gives the values thus far obtained by Feather, and by the writers, is more extensive than that published earlier, but still indicates the probability of the existence of definite nuclear energy states.

79. The Angular Distribution of Protons Projected by Neutrons. F. N. D. KURIE, *Yale University.*—A small brass cylinder containing a piece of beryllium, through which a hole has been bored, is placed with this hole along the axis of an automatic cloud chamber. A linear polonium source (about 2 millicuries) in this hole produces neutrons on striking the beryllium. We thus have a "neutron gun" which fires neutrons radially into the cloud chamber. From the surface of the brass cylinder, which is covered with paraffin, protons are projected. A special reprojection scheme enables one to measure the angle between a radius vector (direction of motion of the neutron) and the direction of motion of the proton. The distribution-in-angle of these protons exhibits a maximum at 0° though some are found up to 75°. A correction is applied for the volume of the paraffin effective in scattering, but no account is taken of the possibility of the neutron being scattered before it projects the proton observed. This distribution is in disagreement with the behavior of both the models of the neutron considered by H. S. W. Massey (Proc. Roy. Soc. **A138**, 460 (1932)). One is tempted to conclude that the neutron is not a proton-electron combination. The high speed β-particles noticed by Auger are observed.

80. The Discovery of the Free Positive Electron. CARL D. ANDERSON, *California Institute of Technology.*—A group of 1300 photographs of cosmic-ray tracks, in a vertical Wilson chamber in a homogeneous magnetic field of 15,000 gauss, included 15 tracks of particles of positive charge which could not have a mass as great as that of the proton. It is concluded from an examination of the energy loss and ionization that the charge is less than twice, and is probably exactly equal to, that of the proton. The curvatures and ionization produced require the mass to be small compared to that of the proton, and of the same order of magnitude as that of the free negative electron. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

81. Relative Abundance of the Neon Isotopes. WALKER BLEAKNEY, *Princeton University.*—The neon isotopes were examined with a mass-spectrograph in both the first and second "orders." Rare gas hydrides seem never to exist in the doubly ionized state, hence the results in the second order are probably free from the uncertainties usually experienced with singly charged ions. At the very low pressures used in this experiment even the formation of singly charged hydrides is very unlikely as shown by the agreement of the relative intensities in the two orders. The intensities found were $^{20}Ne^{+} : {}^{22}Ne^{+} : {}^{21}Ne^{+} = 100 : 8.2 : 0.28$ and $^{20}Ne^{++} : {}^{22}Ne^{++} : {}^{21}Ne^{++} = 100 : 9.2 : 0.30$. The occurrence of $^{21}Ne^{++}$ here and in earlier work by Bainbridge would seem to establish the existence of this isotope beyond any doubt. However, no trace of an ion of mass 23 could be found in either order, whereas 1 part in 10,000 of the other neon isotopes should have been detected. This result is in contradiction to the recent work of Kallmann and Lasareff, where evidence was found for an isotope of mass 23. The results presented here would indicate that they were observing $^{22}Ne\,{}^{1}H^{+}$ also of mass 23.

82. The Isotopic Constitutions of Krypton, Mercury, Selenium, Cadmium and Germanium. K. T. BAINBRIDGE, *Bartol Research Foundation of the Franklin Institute.*—The results for Kr, Se, and Cd check with Aston's analyses. There is no evidence for 9 percent of Se^{81} as Johnston has suggested. Cd^{108} and Cd^{118}, found in band spectra, have not yet been observed, probably owing to the technical difficulties encountered in taking sufficiently long exposures with cadmium and the small abundance of these isotopes with respect to the others. The spectra of mercury could not show Hg^{196}, Hg^{197}, and Hg^{203} as sufficiently long exposures to test for their existence have not yet been made. Lines of mass 71, 75 and 77 attributed to Ge^{71}, Ge^{75}, and Ge^{77} by Aston are mainly if not entirely hydrides of Ge^{70}, Ge^{74}, and Ge^{76}.

83. A Statistical Investigation of an Assembly in Equilibrium in Which All Possible Transmutations Can Occur. T. E. STERNE, *National Research Fellow, Harvard University and the Mass. Inst. of Technology.*—If (a) all nuclei are made of protons and neutrons, and if the electron is a third independent particle, then there are four parameters which determine the statistical state of such an assembly: ξ', η, ξ and T (the absolute temperature). If (b) all nuclei and neutrons are made of electrons and protons, then ξ and ξ' are identical. In cases (a) and (b), for any composite nucleus of mass number M_s and atomic number N_s there exists a parameter μ_s given by

$$\mu_s = \xi^{M_s - N_s} \eta^{M_s} e^{-Q_s/kT};$$

where

$$Q_s = c^2[m_s - (M_s - N_s)m_{el} - M_s m_P]$$

and is in case (b) the energy required to build the nucleus out of electrons and protons. The mean number $\overline{X}_s$ of nuclei of the sort s can be calculated by the equation

$$\overline{X}_s = \mu_s(\partial/\partial\mu_s)_s \Sigma_r \log f[\mu_s \exp(-\epsilon_r/kT)].$$

It may happen that equilibria of this type are established nearly perfectly in the interiors of stars; and the large liberation of energy by such an assembly when the temperature is lowered may be the chief source of the energy radiated by certain stars.

84. Electron Diffraction by Stibnite and Galena. A. G. EMSLIE, *Cornell University.* (*Introduced by* C. C. MURDOCK.)—In scattering 20–60 kv electrons by cleavage faces of stibnite and galena, the diffraction pattern consists of a part which depends on the direction of the incident beam and a part which does not. The first includes groups of

sharp spots lying on concentric circles, in good agreement with the lattice arrangement of the crystal surface and independent of the inner potential of the crystal. This indicates large mosaic units and short "free path" for elastic collisions. The second part of the pattern consists of intersecting bands of different breadths and directions. These arise from ejection of those inelastically scattered electrons which try to move in the forbidden ranges of direction discussed in Morse's theory of electrons in crystals. According to the theory, each band should enclose the line to which it collapses in the limit of uniform crystal potential. Instead, one edge of the band is found to coincide with the line. In addition to the bands, and also independent of the incident beam, there is a system of rings concentric with the circles of spots. This appears to result from inelastic collisions followed by glancing angle scattering by a one-dimensional grating and involves the effect of a refractive index.

85. Electron Diffraction by Transmission Through Liquids. LOUIS R. MAXWELL, *Bureau of Chemistry and Soils, Washington, D. C.*—A new type of electron interference has been obtained by the transmission of electrons (14 kv to 30 kv) through thin liquid films. The films were mounted on a small wire loop and thinned down almost to the stage where optical interference rings become visible. The substance used was ordinary vacuum pump oil because of its low vapor pressure and suitable surface tension. The films would break after one to two minutes bombardment by the electrons; this however was ample time to obtain several photographs without setting up a new film. The diffraction patterns consisted of three complete rings whose intensity decreased with increasing angle, and whose values of $(1/\lambda)\sin\theta/2$ were found to be 0.11, 0.23, and 0.40. At very small angles (0°–1°) the pattern was characteristic of the Debye, Scherrer, Hull patterns consisting of a region of low intensity surrounding the central spot before the appearance of the first ring, in contrast to an intense scattering at these angles that would be expected if the molecules were entirely free. This suggests that the diffraction is produced by groups of molecules in random orientations as postulated by G. W. Stewart for x-ray diffraction from liquids. These electron diffraction "rings" are narrower than the usual "halos" obtained in x-ray diffraction from liquids. This is possibly due to the effect of nuclear scattering for electrons which is unimportant in x-ray scattering.

86. The Reflection of Electrons from Liquid Mercury. ROBERT B. BRODE AND EDWARD B. JORDAN, *University of California.*—A beam of electrons was directed against a mercury surface and the number of electrons which were reflected elastically were measured as a function of their direction. The surface was renewed with freshly distilled mercury about once a minute. A liquid air trap a few centimeters from the surface aided in keeping the surface clean. When the initial beam struck the surface normally the reflected current was symmetrical on both sides. The reflected intensity per unit solid angle was not proportional to the cosine of the angle but showed a preference toward backward reflection in the direction of the initial beam. Measurements were made from 14 to 70 volts. The total number of elastically reflected electrons decreased rapidly with increasing velocity. However the angular variation of intensity did not change much with the speed of the electrons. The intensity and the angular variation of the reflected electron current are in good agreement with the values calculated from the scattering by single mercury atoms. Measurements made with the incident beam at other angles than normal to the surface are also explained by this same assumption.

87. Liquid Structure in Ionic Solutions. G. W. STEWART, *State University of Iowa.*—The accumulation of information concerning x-ray diffraction in liquids, and its interpretation, have made possible certain conclusions from similar diffraction experiments in ionic solutions. The investigations have thus far covered the ionic solutions of lithium chloride in *n*-methyl, *n*-ethyl, *n*-propyl and *n*-butyl alcohols. They show that the cybotactic groups in the alcohols are modified by lithium chloride ions, much as if there were added a lithium chloride alcoholate to the alcohol. Moreover the relatively heavy chlorine ions show coherent scattering, indicating that they cannot be at random relative to one another. These conclusions suggest a description of the ionic atmospheres. Following the crude analogy found between the structure in a liquid and its crystal form, it may be said that the ionic solutions examined show a structure analogous to an ionic crystal or solid solution of the normal alcohol and lithium chloride alcoholate. The Debye-Hückel theory, based upon electric forces, gives an atmosphere around each ion which is preponderately of the opposite sign. The present conclusions point out the additional spatial dependence of the ions and describe the structure as analogous to an ionic crystal of lithium chloride alcoholate and normal alcohol. It indicates the manner by which both the association of ions and of ions with solvent molecules occur.

88. A New Method for Determining Thermionic Work Functions of Metals and Its Application to Nickel. GERALD W. FOX AND ROBERT M. BOWIE, *Iowa State College, Ames, Iowa.*—A new method for determining thermionic work functions of metals has been developed and may be stated as follows: The metal sample has the approximate form of a sphere and is heated by electron bombardment from an auxiliary filament that is disconnected when measurements are made. As the sample cools the electron emission from it charges a condenser, which, at predetermined times, is discharged through a ballistic galvanometer. A Pt, Pt+10 percent Rh thermocouple, spot-welded to the sample, measures its temperatures. The values of the thermionic constants are obtained by use of the equation:

$$\log_{10}(T^2/SQ) = \log_{10}(2.3/aA) + \Phi/(1.988\times10^{-4}T)$$

where Q is the quantity of charge yet to flow on cooling the sample to absolute zero and $-S$ is the slope of the log Q-time curve. This equation is derived from Richardson's. For thoroughly outgassed nickel, this method yields $\Phi = 5.03 \pm .05$ volts and $A = 1.38\times10^{3}$ amp./cm² degree². The value for Φ agrees with photoelectric determinations.

89. Thermionic and Adsorption Characteristics of Platinum on Tungsten. R. W. SEARS AND J. A. BECKER, *Bell Telephone Laboratories, New York, N. Y.*—As the amount (A) of platinum adsorbed on a tungsten surface increases, the thermionic activity decreases rapidly up to one layer and then more slowly until at about ten layers it approaches platinum activity. There is no minimum analogous to the maximum observed for electropositive adatoms. The rate of evaporation E increases with both A and temperature T. For a fixed A, log E varies linearly with $1/T$. For T fixed, log E increases with A, rapidly at first and then more slowly until it reaches the value for bulk platinum at about ten layers. In the region of one or two layers maxima have been found. When platinum is deposited on one side of a tungsten ribbon it migrates over the surface at temperatures at which evaporation is negligible. The rate of migration increases not only with temperature and concentration gradient but also with the concentration. Hence the mechanism of migration is not like that of ordinary diffusion but is similar to the mechanism of evaporation. Very probably migration results from adatoms which leave the surface at one point and return to it at another point.

90. The Temperature Dependence of the Work Function for Composite Surfaces. W. H. BRATTAIN AND J. A. BECKER, *Bell Telephone Laboratories, New York, N. Y.*—Experiment shows that as the amount of adsorbed material on a surface is increased from 0 to about one layer, the various Richardson lines are straight in the observed temperature range and that they all intersect at a common point corresponding to an extrapolated critical temperature T_c. This is necessarily so if the slopes of these Richardson lines are linearly related to the intercepts at $1/T=0$ or else to the logarithms of the emission currents at a fixed temperature. The slope of any such line is $-w_{fT}+Tdw/dT$ where w_{fT} = work function when the surface is covered by an amount f and is at a temperature T. Since the lines are straight and intersect in a common point, it follows that throughout the experimental temperature range $dw/dT=(w_0-w_{fT})/(T_c-T)$; $w_0=w$ for the clean surface for which $dw_0/dT=0$. Hence $w_{fT}=w_{f0}+\alpha_f T$ where $\alpha_f=(w_0-w_{f0})/T_c$. For positive adions $w_0>w_{f0}$ and α is positive; while for negative adions $w_0<w_{f0}$ and α is negative.

91. The Dependence of Adsorbed Ions on Temperature. J. A. BECKER AND W. H. BRATTAIN, *Bell Telephone Laboratories, New York, N. Y.*—In the preceding abstract it is shown that the work function of a composite surface is given by

$$w_{fT}=w_{f0}+[(w_0-w_{f0})/T_c]T.$$

This can be written

$$w_0-w_{fT}=(w_0-w_{f0})(1-T/T_c).$$

Now $w_0-w_{fT}=4\pi eI_{fT}l_T$ and $w_0-w_{f0}=4\pi eI_{f0}l_0$ where I_{fT} = adions per cm² when the concentration of adsorbed material is f and the temperature is T; $I_{f0}=I_{fT}$ when $T=0$; l_T is the average distance that the ionic charge e is above the surface at temperature T.

$$\therefore\ l_TI_{fT}=l_0I_{f0}(1-T/T_c)=l_0I_{f0}(1-\gamma T)\doteq l_0I_{f0}\epsilon^{-\gamma T},$$

where $\gamma=1/T_c$ which is independent of f. This equation states that when the temperature of a given composite surface (f = constant) is changed, the ion moment varies linearly with the temperature in such a way that it would become zero at some critical temperature. This critical temperature is characteristic of the surface and the adsorbed particles, but is the same for all amounts of adsorbent less than one layer. $1/T_c$ or γ might be called the ionization coefficient. Since l_T very likely increases with T, I_{fT} very likely decreases with T more rapidly than l_T increases.

92. The Effect of Temperature on the Emission of Electron Field Currents from Molybdenum and Tungsten. A. J. AHEARN, *Bell Telephone Laboratories, New York, N. Y.*—Electron field currents from the central portion of long molybdenum and tungsten filaments about 2.7×10^{-3} cm in diameter have been studied. The field currents were first made stable to about 5 percent by long continued conditioning treatments of temperature and high voltage under high vacuum conditions. Thermionic emission measurements gave the values 4.32 and 4.58 volts for the work function of the molybdenum and tungsten, respectively, in good agreement with the accepted values for the clean metals. Emission measurements were then made at fields varying from about 5×10^5 v/cm to about 1×10^6 v/cm and at temperatures varying from 300°K to about 2000°K. Down to about 1600°K the thermionic currents completely masked the field currents. Thermionic emission values below 1600°K were obtained by extrapolation. Thus the field currents at the lower temperatures were separated from the thermionic currents. Where necessary, corrections were made for the decrease in the voltage gradient accompanying the thermal expansion of the filament. The field currents were found to be independent of temperature to within 5 percent from 300°K to 1400°K. At temperatures higher than 1400°K the data are consistent with the assumption that the current consists of a thermionic current plus a current which is independent of temperature. However, due to the exponential change of thermionic current with temperature and the relative magnitudes of the two currents a small effect of temperature on the field current could not be distinguished at temperatures higher than 1400°K.

93. Effect of High Series Resistance on Cold Emission. RUSSELL W. MEBS, *Ohio State University.* (*Introduced by* WILLARD H. BENNETT.)—Several well outgassed extremely fine points of tungsten failed to give measurable emission at fields exceeding ten million volts per centimeter. Other points similarly prepared gave emission. When a series resistance of 1.8×10^9 ohms was used, a typical breakdown occurred but no rupture of the point could be found afterwards, using the microscope. When resistance of 2×10^6 was used, rupture of the point occurred at the same value of current, and the deformation of the cathode was easily seen. Emission could never be drawn from copper cathodes without severe rupture when resistance of 10^6 ohms was used, but when resistance of 10^9 ohms was used, no rupture could be found. Hence, it is proven that breakdown due to

disruption of the cathode surface is not produced by the electric stresses due to the field, but instead is a function of the current.

94. An Absolute Determination of the Ampere. HARVEY L. CURTIS AND ROGER W. CURTIS, *Bureau of Standards.*—The ampere has been measured in absolute units by a current balance, in which the force between coils carrying a current is measured. The current balance originally used by Rosa, Dorsey and Miller (Bulletin of the Bureau of Standards **8**, 269–393 (1912)) has been set up. The apparatus has been modified in many important respects. Four sets of coils have been used. The important constant used in computing the current from the force between coils is the ratio of their radii. Eight experimental determinations of these ratios have been made giving results which are self-checking and which show no errors larger than ± 6 parts in a million. Four series of determinations of the force give results in which the average deviation from the mean is about ± 3 parts in one hundred thousand. The results may be expressed as 1 B.S. International Ampere $= 0.99994$ Absolute Ampere. The present paper will appear in full in the Bureau of Standards Journal of Research.

95. The Determination of e or λ by the Dispersion of X-Rays. J. A. BEARDEN AND J. A. WHEELER, *Johns Hopkins University.*—Shiba (Inst. of Chem. and Phys. Tokyo **19**, 97 (1932)) has calculated from the dispersion measurements on quartz of one of the authors (Phys. Rev. **39**, 1 (1932)) the value of the charge on the electron to be $e = 4.805 \times 10^{-10}$ e.s.u. which is in agreement with the ruled grating measurements of x-ray wave-lengths. Shiba has used a value of $N_k = 1.3$ in place of $N_k = 2$ and also neglected the effect of the K absorption limit of oxygen. The value of N_k has now been calculated from some of the most precise absorption measurements and plotted as a function of the atomic number. It is the Pauli principle which makes N_k less than 2. A rough theory taking account of the exclusion principle and the interaction between electrons through the use of the Hartree-Slater functions gives good agreement with the experimental curve for atomic numbers below 40 but deviates increasingly for heavier elements. From this curve N_k for silicon is 1.6 and for oxygen is 1.8. A similar calculation for N_L of silicon leads to a value of 8.0. From these results we find values of λ which are in agreement with the crystal determinations. If we assume perfect crystals this gives the usual value of $e = 4.77 \times 10^{-10}$ e.s.u. instead of Shiba's value.

96. A Photographic Determination of the Half-Period of Thorium B. T. R. WILKINS AND S. D. BURDICK, *University of Rochester.*—Extensive studies of the photographic effects of alpha-rays by T. R. Wilkins and R. N. Wolfe have indicated a linear relation between the photographic density and the number of alpha-rays striking an emulsion over a considerable range before the curve flattens out. This linear relation has been used in the present work to measure the period of Th B. Although Th B itself does not emit alpha-rays, its period controlled the rate of decay of the active deposit. The source was moved over a photographic plate by a synchronous motor at a speed of 6.07 mm per hr. The source was of sufficient strength to give photographic densities varying from about 1 at the beginning to about 0.3 at the end of the record. Some plates with beautifully graded records about 7 mm wide and 120 mm long (corresponding to a 20 hr. exposure) give a value for the period of Th B of about 10.2 hrs. as compared with the accepted value of 10.6 hrs. based on ionization measurements.

97. Hemi-Alpha Groups in Atomic Nuclei as Related to the Energy of Binding. WILLIAM D. HARKINS, *University of Chicago.*—In 1921 the writer assumed the existence of hydrogen of mass two, whose nucleus may be called the η-particle. This nucleus was considered of importance as a constituent of other nuclei, which are represented in general by the formula of Harkins and Masson as follows: $\eta_Z n_I$. If Z is even the hemi-alpha groups are supposed to be associated in pairs as α-particles. Recently this formula has been adopted by Heisenberg. If I, the isotopic number, is zero, then Z has never been found to be greater than 7 for any atomic species for which Z is odd. That is, if the number of α-particles in the nucleus is greater than 3, an odd hemi-alpha (η) group has never been found to be present. If such an odd hemi-alpha group is present the addition of a second hemi-alpha group is found to give 23, 21, and 20 million electron-volts in the three cases for which the data are known. This represents the completion of an α-particle in the nucleus. On the other hand, the addition of an odd hemi-α group gives only 3.4 and 8.5 million e.v. in the two cases for which the data are available. Other energy relations obtained from mass values will be discussed.

98. Estimates of Probable Error Using Prior Information. W. EDWARDS DEMING, *Bureau of Chemistry and Soils, Washington, D. C.*—Mathematically manageable forms for the prior existence curves of the unknown mean and standard deviation of a parent population of observations were given and used by Molina and Wilkinson in the Bell System Technical Journal, November 1929. By means of these it is possible to form posterior estimates of the probable error of a set of measurements, provided the prior information is definite enough to permit an assignment of values to the three arbitrary constants that occur in the prior existence forms. This situation is sufficiently realized in some measurements on the densities of 10 crystalline substances that were carried out here by Dr. Sterling B. Hendricks. From 7 to 10 measurements were made on each substance. The posterior estimates of the 10 probable errors are compared with the maximum likelihood estimates (the usual formula). The former estimates show about half as much fluctuation as the latter. This is to be expected, since the prior information acts as a stabilizer for the statistical fluctuations in the standard deviations of samples, so that a posterior estimate is not proportional to the standard deviation of the sample. The posterior method gives the more reliable estimates, and the illustration here presented shows a possible scheme of application.

99. The Azimuthal Asymmetry of the Cosmic Radiation at Mexico City. THOMAS H. JOHNSON, *The Bartol Research*

Foundation and The Carnegie Institution of Washington.—With a train of three Geiger-Mueller counters connected for recording coincident discharges, the intensities of the cosmic radiation on the east and west sides of the magnetic meridian have been compared at Mexico City, magnetic latitude 31° N, elevation 2250 meters. At the same angular distance from the zenith the intensity in the west is greater than that in the east. Preliminary values for the percentage differences between the east and west intensities at various angles from the vertical are as follows:

Distance from the zenith	25°	35°	45°	55°	65°
Percentage difference E–W	1%±2%	5%±2%	10%±2%	13%±4%	25%±10%

These results are just those to be expected on the basis of the theory of the latitude intensity variations of Lemaitre and Vallarta, and they show that the principal corpuscular component of the primary cosmic radiation is positively charged.

100. The Luminescence from Solidified Gases and its Interpretation. L. VEGARD, *Physical Institute, Oslo, Norway.* (*Introduced by* JOHN A. FLEMING.)—Investigations on the luminescence of solidified gases, commenced at Leiden and continued at Oslo, have shown a great variety of bands and lines, a large number of which have been classified into vibrational series and series systems not known from the gas spectra. Most of these appearing in the afterglow are emitted from α-nitrogen either in the pure state or in mixtures with the inert gases neon, argon and krypton. In pure nitrogen the luminescence is dominated by the complex bands N_1, N_2, N_3, N_4 in green and red, and sometimes in the blue part by series called A, B, C, D, F, ζ and the principal series of the ϵ-system. The bands N_2–N_4 and N_1–N_3 are components of vibrational series belonging to the neutral N_2-molecule and the molecular ion N_2^+, respectively. With decreasing N_2 concentration in neon mixtures the complex N_1 band gradually turns into a single line with wave-length 5577. In mixtures of N_2 with inert gases the following vibrational band-systems were observed: the ϵ-family including the ϵ-system and the $B, b, b', D, d, \varkappa, F, f$-series, the ϑ-family including the ϑ, δ, π and χ-series, the A-family including the A, α and ψ-series and the C and ζ-series. The complex structure is reduced with increasing atomic weight of the inert gas, and is explained from vibrations of the molecules in the crystal lattice of α-nitrogen, the structure of which was determined. The ϵ-system and the series referred to it, are found to correspond to the forbidden transition from the A-state towards the normal state of the neutral N-molecule. The electronic transitions for the other series are not yet definitely fixed. A number of infrared bands have been observed and measured, but they are not yet classified.

101. Mass-Spectra of Neon Fractionated by G. Hertz and the Nonexistence of Ne^{23}. K. T. BAINBRIDGE, *Bartol Research Foundation of the Franklin Institute.*—An independent check of Bleakney's measurement of the nonexistence of Ne^{23} to one part in 10,000 has been made possible through the generous gift by Hertz of some fractionated neon containing Ne^{22} in the same abundance as Ne^{20}. The enrichment of the isotopes Ne^{21} and Ne^{22} is approximately three and nine times their abundance in atmospheric neon. Ne^{23} if present would have been enriched twenty-seven times. Spectra have been secured of second order neon which would permit the observation of a line of mass 11.5 (Ne^{23++}), one-third to one-fourth of the intensity of the Ne^{21++} line, corresponding to an abundance of Ne^{23} of one in three hundred to one in four hundred in the fractionated neon or one in eight to ten thousand in the original atmospheric neon. No line has been observed at the position corresponding to Ne^{23++}. A first order spectrum containing Ne^{22} H will be shown together with a second order spectrum taken under the same conditions. From data from Bleakney's and Kallman's papers on He^6 it is possible to show that if helium fractionated by Hertz were analyzed, one part of He^6 in *seven million* in the unfractionated gas should be a lower limit for detection with their apparatus.

102. X-Ray Diffraction of Liquid Films. K. LARK-HOROVITZ AND E. P. MILLER, *Purdue University.*—With Cu$K\alpha$ radiation from a tube as described recently (Phys. Rev. **42**, 915A (1932)), the diffraction patterns of very thin layers of liquids have been investigated. The liquids were held by surface tension in a diaphragm 5 mm in diameter which could be kept at different temperatures down to liquid air temperature. In this way the diffraction patterns of glycerin, different fractions of paraffin oil and commercial mineral oil have been investigated at different temperatures. The pattern for glycerin consists of one intense ring corresponding to $d=4.55$ A.E. at 20°C which increases and becomes sharper at decreasing temperature; also the scattering at small angles decreases. Even at liquid air temperature the structure of glycerin film remained quasi-liquid and the pattern does not show the formation of any crystals or any orientation. Besides the intense inner ring, a faint ring farther out can be observed on some of the plates. The different mineral oils show one or two outer rings besides the most intense inner ring corresponding to $d=5.02$ A.E. Castor oil, under the same conditions, shows one intense ring $d=7.88$ A.E., which seems to be narrower than corresponding rings in mineral oils.

AUTHOR INDEX TO ABSTRACTS OF THE WASHINGTON MEETING

Adel, Arthur and David M. Dennison—No. 13
Ahearn, A. J.—No. 92
Allen, A. J., Rachel Franklin and Ellice McDonald—No. 2
—— see Franklin
Almy, G. M. and F. M. Sparks—No. 14
Anderson, Carl D.—No. 80

Bacher, R. F.—No. 22
—— see Goudsmit
Bainbridge, K. T.—Nos. 82, 102
Barnes, S. W. and L. D. Palmer—No. 51
—— see Richtmyer
Bearden, J. A. and C. H. Shaw—No. 50
—— and J. A. Wheeler—No. 95
Becker, J. A.—see Brattain
—— and W. H. Brattain—No. 91
—— see Sears
Beij, K. H.—No. 32
Bleakney, Walker—No. 81
Bock, Charles D.—No. 69
Bowie, Robert M.—see Fox
Bradbury, N. E.—see Young
Brattain, W. H. and J. A. Becker—No. 90
—— see Becker
Breit, G. and Lawrence A. Wills—No. 21
—— see Johnson
Brickwedde, F. G.—see Silsbee
Brode, Robert B. and Edward B. Jordan—No. 88
Brown, F. W. and C. G. Dunn—No. 38
Bruce, C. W.—No. 17
Burdick, S. D.—see Wilkins

Cohen, V. W.—see Rabi
Compton, K. T. and E. S. Lamar—No. 72
Cook, J. W.—see Silsbee
Curtis, Harvey L., Charles Moon and C. Matilda Sparks—No. 4
—— and Roger W. Curtis—No. 94
Curtis, Roger W.—see Curtis, Harvey L.

Dahl, O.—see Tuve
Deming, Lola S.—see Deming, W. Edwards
Deming, W. Edwards—No. 98
—— and Lola S. Deming—No. 58
Dennison, David M.—see Adel
Dryden, Hugh L.—No. 29
—— W. C. Mock, Jr., and G. B. Schubauer—No. 9
Duffendack, O. S. and R. W. Smith—No. 65
DuMond, Jesse W. M.—No. 45
Dunn, C. G.—see Brown

Eaton, H. N.—No. 33
Ellwood, W. B.—No. 6
Emslie, A. G.—No. 84

Fankuchen, I.—No. 43
Fox, Gerald W. and Robert M. Bowie—No. 88
Franklin, Rachel, A. J. Allen and Ellice McDonald—No. 1
—— see Allen
Fredrickson, W. R. and Carl R. Stannard—No. 11

Gans, David M.—see Harkins
Golay, M. J. E.—see Hershberger
Goudsmit, S. and R. F. Bacher—No. 23
Gray, Newton M.—No. 20
Grimminger, G.—No. 8
Gunn, Ross—No. 74

Hafstad, L. R.—see Tuve
Harkins, William D.—No. 97
—— David M. Gans and Henry W. Newson—No. 78
Harrison, Louis P.—see No. 10
Hershberger, W. D., H. A. Zahl and M. J. E. Golay—No. 7
Heydenberg, N. P.—No. 24
Hirsh, F. R., Jr., and F. K. Richtmyer—No. 48
Holzer, Robert E.—No. 67
Hootman, James A.—No. 18
Hudson, J. C. and H. G. Vogt—No. 46

Johnson, M. H., Jr., and G. Breit—No. 37
Johnson, Thomas H.—No. 99
Jordan, Edward B.—see Brode

von Karman, Th.—No. 28
Keulegan, G. H.—No. 31
Kneser, H. O.—No. 57
Knudsen, V. O.—No. 56
Koehler, James F.—No. 16
Kunz, Jakob—No. 63
Kurie, F. N. D.—No. 79

Lamar, E. S.—see Compton
Langer, R. M. and R. W. Raitt—No. 77
Lark-Horovitz, K. and E. P. Miller—No. 102
Lindsay, R. B.—No. 55
Luhr, Overton—No. 68

Manning, K. V.—No. 52
Margenau, Henry and William W. Watson—No. 66
Maxwell, Louis R.—No. 85
McDonald, Ellice—see Franklin
—— see Allen
Mebs, Russell W.—No. 93
Miller, Dayton C.—No. 73
Miller, E. P.—see Lark-Horovitz
Mock, W. C., Jr.—see Dryden
Mohler, F. L.—No. 64
Moon, Charles—see Curtis

Newson, Henry W.—see Harkins

O'Brien, Brian and Vernon L. Parks—No. 61

Palmer, L. D.—see Barnes
Parks, Vernon L.—see O'Brien
Penney, W. G.—No. 40
Pool, M. L. and S. J. Simmons—No. 25

Rabi, I. I. and V. W. Cohen—No. 19
Raitt, R. W.—see Langer
Randall, H. M. and Norman Wright—No. 12
Richtmyer, F. K. and S. W. Barnes—No. 47
—— see Hirsh
Ruedy, J. E.—No. 26

Sandow, A.—No. 3
Schubauer, G. B.—see Dryden
Schureman, P.—No. 30
Scott, R. B.—see Silsbee
Sears, R. W. and J. A. Becker—No. 89
Seitz, F.—see Wigner
Shaw, C. H.—see Bearden
Shaw, R. William—No. 15
Shelton, S. M.—see Van Dusen
Shortley, George H.—No. 35
Silsbee, F. B., R. B. Scott, F. G. Brickwedde and J. W. Cook—No. 54
Simmons, S. J.—see Pool
Smith, P. T.—see Tate
Smith, R. W.—see Duffendack
Sparks, C. Matilda—see Curtis
Sparks, F. M.—see Almy
Stannard, Carl R.—see Fredrickson
Sterne, T. E.—No. 83
Stewart, G. W.—No. 87
Street, J. C.—No. 75

Tate, John T., P. T. Smith and A. L. Vaughan—No. 71
Trawick, M. W.—No. 34
Tuckerman, L. B.—No. 27
Turner, Louis A.—No. 53
Tuve, M. A., L. R. Hafstad and O. Dahl—No. 76

Ufford, C. W.—No. 36

Van Dusen, M. S. and S. M. Shelton—No. 59
Van Vleck, J. H.—No. 39
Vaughan, A. L.—see Tate
Vegard, L.—No. 100
Vogt, H. G.—see Hudson

Warren, B. E.—No. 44
Waterman, A. T.—No. 41
Watson, William W.—see Margenau
Wheeler, J. A.—see Bearden
Wigner, E. and F. Seitz—No. 42
Wilkins, T. R. and S. D. Burdick—No. 96
—— **and R. N. Wolfe**—No. 60
Williams, John H.—No. 49
Wills, Lawrence A.—see Breit
Wold, P. I.—No. 5
Wolfe, R. N.—see Wilkins
Wood, R. W.—No. 62
Wright, Norman—see Randall

Young, L. A. and N. E. Bradbury—No. 70

Zahl, H. A.—see Hershberger

AUTHOR INDEX TO VOLUME 43

References with (A) are to abstracts of papers presented at meetings of the Physical Society and references with (L) are to Letters to the Editor.

Abramson, H. A. (see Wintersteiner, O.)—372(A)

——— **and Janet Daniel.** Influence of the dielectric constant of the medium on the potential and charge of a protein surface in a liquid—773(A)

Adel, Arthur and David M. Dennison. Infrared spectrum of carbon dioxide. Part I.—716; 1043(A)

Ahearn, A. J. Effect of temperature on the emission of electron field currents from molybdenum and tungsten —1058(A)

Albers, V. M. (see Knorr, H. V.)—379(A)

Albertson, Walter. Application of the mechanical interval recorder to analysis of the spectrum of Os I—501(A)

Allen, A. J. (see Franklin, Rachel)—1041(A)

——— **Rachel Franklin and Ellice McDonald.** Microphotography and some radiation effects on living organisms with various wave-lengths of monochromatic ultraviolet light—1041(A)

Allen, Mildred. Effect of tension on the electrical resistance of the single trigonal crystals, bismuth and antimony—502(A)

——— Effect of tension on the electrical resistance of single antimony crystals—569

Allison, Fred (see Goslin, Roy)—49

——— **and Edna R. Bishop.** Bismuth isotopes—47

Almy, G. M. and F. M. Sparks. Absorption spectrum of diatomic bismuth—1043(A)

Alvarez, Luis and Arthur H. Compton. Positively charged component of cosmic rays—835(L)

Amdur, I. Recombination of atomic hydrogen—208(L)

Anderson, Carl D. Cosmic-ray bursts—368(L)

——— Discovery of the free positive electron—1056(A)

——— Energy-loss and scattering of cosmic-ray particles —381(A)

——— Positive electron—491; 1056(A)

——— **and Seth H. Neddermeyer.** Positrons from gamma-rays—1034(L)

Andrews, Donald H., V. Deitz and S. Goldheim. New method for calculating molecular vibration frequencies —373(A)

Arkadiew, W. On the permeability of iron at ultra-radio frequencies—671(L)

Ashley, Muriel F. Band spectrum of the H^1H^2 molecule—770(L)

——— (see Giauque, W. F.)—81(L)

——— (see Lewis, Gilbert N.)—837(L)

Bacher, R. F. Interaction of configurations: $sd-p^2$—264

——— Note on the magnetic moment of the nitrogen nucleus—1001; 1044(A)

——— (see Goudsmit S.)—894; 1045(A)

Badger, Richard M. and Lyman G. Bonner. Infrared spectrum and the molecular structure of ozone and sulfur dioxide—305(L)

Bainbridge, K. T. Comparison of the masses of He and H^1 on a mass-spectrograph—103

——— Isotopic constitutions of krypton, mercury, selenium, cadmium and germanium—1056(A)

——— Mass of Be^9 and the atomic weight of beryllium —367(L)

——— Masses of Ne^{20} and B^{11}. The mass of Ne^{22} and the disintegration of F^{19}—424

——— Mass-spectra of neon fractionated by G. Hertz and the non-existence of Ne^{22}—1060(A)

——— Measurement of the masses of He and H^1, Ne^{20}, Ne^{22}, B^{11}, Cl^{35} and Cl^{37} with a mass-spectrograph—378(A)

Ballard, Stanley S. and H. E. White. Isotope effect in the Lyman series of hydrogen—941(L)

Baragar, A. E. (see Marvin, H. H.)—963

Barnes, R. Bowling. Notes on the preparation of rest-strahlen plates and the reflection power of powders—31

——— Plasticity of rocksalt and its dependence upon water—82(L)

Barnes, S. W. (see Richtmyer, F. K.)—1049(A)

——— **and L. D. Palmer.** Rocking curve correction in the determination of the widths of x-ray lines—1050(A)

Barnett, S. J. Gyromagnetic ratios for nickel and cobalt—384(A)

——— Search for a new gyromagnetic effect: Magnetization by the rotation of a magnetic field—384(A)

Barton, H. A. (see Van Atta, L. C.)—158

Bateman, H. Schroedinger's rule and hydrodynamics—363(L)

——— Variational principles in electromagnetism—481

Bayley, P. L. Dielectric losses in rocksalt—355

——— Raman spectra of five higher alcohols—373(A)

Beams, J. W. Field electron currents from liquid mercury surfaces—781(A)

——— Initiation of discharges in high vacua—382(A)

Bearden, J. A. α-particles from ionization chamber materials—376(A)

——— Wave-lengths of the silver, molybdenum, copper, iron and chromium $K\alpha_1$ lines—92

——— **and C. H. Shaw.** Wave-lengths and shapes of the K-series lines of Cu, Ni, Co, Fe and Cr—1050(A)

——— **and J. A. Wheeler.** Determination of e or λ by the dispersion of x-rays—1059(A)

Becker, Joseph A. (see Brattain, Walter H.)—428; 1058(A)

——— (see Sears, R. W.)—1058(A)

——— **and W. H. Brattain.** Dependence of adsorbed ions on temperature—1058(A)

Beij, K. H. Capacity of overflow troughs—1046(A)
Bell, E. T. Remarks on the preceding note on many-valued truths—1033(L)
Benedict, William S. Molecular structure of ozone—580(L)
Bennett, R. D. (see Stearns, J. C.)—1038(L)
Benson, A. N. and R. A. Sawyer. Lack of observed hyperfine structure in strontium—766(L)
Birge, Raymond T. Value of e/m—211(A)
Bishop, Edna R. Radioactive families—38
——— (see Allison, Fred)—47
——— **and C. B. Dollins.** Radium isotopes—48
——— **Margaret Lawrenz and C. B. Dollins.** Lead isotopes—43
Bitter, F. Dynamic aspect of ferromagnetism—377(A)
——— On the magnetization of ferromagnetic crystals—655
Blair, Julian M. Herschel effect with x-rays and with blue light—85(L)
Blake, F. C. Critical study of the intensity formula for the powder method of crystal analysis—376(A)
Bleakney, Walker. Probability function for the production of He^{++} by single electron impact—378(A)
——— Relative abundance of the neon isotopes—1056(A)
——— (see Taylor, Hugh S.)—496(L)
Bock, Charles D. Ions in oxygen and hydrogen—1053(A)
Boeker, Gilbert F. Diamagnetism of carbon tetrachloride, benzene and toluene at different temperatures—756
Bonner, Lyman G. (see Badger, Richard M.)—305(L)
Bonner, T. W. Ionization of gases at high pressures by neutrons—871
Botset, H. G. (see Wyckoff, R. D.)—373(A)
Bowen, I. S. and R. A. Millikan. Cosmic-ray intensities in the stratosphere—695
Bowie, Robert M. (see Fox, Gerald W.)—1057(A)
Bradbury, Norris E. Energy distribution of photoelectrons from zinc surfaces—502(A)
——— (see Young, Lloyd A.)—34; 1054(A)
——— **and Lloyd A. Young.** Passage of photoelectrons through mica—84(L)
Bramhall, E. H. Demonstration of an electrostatic high voltage generator—502(A)
——— **and R. J. Van de Graaff.** 1,500,000 volt portable electrostatic generator of rugged design—382(A)
Bramley, Arthur. Continuous beta-ray spectrum—767(L)
Brasefield, Charles J. Ionization of argon, neon and helium by A, Ne and He atoms—776(A); 785
Brattain, W. H. (see Becker, J. A.)—1058(A)
——— **and Joseph A. Becker.** Temperature dependence of the work function for composite surfaces—1058(A)
——— **and Joseph A. Becker.** Thermionic and adsorption characteristics of thorium on tungsten—428
Breit, G. (see Johnson, M. H., Jr.)—1047(A)
——— **and Lawrence A. Wills.** Hyperfine structure in intermediate coupling and nuclear magnetic moments—1044(A)
Brickwedde, F. G. (see Silsbee, F. B.)—1050(A)
Brindley, G. W. Diamagnetic susceptibilities and polarizabilities of ions—1030(L)
Brode, Robert B. (see Jordan, Edward B.)—112; 211(A)
Brode, Robert B. and Edward B. Jordan. Reflection of electrons from liquid mercury—1057(A)
Brown, F. W. and C. G. Dunn. Atomic wave functions—1047(A)
Brown, Frederick L. Index of refraction of carbon dioxide as a function of the density—373(A)
Broxon, James W., George T. Merideth and Louis Strait. Fluctuations of cosmic-ray ionization—687
Bruce, C. W. Electro-optical Kerr effect in gases—1044(A)
Burbank, Cecil J. Anomaly in the columbium K-absorption limit—83(L)
Burdick, S. D. (see Wilkins, T. R.)—1059(A)

Cady, Willoughby M. Spectra of the calcium I-like ions: manganese VI, iron VII, cobalt VIII and nickel IX—214(A); 322
Callihan, Dixon (see Salant, E. O.)—590(L)
Carr, Emma P. (see Stücklen, H.)—944(L)
Cashman, R. J. and W. S. Huxford. Photoelectric sensitivity of magnesium—811
Chamberlain, Katherine and Harold B. Cutter. New lines in the electronic band spectrum of neutral OH—771(L)
Chambers, C. C. Surfaces from which cold emission currents appear only at very high field gradients—768(L)
Clark, George L. and Howard A. Smith. Occurrence of $CuAl_2$ in duralumin—305(L)
Clark, H. T. Portable high-tension voltmeter—382(A)
Coates, Wesley M. Production of x-rays by fast mercury ions—837(L)
——— **and David H. Sloan.** High velocity mercury ions—212(A)
Cohen, V. W. (see Rabi, I. I.)—377(A); 1044(A); 582(L)
Colby, M. Y. and Sidon Harris. Effect of etching on the relative intensities of the components of double Laue spots obtained from a quartz crystal—562
Colwell, R. C. Related figures on circular plates—782(A)
——— **and I. O. Myers.** Reflecting layers of the upper atmospheres—774(A)
Compton, Arthur H. Geographic study of cosmic rays—387
——— Some evidence regarding the nature of cosmic rays—382(A)
——— (see Alvarez, Luis)—835(L)
Compton, K. T. (see Van de Graaff, R. J.)—382(A); 149
——— **and E. S. Lamar.** Test of the "momentum transfer" theory of accommodation coefficients of ions at cathodes—1054(A)
Condon, E. U. Notes on the Stark effect—648
Cook, J. W. (see Silsbee, F. B.)—1050(A)
Cooksey, C. D. and S. T. Stephenson. Secondary absorption and fine structure of x-rays as observed with a double crystal spectrometer—670(L)
Coolidge, Albert Sprague (see James, Hubert M.)—588(L)
Crane, Richard and C. C. Lauritsen. High potential porcelain x-ray tube—212(A)
Crawford, F. H. and W. A. Shurcliff. Band spectrum of CS—766(L)
Curtis, Harvey L., Charles Moon and C. Matilda Sparks. Absolute determination of the ohm—1041(A)
——— **and Roger W. Curtis.** Absolute determination of the ampere—1059(A)

Curtis, Roger W. (see Curtis, Harvey L.)—1059(A)
Cutter, Harold B. (see Chamberlain, Katherine)—771(L)

Dahl, O. (see Tuve, M. A.)—942(L); 1055(A)
Danforth, W. E., Jr. (see Swann, W. F. G.)—778(A)
Daniel, Janet (see Abramson, Harold A.)—773(A)
Davis, Bergen. Conquest of the physical world—379(A)
Davisson, C. J. Electron optics—777(A)
Dawson, David H. (see Johnston, Herrick L.)—374(A); 473; 580(L)
——— **and Herrick L. Johnston.** Further studies in the spectrum of the OH molecule: a new (2,2) band: satellite series in λ3122; Λ-type doubling and electronic spin doubling—980
Deitz, V. (see Andrews, Donald H.)—373(A)
Deming, Lola S. (see Deming, W. Edwards)—1051(A)
Deming, W. Edwards. Estimates of probable error using prior information—1059(A)
——— **and Lola S. Deming.** Entropies of nitrogen, carbon monoxide, and hydrogen from 1 to 1200 atmospheres and from −70°C to 400°C—1051(A)
Dempster, A. J. and R. E. Holzer. Light excitation in neon by lithium ions—365(L)
Dennison, David M. (see Adel, Arthur)—716; 1043(A)
——— (see Gerhard, Sherman L.)—197
Dieke, G. H. Internal resonance in the CO molecule—780(A)
——— **and J. W. Mauchly.** Structure of the third positive group of CO bands—12
Dixon, J. K. Ultraviolet absorption bands of ammonia—711
Dollins, C. B. (see Bishop, Edna R.)—43, 48
Dow, R. B. Viscosity of mixtures of liquids at high pressures—502(A)
Dryden, Hugh L. Studies of boundary layer flow at the Bureau of Standards—1046(A)
——— **W. C. Mock, Jr. and G. B. Schubauer.** Effect of turbulence on the rating of cup anemometers—1042(A)
DuBridge, Lee A. Theory of the energy distribution of photoelectrons—727
Duffendack, O. S. and R. W. Smith. Effect of a small admixture of a foreign gas on the conductivity of a rare gas irradiated by its own resonance radiation—374(A)
——— **and R. W. Smith.** Conductivity of rare gases irradiated by their own resonance radiation—586(L); 1053(A)
——— **and K. Thomson.** Some factors affecting action cross section for collisions of the second kind between atoms and ions—106
DuMond, Jesse W. M. Method of doubling the energy of electrons obtainable in a Sloan type tube and of measuring e/m at a million volts—1049(A)
Dunn, C. G. (see Brown, F. W.)—1047(A)
Dunning, John R. Detection of corpuscular radiation by vacuum tube methods—380(A)
——— **and G. B. Pegram.** Scattering and absorption of neutrons—497(L); 775(A)
Dunnington, Frank G. Determination of e/m for an electron by a new deflection method—211(A); 404
Duveneck, F. B. (see Webster, D. L.)—384(A); 385(A); 839

Eaton, H. N. National Hydraulic Laboratory—1046(A)
Edwards, Hiram W. Aluminum-magnesium mirrors—205(L)
Edwards, J. E. (see Heil, L. M.)—1
Eliason, A. Y. Multiplets in the spectra of Cb III and Mo IV—745
Ellwood, W. B. Sensitive ballistic galvanometer operating in high vacuum—1042(A)
Emslie, A. G. Electron diffraction by stibnite and galena—1056(A)
Epstein, Paul S. Magnetization in strong fields as a function of temperature—216(A)
Estabrook, G. B. Effect of high electrostatic fields upon the vaporization of molybdenum—383(A)
Evans, J. F. (see Schriever, William)—372(A)
Evans, Robley D. Direct fusion method for determining the radium content of rocks—211(A)
Evjen, H. M. New analysis of Slater's compressibility data—215(A)
Ewell, Arthur W. Decomposition of ozone—779(A)

Failla, G. Vacuum tube instrument for measuring very small unidirectional currents and voltages—774(A)
Fankuchen, I. Crystal structure of sodium uranyl acetate—1048(A)
Farnsworth, H. E. Diffraction of low-speed electrons—778(A)
——— **Fine structure of electron diffraction beams from a gold crystal and from a silver film on a gold crystal**—900
——— **and B. A. Rose.** Measurements on contact potential difference between different faces of copper single crystals—375(A)
Fountain, C. R. Absorption and reradiation of short electric waves—384(A)
Fox, Gerald W. and Robert M. Bowie. New method for determining thermionic work functions of metals and its application to nickel—1057(A)
Franklin, Rachel (see Allen, A. J.)—1041(A)
——— **A. J. Allen and Ellice McDonald.** Some micro-moving-pictures showing the lethal effects of ultraviolet radiation on certain living protozoa—1041(A)
Frazier, Richard H. Precise determination of the thermal diffusivity of zinc—135
—— Purity of zinc for which thermal diffusivity recently was reported—762(L)
Frederickson, W. R. and Carl R. Stannard. Magnetic rotation spectrum of the red bands of sodium—1043(A)
Frenkel, J. On a possible explanation of superconductivity—907
Frische, Carl A. Ionization and scattering accompanying positive ion impact in gases—160
Furry, Wendell H. C state of Li_2—361(L)

Gagge, A. P. Magnetic focussing method for the angular distribution of electrons after elastic and inelastic collisions with gases—776(A)
Gans, David M. (see Harkins, William D.)—208(L); 584(L); 1055(A)
Gerhard, Sherman L. and David M. Dennison. Envelopes of infrared absorption bands—197

Germer, L. H. Diffraction of electrons by metal surfaces—724; 777(A)
Giauque, W. F. and Muriel F. Ashley. Molecular rotation in ice at 10°K. Free energy of formation and entropy of water—81(L)
——— **and D. P. MacDougall.** Attainment of temperatures below 1° absolute by demagnetization of $GD_2(SO_4)_3 \cdot 8H_2O$—768(L)
Gideon, Edith (see Henderson, Jos. E.)—601
Ginsburg, N. (see Inglis, D. R.)—194
Gleason, Paul R. Response of barrier-layer photo-cells to x-rays—775(A)
Goetz, A. (see Ho, T. L.)—213(A)
——— (see Wolf, A.)—213(A)
——— **and R. B. Jacobs.** Effect of temperature on the reflection of x-rays from bismuth crystals—213(A)
Golay, M. J. E. (see Hershberger, W. D.)—1042(A)
Goldheim, S. (see Andrews, Donald H.)—373(A)
Gorter, C. J. (see de Haas, W. J.)—81(L)
Goslin, Roy and Fred Allison. Isotopes of uranium, thorium and thallium—49
Goudsmit, S. Nuclear magnetic moments—636
——— (see Wu, Ta-You)—496(L)
——— **and R. F. Bacher.** Anomalies in hyperfine structure—894; 1045(A)
Gould, Austin J. (see Taylor, Hugh S.)—496(L)
Grace, Norman S. Hyperfine structure and nuclear moment of cobalt—762(L)
——— **and H. E. White.** Hyperfine structure and isotopic constitution of tungsten—1039(L)
Gray, Newton M. Spin of Li^7 from hyperfine structure data—1044(A)
Green, J. B. and R. A. Loring. Zeeman effect of lead II, III and IV—459
Grimminger, G. Principle of dimensional homogeneity and aerodynamical resistance, with application to the cup anemometer—1042(A)
Grinfeld, Rafael (see Jenkins, F. A.)—943(L)
Grosse, A. v. Mass of the neutron and the constitution of atomic nuclei—143(L)
Gunn, Ross. High speed ions of stellar origin—380(A)
——— Radiation reaction forces and the expanding universe—764(L); 1054(A)
Gvosdover, S. D. Effect of secondary emission from a metal collector immersed in a discharge—146(L)

de Haas, W. J., J. v. d. Handel and C. J. Gorter. Paramagnetic saturation in a single crystal—81(L)
Hafstad, L. R. (see Tuve, M. A.)—942(L); 1055(A)
Halpern, O. On the statistical foundation of Debye's theory of electrolytes—495(L)
——— On a thermodynamic relation in the theory of electrophoresis—372(A)
Handel, J. v. d. (see de Haas, W. J.)—81(L)
Hansen, W. W. (see Webster, D. L.)—384(A); 385(A); 839
——— **and K. B. Stoddard.** Relation between line and continuous x-ray spectra—701
Hare, Donald G. and Paul Kirkpatrick. Lateral distribution of photoelectrons and the measurement of x-ray polarization—1037(L)
Harkins, William Draper. Emission of γ-rays by nuclei excited by neutrons, and nuclear energy levels—362(L)
——— Hemi-alpha groups in atomic nuclei as related to the energy of binding—1059(A)
——— **David M. Gans and Henry W. Newson.** Atomic disintegration by a relatively slow neutron—208(L)
——— **David M. Gans and Henry W. Newson.** Disintegration of atomic nuclei by neutrons—1055(A)
——— **David M. Gans and Henry W. Newson.** Neutron of high velocity, and energy relations for nuclear disintegration by non-capture—584(L)
Harris, Sidon (see Colby, M. Y.)—562
Harrison, George R. Improvements in the mechanical interval sorter for complex spectra—779(A)
Harrison, Louis P. Review of observation and theory of turbulence in the atmosphere—1042(A)
Harvey, G. G. Diffuse scattering of x-rays from sylvine. II—591
——— Diffuse scattering of x-rays from sylvine. III. Scattering at the temperature of liquid air—707
Havens, Glenn G. Magnetic susceptibilities of some common gases—992
Heaps, C. W. Discontinuities of magnetoresistance—763(L)
——— Electromotive forces associated with Barkhausen discontinuities—945(L)
Heil, L. M. and J. E. Edwards. Measurement of x-ray absorption coefficients by the use of the FP-54 Pliotron—1
Henderson, Jos. E. and Edith Gideon. Cathodic disintegration of platinum by mercury ions—601
Henderson, M. C. Disintegration of lithium by protons of high energy—98; 380(A)
Hershberger, W. D., H. A. Zahl and M. J. E. Golay. Luminous discharge between Lecher wires in a partially evacuated tube—1042(A)
Heydenburg, N. P. Paschen-back effect of hyperfine structure and polarization of resonance radiation, cadmium $(6^1P_1 - 5^1S_0)$—640
——— Paschen-back effect and the polarization of resonance radiation. Sodium $(5^2P_{\frac{1}{2},\frac{3}{2}} - 3^2S_{\frac{1}{2}})$—1045(A)
Hill, E. L. Conservation theorems in quantum mechanics—780(A)
——— Perturbation theory and electron scattering—303(L)
Hirsch, F. R., Jr. and F. K. Richtmyer. Measurement of the relative intensity of x-ray satellites in cathode-ray and in fluorescence excitation—1049(A)
Ho, T. L. and A. Goetz. Thermal expansion of Bi single crystals—213(A)
Hoffmann, Banesh. Projective relativity and the Einstein-Mayer unified field theory—615; Erratum—946(L)
Holzer, Robert E. Light excitation by alkali ions in the Noble gases—1053(A)
——— (see Dempster, A. J.)—365(L)
Hootman, James A. Kerr's law at high field strengths—749; 1044(A)

Hopfield, John J. Use of argon in the ionization method of measuring cosmic rays and gamma-rays—675
Howell, Lynn G. (see Mott-Smith, Lewis M.)—381(A)
——— **and Lewis M. Mott-Simth.** Airplane method of obtaining cosmic-ray intensity data—381(A)
Howey, J. H. Effect of spin interaction in the diffraction and polarization of electrons—499(L)
Hudson, J. C. and H. G. Vogt. *K*-series spectrum of tungsten—1049(A)
Huggins, M. L. and J. E. Mayer. Interatomic distance calculations for ionic crystals—373(A)
Hughes, A. L. and J. H. McMillen. Elastic electron scattering in neon—875
Hull, A. W. Characteristics and functions of Thyratrons—379(A)
Huxford, W. S. (see Cashman, R. J.)—811

Ikehara, S. (see Rosen, N.)—5
Inglis, D. R. and N. Ginsburg. Atomic energy levels and Zeeman effect—194
Ireland, Clarence. Quantum mechanics of beryllium hydride—329

Jacobs, R. B. (see Goetz, A.)—213(A)
James, Hubert M. Influence of inner shells on atomic interactions—589(L)
——— **and Albert Sprague Coolidge.** Improved calculation of ground state of H_2—588(L)
Jauncey, G. E. M. and Ford Pennell. Scattering of x-rays from powdered crystals—505
Jefferson, M. E. (see Maxwell, Louis R.)—777(A)
Jen, C. K. Continuous electron affinity spectrum of hydrogen—540
Jenkins, F. A. and Rafael Grinfeld. Spectrum of MgF—943(L)
Jeppesen, C. Rulon. Extreme ultraviolet spectrum of molecular hydrogen—214(A)
Joffe, J. and Harold C. Urey. Spin of the sodium nucleus—761(L)
Johnson, M. H., Jr. Note on almost closed shells—632
——— Vector model and the Pauli principle—627
——— **and G. Breit.** Interaction of a valence electron with a closed shell—1047(A)
Johnson, Thomas H. Azimuthal asymmetry of the cosmic radiation—834(L)
——— Azimuthal asymmetry of the cosmic radiation at Mexico City—1059(A)
——— Comparison of the angular distributions of the cosmic radiation at elevations 6280 ft. and 620 ft.—307; 385(A)
——— Cosmic-ray hodoscope and a circuit for recording multiply coincident discharges of Geiger-Mueller counters—379(A)
——— **and E. C. Stevenson.** Angular distribution of low-energy cosmic radiation and interpretation of angular distribution curves—583(L); 775(A)
——— **and J. C. Street.** Variation of cosmic-ray intensities with azimuth on Mt. Washington, N. H.—381(A)
Johnston, Herrick L. (see Dawson, David H.)—980
——— **and David H. Dawson.** Identification of the (2,2) band of neutral OH. Satellite series of the (1,1) band. Extensions of the (0,0) band—580(L)
——— **David H. Dawson and Margery K. Walker.** New band in the spectrum of the OH molecule—374(A); 473
Jordan, Edward B. (see Brode, Robert B.)—1057(A)
——— **and Robert B. Brode.** Elastic scattering of electrons by mercury atoms—112; 211(A)

Kabakjian, D. H. Decay of luminescence and light absorption in phosphorescent materials—378(A)
Kaplan, Carl. On some applications of the absolute differential calculus to physics—137
Kaplan, Joseph. Auroral spectrum—214(A)
von Karman, Th. Our present knowledge of turbulent flow—1046(A)
Kassel, Louis S. Thermodynamic calculations from spectroscopic data—364(L)
Kempf, L. W. Occurrence of $CuAl_2$ in duralumin—942(L)
Kennard, E. H. Orthogonality of scattered wave functions—378(A)
——— Torque on Faraday's magnet—587(L)
Kennard, R. B. Variation with height of convective heat flux from parallel vertical plates—372(A)
Kenty, Carl. Photoelectric and metastable atom emission of electrons from surfaces in the rare gases—181
——— Variation of photoelectric efficiency with work function in the extreme ultraviolet—776(A)
Keulegan, G. H. Present status of knowledge of pressure loss for flow in curved pipes—1046(A)
King, Arthur S. Electric-furnace ionization effect—214(A)
King, P. B. (see Mimno, Harry R.)—501(A)
Kirkpatrick, Paul and P. A. Ross. Absolute x-ray reflectivities of single crystals of calcite, rocksalt, Rochelle salt and barite—596
——— (see Hare, Donald G.)—1037(L)
——— (see Ross, P.A.)—1036(L)
Knauss, Harold P. Production of specified wave forms with the photoelectric siren—375(A)
Kneser, H. O. Sound absorption in molecular gases—215(A)
——— Transfer of vibrational energy between molecules—1051(A)
Knopf, G. D. (see Kurie, F. N. D.)—311; 781(A)
Knorr, H. V. and V. M. Albers. Fluorescence of solutions of chlorophyll—379(A)
Knudsen, V. O. Absorption of sound in air and water vapor—1051(A)
Koehler, James F. Quenching of iodine fluorescence by iodine and argon—1044(A)
Kretschmar, G. G. Determination of e/m by means of photoelectrons excited by x-rays—417
Krutter, H. M. (see Warren, B. E.)—500(A)
Kunz, Jakob. Photoelectric effect of caesium vapor—1052(A)
——— Problem of the intrinsic magnetic field by P. Weiss—377(A)
Kurie, Franz N. D. Angular distribution of protons projected by neutrons—1056(A)
——— Collisions of neutrons with protons—672(L)

Kurie, Franz N. D. Nitrogen disintegration by a very fast neutron—771(L)
——— **and G. D. Knopf.** Measurement of the range of alpha-particles from thorium with the Wilson chamber—311; 781(A)

Ladenburg, R. and C. C. Van Voorhis. Continuous absorption of oxygen between 1750 and 1300A and its bearing upon dispersion—315
Lamar, E. S. Momentum transfer to cathode surfaces by impinging positive ions in a helium arc—169
——— (see Compton, K. T.)—1054(A)
La Mer, Victor K. and M. L. Miller. Index of refraction of water and its thermal history—207(L)
Landé, Alfred. Neutrons in the nucleus. I—620; II—624
Langer, R. M. Latitude effect on secondary electrons due to cosmic rays—215(A)
——— **and R. W. Raitt.** Radioactivity of beryllium—1055 (A)
——— **and R. W. Raitt.** New kind of radioactivity—585 (L)
Langmuir, Irving. Nature of adsorbed films of caesium on tungsten. Part I. The space charge sheath and the image force—224
Laporte, Otto. Note on Kowalewski's top in quantum mechanics—548; 782(A)
Lark-Horovitz, K. and E. P. Miller. X-ray diffraction of liquid films—1060(A)
——— **and H. J. Yearian.** New electron diffraction rings in zinc oxide and the interpretation—376(A)
Lauritsen, Charles C. Portable roentgenmeter—212(A)
——— (see Crane, Richard)—212(A)
Lawrence, Ernest O. (see Livingston, M. Stanley)—212 (A); 369(L)
——— (see White, Milton G.)—304(L)
Lawrenz, Margaret (see Bishop, Edna R.)—43
Leberknight, C. E. Infrared absorption of benzene in the liquid, solid and vapor states—967
Lemaitre, G. Uncertainty of the electromagnetic field of a particle—148(L)
——— **and M. S. Vallarta.** On Compton's latitude effect of cosmic radiation—87
Lewis, Gilbert N. and Muriel F. Ashley. Spin of hydrogen isotope—837(L)
——— **and Frank H. Spedding.** Spectroscopic search for H^3 in concentrated H^2—964
Lindsay, R. B. Filtration of elastic waves in solid rods—1051(A)
——— Modification of Brillouin's unified statistics—780(A)
Livingston, M. Stanley. Attainment of high vacua in large metal chambers—214(A)
——— **and Ernest O. Lawrence.** Disintegration of aluminum by swiftly moving protons—369(L)
——— **and Ernest O. Lawrence.** Production of 4,800,000 volt hydrogen ions—212(A)
Locher, Gordon L. Expansion chamber data on cosmic-ray ionization—381(A)
Loring, R. A. (see Green, J. B.)—459
Lozier, W. Wallace. Dissociation products of carbon monoxide formed by primary electron impact—776(A)
Luhr, Overton. Nature of gas ions—1053(A)

MacDonald, J. K. L. Successive approximations by the Rayleigh-Ritz variation method—830
Macdonald, R. T. and G. K. Rollefson. Effect of foreign gases upon the relative intensities of the mercury triplet $2^3P_{012}-2^3S_1$ under conditions of optical excitation—177
MacDougall, D. P. (see Giauque, W. F.)—768(L)
Mais, W. H. and I. I. Rabi. Direct measurement of the scattering of potassium atoms by other atoms and molecules—378(A)
Manning, K. V. Effect of etching on the rocking curve widths of calcite crystals—1050(A)
Margenau, Henry. Pressure broadening of spectral lines. II—129, 374(A)
——— **and William W. Watson.** Pressure effects of foreign gases on the sodium resonance lines—1053(A)
Marvin, H. H. and A. E. Baragar. Zeeman effect in the arc spectrum of nickel—973
Mauchly, J. W. (see Dieke, G. H.)—12
Maxwell, Louis R. Electron diffraction by transmission through liquids—1057(A)
——— **M. E. Jefferson and V. M. Mosley.** Diffraction of electrons by single molecules—777(A)
Mayer, J. E. (see Huggins, M. L.)—373(A)
——— **and Maria Goeppert Mayer.** Polarizabilities of ions from spectra—605
Mayer, Maria Goeppert (see Mayer, J. E.)—605
McDonald, Ellice (see Allen, A. J.)—1041(A)
——— (see Franklin, Rachel)—1041(A)
McKeehan, L. W. Magnetic dipole fields in unstrained cubic crystals—913
——— Magnetic dipole fields in dislocated cubic crystals—924
——— Magnetic dipole energy in homogeneously strained cubic crystals—1022
——— Magnetic dipole energy in hexagonal crystals—1025
McKellar, Andrew. Mass ratio of the lithium isotopes from the spectrum of Li_2—215(A)
McMillen, J. H. (see Hughes, A. L.)—875
Mebs, Russell W. Effect of high series resistance on cold emission—1058(A)
Menzel, D. H. Physics of the sun—502(A)
Merideth, George T. (see Broxon, James W.)—687
Miller, Dayton C. Absolute motion of the solar system and the orbital motion of the earth determined by the ether-drift experiment—1054(A)
Miller, E. P. (see Lark-Horovitz, K.)—1060(A)
Miller, M. L. (see La Mer, Victor K.)—207(L)
Millikan, R. A. (see Bowen, I. S.)—695
——— New techniques in the cosmic-ray field and some of the results obtained with them—661
——— **and H. Victor Neher.** New technique in the cosmic-ray field and some of the results obtained with it—381(A)
——— **and H. Victor Neher.** Sun and cosmic rays—211(A)
Mimno, Harry Rowe and P. H. Wang. Balanced receiving circuit for Kennelly-Heaviside layer observations—774 (A)

Mimno, Harry Rowe and P. H. Wang. Double-refraction effects in the Kennelly-Heaviside layers—769(L)
——— **P. H. Wang and P. B. King.** New modulator for use in Kennelly-Heaviside layer recording—501(A)
Mitchell, Allan C. G. Effect of hyperfine structure on magnetic depolarization of resonance radiation—780(A)
——— Hyperfine structure and the polarization of resonance radiation. II. Magnetic depolarization and the determination of mean lives—887
Mock, W. C., Jr. (see Dryden, Hugh L.)—1042(A)
Mohler, F. L. Recombination spectra in the positive column of a caesium discharge—374(A)
——— Spontaneous recombination and the effect of vapor pressure in a caesium discharge—1052(A)
Montgomery, Carol G. (see Swann, W. F. G.)—782(A)
——— **and William H. Ross.** Magnetic susceptibilities of lead, silver and their alloys—358
Moon, Charles (see Curtis, Harvey L.)—1041(A)
Morse, P. M. (see Vinti, J. P.)—337
——— **and L. A. Young.** Variational atomic wave functions—501(A)
Mosley, V. M. (see Maxwell, Louis R.)—777(A)
Mott-Smith, Lewis M. (see Howell, Lynn G.)—381(A)
——— **and Lynn G. Howell.** Airplane cosmic-ray intensity measurements—381(A)
Mueller, D. W. (see Smyth, H. D.)—116; 121
Müller, Hans. Weiss law for Rochelle salt—500(A)
Mulliken, Robert S. Electronic structures of polyatomic molecules and valence. IV. Electronic states, quantum theory of the double bond—279
——— Electronic structures of polyatomic molecules and valence. Magnetism of B_2H_6—765(L)
Munro, George C. Theory of the inverse first power electrostatic field—213(A)
Myers, I. O. (see Colwell, R. C.)—774(A)

Nathanson, J. B. Interference of light in transparent films of manganese—383(A)
Neddermeyer, Seth H. (see Anderson, Carl D.)—1034(L)
Neher, H. Victor (see Millikan, Robert A.)—211(A); 381(A)
Newson, Henry W. (see Harkins, William D.)—208(L); 584(L); 1055(A)
Nielsen, H. H. (see Sprague, A. D.)—375(A)
Nishikawa, S., Y. Sakisaka, I. Sumoto. X-ray examination of the harmonic thickness vibration of piezoelectric quartz plates—363(L)
Norris, Will V. and Hilbert J. Unger. Infrared absorption bands of methane—467

O'Brien, Brian and Vernon L. Parks. Influence of frequency of flash on the photographic intermittency effect—1052(A)
Oldenberg, O. Attempts to observe an electron affinity spectrum—374(A); 534
——— On a molecular spectrum emitted from atomic iodine vapor—501(A)
Oncley, J. L. and John Warren Williams. Frequency variation of the dielectric constant of dilute non-aqueous solutions—341
Oppenheimer, J. R. Disintegration of lithium by protons—380(A)
Osterberg, H. Multiple interferometer for analyzing the vibrations of a quartz plate—819
——— (see Roebuck, J. R.)—60

Palmer, L. D. (see Barnes, S. W.)—1050(A)
Parker, Allan E. Partial resolution of the Be I line λ4572.69 and probable spin of the Be^9 nucleus—1035(L)
——— Zeeman effect for perturbed N_2^+ terms—780(A)
Parks, Vernon L. (see O'Brien, Brian)—1052(A)
Pegram, G. B. (see Dunning, J. R.)—497(L); 775(A)
Pennell, Ford (see Jauncey, G. E. M.)—505
Penney, W. G. Crystalline fields of Pr, Nd and Yb from paramagnetic susceptibilities—485
——— Structure of ethylene and the theory of the double bond—1048(A)
Pickering, W. (see Sparks, B.)—214(A)
Pierce, W. C. Scattering of x-rays by the gaseous dichlorbenzenes—145(L)
Piggot, Charles Snowden. Isotopes of uranium, thorium and lead, and their geophysical significance—51
Pike, Eugene W. Efficiency of electron release by metastable atoms—144(L)
Plumley, Harold. Resonance spectrum of bromine—495(L)
Pool, M. L. and S. J. Simmons. Absorption and emission of hyperfine structure in optically excited mercury vapor—1045(A)
Poole, Cyrus A. and W. S. Werner. High voltage x-ray tube and constant potential transformer equipment—774(A)
Potapenko, G. and R. Sanger. New method of measuring electric and magnetic properties of metals in the region of ultra-short electromagnetic waves—210(A)
Powell, W. M. Intensity measurements between 1650A and 1240A with a photoelectric cell—779(A)

Rabi, I. I. (see Mais, W. H.)—378(A)
——— Effective neutron collision radius—838(L)
——— **and V. W. Cohen.** Magnetic velocity selector for molecular beams—377(A); 1044(A)
——— **and V. W. Cohen.** Nuclear spin of sodium—582(L)
Raitt, R. W. (see Langer, R. M.)—585(L); 1055(A)
Randall, H. M. and Norman Wright. Pure rotation absorption spectra of NH_3 and PH_3 gases under high resolving power—1043(A)
Rashevsky, N. Physico-mathematical aspects of the conduction of nervous impulse—372(A)
Rice, O. K. Predissociation and the crossing of molecular potential energy curves—501(A)
Richtmyer, F. K. (see Hirsch, F. R., Jr.)—1049(A)
——— **and S. W. Barnes.** Widths of the lines of the *K*-series of tungsten—1049(A)
Robertson, H. P. (see Tolman, R. C.)—564
Roebuck, J. R. and H. Osterberg. Joule-Thomson effect in helium—60
Rojansky, V. On a relativistic particle with a *q*-number charge—207(L)
Rollefson, G. K. (see Macdonald, R. T.)—177
Rose, B. A. (see Farnsworth, H. E.)—375(A)

Rosen, N. Potential energy of diatomic molecules—671(L)
——— **and S. Ikehara.** Interaction between atoms with *s*-electrons—5
Rosenthal, Jenny E. (see Salant, E. O.)—581(L)
Ross, P. A. and Paul Kirkpatrick. Weak x-ray lines of columbian and antimony—1036(L)
——— (see Kirkpatrick, Paul)—596
Ross, William H. (see Montgomery, Carol G.)—358
Ruark, Arthur E. (see Western, Forrest)—205(L); 781(A)
Ruedy, J. E. Arc spectrum of sulfur in the ultraviolet—1045(A)

Sabine, George B. (see Williams, Robley C.)—362(L)
Sakisaka, Y. (see Nishikawa, S.)—363(L)
Salant, E. O. and Dixon Callihan. Modified scattering by crystalline hydrogen halides—590(L)
——— **and Jenny E. Rosenthal.** Vibrational isotope effects in polyatomic molecules. II—581(L)
Sandow, A. Diffraction spectra from striated muscle and the mechanism of contraction—1041(A)
Sanger, R. (see Potapenko, G.)—210(A)
Sawyer, R. A. (see Benson, A. N.)—766(L)
Schmidt, A. R. (see Woodrow, Jay W.)—379(A)
Schneider, Edwin G. Note on photographic intensity measurement in the Schumann region—779(A)
Schoepfle, George Kern. Extension of the Pt I-like isoelectronic sequence tellurium IV, lead V and bismuth VI—374(A)
——— Extension of the Pd I-like isoelectronic sequence to antimony VI and tellurium VII—742
Schriever, William and J. F. Evans. Motion of an air-segment in a water-filled capillary tube—372(A)
Schubauer, G. B. (see Dryden, Hugh L.)—1042(A)
Schureman, P. Tides—1046(A)
Scott, R. B. (see Silsbee, F. B.)—1050(A)
Sears, R. W. and J. A. Becker. Thermionic and adsorption characteristics of platinum on tungsten—1058(A)
Seitz, F. (see Wigner, E.)—804; 1048(A)
Serber, Robert. Calculation of statistical averages for perturbed systems—1011
——— Theory of depolarization, optical anisotropy and the Kerr effect—1003
Shaw, C. H. (see Bearden, J. A.)—1050(A)
Shaw, R. William. Vibrational analysis of the GeO emission band spectrum—1043(A)
Shelton, S. M. (see Van Dusen, M. S.)—1052(A)
Shonka, J. J. Effect of temperature on the intensity of x-rays scattered by powdered sodium fluoride—947
Shortley, George H. Configuration p^5p of the rare gases—1047(A)
——— Transformations in the theory of complex spectra—451
Shrader, J. E. New high tension electrostatic voltmeter—774(A)
Shurcliff, W. A. (see Crawford, F. H.)—766(L)
Silsbee, F. B., R. B. Scott, F. G. Brickwedde and J. W. Cook. Superconductivity of tin at radiofrequencies—1050(A)
Simmons, S. J. (see Pool, H. L.)—1045(A)
Sixtus, K. J. (see Tonks, L.)—70; 931
Slichter, L. B. Potential problem for a semi-infinite medium of variable conductivity—373(A)
Sloan, David H. High voltage vacuum tube—213(A)
——— (see Coates, Wesley M.)—212(A)
Smekal, Adolf G. Plasticity of rocksalt and its dependence upon water—366(L)
Smith, Howard A. (see Clark, George L.)—305(L)
Smith, Philip T. (see Tate, John T.)—672(L); 1054(A)
Smith, R. W. (see Duffendack, O. S.)—374(A); 586(L); 1053(A)
Smith, Sinclair. Sensitive stellar photometer—211(A)
Smyth, H. D. and D. W. Mueller. Ionization of water vapor by electron impact—116
——— **and D. W. Mueller.** Ionization of sulphur dioxide by electron impact—121
Sonkin, Simon. Action of mercury metastable atoms on a tungsten surface—788
——— **and Harold W. Webb.** Action of mercury metastable atoms on metal surfaces—776(A)
Sparks, B. and W. Pickering. Some new coincidence measurements on the cosmic radiation—214(A)
Sparks, C. Matilda (see Curtis, Harvey L.)—1041(A)
Sparks, F. M. (see Almy, G. M.)—1043(A)
Spedding, F. H. Interpretation of line spectra in crystals—143(L)
——— Zeeman effect in crystals—214(A)
——— (see Lewis, Gilbert N.)—964
Sprague, A. D. and H. H. Nielsen. Infrared rotation vibration spectrum of hydrogen sulphide—375(A)
Sproull, Wayne T. Diffraction of low-speed electrons by a tungsten single crystal—516
Stannard, Carl R. (see Frederickson, W. R.)—1043(A)
Stearns, J. C. and R. D. Bennett. Angular distribution of cosmic-ray particles—1038(L)
Steinberger, R. L. Ferromagnetism of the iron nickel alloys under hydrostatic pressure—502(A)
Stephenson, Reginald J. Fluorescence yield from L_{III} level of uranium—527
Stephenson, S. T. (see Cooksey, C. D.)—670(L)
Sterne, T. E. Equilibrium theory of the abundance of the elements—585(L); 768(L)
——— Statistical investigation of an assembly in equilibrium in which all possible transmutations can occur—1056(A)
Stevenson, E. C. (see Johnson, Thomas H.)—583(L); 775(A)
Stewart, G. W. Liquid structure in ionic solutions—1057(A)
Stierstadt, O. New phenomena in the change of resistance of Bi single crystals in magnetic fields—577
Stoddard, K. B. (see Hansen, W. W.)—701
Strait, Louis (see Broxon, James W.)—687
Street, J. C. Production of secondaries by cosmic rays—1055(A)
——— (see Johnston, Thomas H.)—381(A)
Strong, John. Evaporation technique for aluminum—498(L)
Stücklen, H. and Emma P. Carr. Metastable ^2D-level of the nitrogen atom—944(L)
Sumoto, I. (see Nishikawa, S.)—363(L)

Sutherland, G. B. B. M. Infrared absorption bands of acetylene—883
Swann, W. F. G. Detection of the ionization by individual cosmic rays—775(A)
——— Mechanism of acquirement of cosmic-ray energies by electrons—217
——— On the nature of the primary cosmic radiation—945(L)
——— **and W. E. Danforth, Jr.** Lens for cosmic-ray electrons—778(A)
——— **and C. G. Montgomery.** Non-unidirectional emission of Hoffman's Stösse—782(A)

Tate, John T. and Philip T. Smith. Attempt to observe a helium isotope—672(L)
——— **Philip T. Smith and A. L. Vaughan.** Mass spectrum analysis of the products of ionization by electron impact in nitrogen and acetylene—1054(A)
Taylor, Hugh S., Austin J. Gould and Walker Bleakney. Separation of hydrogen isotopes by fractional desorption—496(L)
Thomassen, L. and J. E. Wilson. Note on the broadening of x-ray lines of cold-worked aluminum—763(L)
Thomson, K. (see Duffendack, O. S.)—106
Tolman, R. C. and H. P. Robertson. On the interpretation of heat in relativistic thermodynamics—564
Tonks, L. and K. J. Sixtus. Propagation of large Barkhausen discontinuities. III. Effect of a circular field with torsion—70
——— **and K. J. Sixtus.** Propagation of large Barkhausen discontinuities. IV. Regions of reversed magnetization—931
Trawick, M. W. Spectra of Cb V and Mo VI—1047(A)
Tuckerman, L. B. Theory and experiment in fluid dynamics—1045(A)
Turner, Louis A. Direct-current amplifier circuit—1050(A)
Tuve, M. A. Recent advances in nuclear physics—502(A)
——— Search by deflection-experiments for the Dirac isolated magnetic pole—770(L)
——— **L. R. Hafstad and O. Dahl.** Disintegration-experiments on elements of medium atomic number—942(L)
——— **L. R. Hafstad and O. Dahl.** Nuclear physics studies using the Van de Graaff electrostatic generator—1055(A)

Uehling, E. A. and G. E. Uhlenbeck. Transport phenomena in Einstein-Bose and Fermi-Dirac gases. I—552
Ufford, C. W. Configuration interaction in complex spectra 1047—(A)
Uhlenbeck, G. E. (see Uehling, E. A.)—552
Unger, Hilbert J. Infrared absorption bands of ammonia—123
——— (see Norris, Will V.)—467
Urey, Harold C. (see Joffe, J.)—761(L)
Utterback, C. L. and E. Allan Williams. Secondary electron emission from tantalum—212(A)

Valasek, Joseph. Effects of chemical combination on the x-ray emission spectrum of sulphur—612
Vallarta, M. S. (see Lemaitre, G.)—87
Van Atta, L. C. (see Van de Graaff, R. J.)—382(A); 149
——— **R. J. Van de Graaff and H. A. Barton.** New design for a high-voltage discharge tube—158
Van de Graaff, R. J. High-voltage methods in the investigation of nuclear structure—502(A)
——— (see Bramhall, E. H.)—382(A)
——— (see Van Atta, L. C.)—158
——— **K. T. Compton and L. C. Van Atta.** Electrostatic production of high voltages for nuclear investigations—382(A); 149
Van Dusen, M. S. and S. M. Shelton. Thermal conductivity of metals in the range 0 to 600°C—1052(A)
Van Vleck, J. H. Perturbations due to spin-orbit forces in CO and other band spectra—1047(A)
Van Voorhis, C. C. (see Ladenburg, R.)—315
Van Voorhis, S. N. Small-angle inelastic scattering of electrons in helium, mercury and hydrogen—777(A)
Vaughan, A. L. (see Tate, John T.)—1054(A)
Vegard, L. Luminescence from solidified gases and its interpretation—1060(A)
Vinti, J. P. (see Witmer, E. E.)—780(A)
——— **and P. M. Morse.** Variable scale atomic wave functions—337
Vogt, H. G. (see Hudson, J. C.)—1049(A)

Wainer, Eugene. Chemical intensification of x-ray spectrograms—303(L)
Walker, Margery K. (see Johnston, Herrick L.)—374(A); 473
Wang, P. H. (see Mimno, Harry Rowe)—769(L); 774(A); 501(A)
Warren, B. E. X-ray diffraction study of vitreous silica and Pyrex—1048(A)
——— **and H. M. Krutter.** X-ray study of crystal structure of Rochelle salt and effect of temperature—500(A)
Waterman, A. T. Note on contact potential difference—1048(A)
Watson, William W. Barium hydride band spectra in the near infrared—375(A); 9
——— (see Margenau, Henry)—1053(A)
Webb, Harold W. (see Sonkin, S.)—776(A)
Weber, Ernst. Electron flow in large direct currents—781(A)
Webster, D. L., W. W. Hansen and F. B. Duveneck. Probabilities of *K*-electron ionization of silver by cathode rays—384(A); 839
——— **W. W. Hansen and F. B. Duveneck.** Relative intensity of the silver *K*-lines from a thick target as a function of voltage and emergence angle—385(A)
Werner, W. S. (see Poole, Cyrus A.)—774(A)
Western, Forrest and Arthur E. Ruark. Actinouranium and the geologic time scale—205(L); 781(A)
Wheeler, John A. Theory of the dispersion and absorption of helium—258
——— (see Bearden, J. A.)—1059(A)
White, H. E. (see Ballard, Stanley S.)—941(L)
——— (see Grace, Norman S.)—1039(L)
White, Milton G. and Ernest O. Lawrence. Disintegration of boron by swiftly moving protons—304(L)
Wigner, E. On the mass defect of helium—252

Wigner, E. and F. Seitz. On the constitution of metallic sodium—804; 1048(A)
Wilkins, T. R. and S. D. Burdick. Photographic determination of the half-period of thorium *B*—1059(A)
——— **and R. N. Wolfe.** Response of a chromic acid desensitized emulsion to alpha-rays—1052(A)
Williams, E. Allan (see Utterback, C. L.)—212(A)
Williams, John H. Relative intensities of the *K*-series lines of the elements 24 to 52 by the ionization chamber method—1049(A)
Williams, John Warren (see Oncley, J. L.)—341
Williams, Robley C. and George B. Sabine. Lack of observed hyperfine structure in the Al III line, 5722.6A —362(L)
Wills, Lawrence A. (see Breit, G.)—1044(A)
Wilson, J. E. (see Thomassen, L.)—763(L)
Wilson, Thomas A. Unit cell of uranium calculated from x-ray powder method data—781(A)
Wintersteiner, O. and H. A. Abramson. Electrical potential of amorphous and crystalline amphoteric surfaces in liquids—372(A)
Witmer, E. E. and J. P. Vinti. Symmetry properties and the indistinguishability of similar particles—780(A)
Wold, P. I. Resistivity of expanded conductors—383(A); 1042(A)
Wolf, A. and A. Goetz. Magnetostriction in bismuth single crystals—213(A)
Wolfe, Hugh C. Interpretation of x-ray satellites—221; 376(A)
Wolfe, R. N. (see Wilkins, T. R.)—1052(A)
Wollan, E. O. Measurements of intensity of Compton modified radiation by means of filters—955
Wood, Lawrence A. Equivalent circuit of a blocking-layer photo-cell—375(A)
Wood, R. W. (a) Raman spectra made with a quartz high potential mercury vacuum tube (Hannovia tube). (b) Exhibition of phase-reversal zone-plates ruled on a specially constructed machine—782(A)
——— Remarkable optical properties of the alkali metals —1052(A)
——— Transmission band of alkali metals in ultraviolet —779(A)
Woodrow, Jay W. and A. R. Schmidt. Fluorescence of some substances containing vitamin A—379(A)
Workman, E. J. Secondary effects in ionization by hard gamma-rays—859
Worthing, A. G. On the effect of high electrostatic fields on the vaporization of metals—383(A)
Wright, Norman (see Randall, H. M.)—1043(A)
Wu, Ta-You and S. Goudsmit. Low states of the heaviest elements—496(L)
Wyckoff, R. D. and H. G. Botset. Experimental study of the motion of particles in systems of complex potential distribution—373(A)

Yearian, H. J. Logarithmic protractor—779(A)
——— (see Lark-Horovitz, K.)—376(A)
Young, Lloyd A. (see Bradbury, Norris E.)—84(L)
——— (see Morse, Philip M.)—501(A)
——— **and Norris E. Bradbury.** Electron capture cross sections—1054(A)
——— **and Norris E. Bradbury.** Photoelectric currents in gases between parallel plate electrodes—34

Zacharias, Jerrold. Temperature dependence of Young's modulus for nickel—377(A)
Zahl, H. A. (see Hershberger, W. D.)—1042(A)
Zisman, W. A. Elastic constants of rocks and their relation to seismic wave speeds—501(A)
Zwicky, F. How far do cosmic rays travel?—147(L)
——— On cooperative phenomena—215(A); 270
——— On a new type of reasoning and some of its possible consequences—1031(L)
——— On the plasticity of crystals—765(L)
Zworykin, Vladimir K. On electron beams in high vacuum —778(A)

ANALYTIC SUBJECT INDEX TO VOLUME 43

References marked (A) are to abstracts of papers presented at meetings of the Physical Society, those marked with (L) are to Letters to the Editor.

Absorption of light (see also **Spectra, absorption**)
By cancer cells, A. J. Allen, R. Franklin, E. McDonald—1041(A)
Of O_2 between 1750 and 1300°A, R. Ladenburg, C. C. Van Voorhis—315
In phosphorescent materials, D. H. Kabakjian—378(A)
Transmission band of alkali metals in ultraviolet, R. W. Wood—779(A)

Accommodation coefficient
Momentum transfer theory, K. T. Compton, E. S. Lamar—1054(A)
Positive ions in an He arc, E. S. Lamar—169

Acoustics
Absorption of sound in air and water vapor, V. O. Knudsen—1051(A)
Figures on circular plates, R. C. Colwell—782(A)
Filtration of elastic waves in rods, R. B. Lindsay—1051(A)
Sound absorption and transfer of vibrational energy, H. O. Kneser—1051(A)
Sound absorption in molecular gases, H. O. Kneser—215(A)

Adsorption
Ions and temperature effects, J. A. Becker, W. H. Brattain—1058(A)
Protein molecules on quartz, O. Wintersteiner, H. A. Abramson—372(A)

Aerodynamics (see also **Hydrodynamics**)
Boundary layer flow, H. L. Dryden—1046(A)
Effect of turbulence on cup anemometers, H. L. Dryden, W. C. Mock, Jr., G. B. Schubauer—1042(A)
Theory of cup anemometer, G. Grimminger—1042(A)
Turbulence in the atmosphere, L. P. Harrison—1042(A)

Affinity
Electron affinity spectrum of I, O. Oldenberg—374(A); 534

Afterglow
From N_2 and mixtures with inert gas, L. Vegard—1060(A)

Alloys
$CuAl_2$ in duralumin, G. L. Clark, H. A. Smith—305(L) L. W. Kempf—942(L)
Magnetic susceptibilities of, C. G. Montgomery, W. H. Ross—358

Alpha-particles (see also **Radioactivity**)
Disintegration of Al by swiftly moving protons, M. S. Livingston, E. O. Lawrence—369(L)
Range from Th, F. N. D. Kurie, G. D. Knopf—311; 781(A)

Arcs (see **Discharge of electricity in gases**)

Astrophysics
Absolute motion and the ether-drift experiment, D. C. Miller—1054(A)
Equilibrium theory of the abundance of the elements, T. E. Sterne—768(L); 585(L); 1056(A)
Radiation reaction forces and the expanding universe, R. Gunn—764(L); 1054(A)
Sensitive stellar photoelectric photometer, S. Smith—211(A)

Atomic form factors
Of K^+ and Cl^- from scattering of x-rays, G. G. Harvey—591; 707
From x-ray scattering, G. E. M. Jauncey, F. Pennell—505

Atomic and molecular beams
Magnetic velocity selector, I. I. Rabi, V. W. Cohen—377(A)
Scattering of K atoms, W. H. Mais, I. I. Rabi—378(A)

Atomic and molecular radii
By measurement of scattering of K atoms, W. H. Mais, I. I. Rabi—378(A)

Atomic structure (see **Energy states of atoms; Spectra**)

Atomic weights (see **Isotopes**)

Aurora
Auroral spectrum, J. Kaplan—214(A)
Luminescence from solidified gases, L. Vegard—1060(A)

Barkhausen effect (see **Magnetic properties**)

Barrier-layer photo-cells
Response to x-rays, P. R. Gleason—775(A)

Biophysics
Absorption spectra of cancer cells, A. J. Allen, R. Franklin, E. McDonald—1041(A)
Conduction of nervous impulse, N. Rashevsky—372(A)
Fluorescence of substances containing vitamin A, J. W. Woodrow, A. R. Schmidt—379(A)
Lethal effects of ultraviolet radiation on protozoa, R. Franklin, A. J. Allen, E. McDonald—1041(A)
Muscle contraction, spectra from striated muscle, A. Sandow—1041(A)
Potential and charge of a protein surface in a liquid; dielectric constant of mediums, H. A. Abramson, J. Daniel—773(A)

Capillary action
Motion of an air-segment, W. Schriever, J. F. Evans —372(A)

Chemical reactions
Decomposition of ozone, A. W. Ewell—779(A)

Cold emission (see also **Field currents**)
Effect of temperature, A. J. Ahearn—1058(A)
From Hg surfaces, J. W. Beams—781(A)
Surfaces which require very high gradients, C. C. Chambers—768(L)

Collision diameter (see **Atomic and molecular radii**)

Collision of second kind
Factors affecting action cross section, O. S. Duffendack, K. Thomson—106
With metastable atoms, E. W. Pike—144(L)

Conductivity, electrical (see **Electrical conductivity and resistance discharge of electricity in gases**)

Conductivity, thermal (see **Thermal conductivity**)

Constants, physical; Standards
Absolute determination of ampere, H. L. Curtis, R. W. Curtis—1059(A)
Absolute determination of the ohm, H. L. Curtis, C. Moon, C. M. Sparks—1041(A)
e by the dispersion of x-rays, J. A. Bearden, J. A. Wheeler —1059(A)
e/m by a new deflection method, F. G. Dunnington—404
e/m by means of photoelectrons excited by x-rays, G. G. Kretschmar—417
Photoelectric determination of h, L. A. DuBridge—727

Contact potentials
Between different faces of Cu single crystals, H. E. Farnsworth, B. A. Rose—375(A)
Theory of, A. T. Waterman—1048(A)

Cosmic radiation
Airplane intensity measurements, L. M. Mott-Smith, L. G. Howell—381(A)
Angular distribution, T. H. Johnson, E. C. Stevenson—583(L); J. C. Stearns, R. D. Bennett—1038(L)
Angular distribution at different altitudes, T. H. Johnson—307; 385(A); At Mexico City—1059(A)
Angular distribution of the low energy rays, T. H. Johnson, E. C. Stevenson—775(A)
Azimuthal asymmetry positively charged component, T. H. Johnson—834(L)
Azimuthal variation of intensity, T. H. Johnson—381(A)
Bursts of ionization, C. D. Anderson—368(L)
Detection by vacuum tube methods, J. R. Dunning—380 (A)
Energy-loss and scattering measurements, C. D. Anderson—381(A)
Evidence regarding nature of rays, A. H. Compton—382(A)
Expansion chamber data, G. L. Locher—381(A)
Fluctuations over 15 day period, J. W Broxon, G. T. Merideth, L. Strait—687
Geographic study of cosmic rays at various altitudes and latitudes, A. H. Compton—387
High speed ions of stellar origin, R. Gunn—380(A)
Hodoscope and a circuit for recording multiple coincident discharges, T. H. Johnson—379(A)
Intensities in the stratosphere, I. S. Bowen, R. A. Millikan—695
Ionization by hard γ-rays, E. J. Workman—859
Ionization by individual rays, W. F. G. Swann—775(A)
Latitude effect calculation of energies, G. Lemaitre, M. S. Vallarta—87
Latitude effect on secondary electrons, R. M. Langer—215(A)
Location of origin, F. Zwicky—147(L)
Mechanism of acquirement of energies, W. F. G. Swann —217
Nature of the primary radiation, W. F. G. Swann—945 (L)
New circuit for coincidence measurements, B. Sparks, W. Pickering—214(A)
New technique and some results, R. A. Millikan, H. V. Neher—381(A)
New techniques and results obtained, R. A. Millikan—661
New type of reasoning and some consequences, F. Zwicky—1031(L)
Non-unidirectional emission of Hoffman's Stösse, W. F. G. Swann, C. G. Montgomery—782(A)
Positive electron, C. D. Anderson—491
Positively charged component of cosmic rays, L. Alvarez, A. H. Compton—835(L)
Secondaries, production of, J. C. Street—1055(A)
Solar influence, R. A. Millikan, H. V. Neher—211(A)
Use of A in measuring cosmic rays and γ-rays, J. J. Hopfield—675

Critical potentials (see **Potentials, critical**)

Crystals and crystal structure
Contact potential difference between different faces of Cu single crystals, H. E. Farnsworth, B. A. Rose—375(A)
Cooperative phenomena, F. Zwicky—215(A) 270
Crystalline fields and paramagnetic susceptibilities, W. G. Penney—485
$CuAl_2$ in duralumin, G. L. Clark, H. A. Smith—305(L); L. W. Kempf—942(L)
Dielectric losses in rocksalt, P. L. Bayley—355
Effect of temperature on the reflection of x-rays, A. Goetz, R. B. Jacobs—213(A)
Ferromagnetic crystals; models, F. Bitter—655
Harmonic thickness vibration of quartz plates, S. Nishikawa, Y. Sakisaka, I. Sumoto 363(L)
Intensity formula for the powder method, F. C. Blake—376(A)
Interatomic distance calculations, M. L. Huggins, J. E. Mayer—373(A)
Magnetic dipole energy, L. W. McKeehan—1022
Magnetic dipole energy in hexagonal crystals, L. W. McKeehan—1025

Magnetic dipole fields in dislocated cubic crystals, L. W. McKeehan—924
Magnetic fields in unstrained cubic crystals, L. W. McKeehan—913
Magnetostriction in Bi single crystals, A. Wolf, A. Goetz—213(A)
Passage of photoelectrons through mica, N. E. Bradbury, L. A. Young—84(L)
Penetration of water into plastic rocksalt, R. B. Barnes—82(L)
Plasticity of crystals, F. Zwicky—765(L)
Plasticity of rocksalt, A. G. Smekal, 366(L)
Resistance of Bi single crystals in magnetic fields, O. Stierstadt—577
Of Rochelle salt and effect of temperature, B. E. Warren, H. M. Krutter—500(A)
Of sodium uranyl acetate, I. Fankuchen—1048(A)
Tension and electrical resistance, M. Allen—569
Thermal expansion of Bi single crystals, T. L. Ho, A. Goetz—213(A)
Unit cell of U, T. A. Wilson—781(A)
Vibrations of a quartz plate, H. Osterberg—819
Of vitreous silica and Pyrex, B. E. Warren—1048(A)
Weiss law for Rochelle salt, H. Müller—500(A)
X-ray lines of cold-worked Al, L. Thomassen, J. E. Wilson—763(L)
Zeeman effect in crystals, F. H. Spedding—214(A)

Dielectric constants (see **Dielectrics and dielectric properties**)

Dielectrics and dielectric properties
Dielectric constant of dilute non-aqueous solutions; frequency variation, J. L. Oncley, J. W. Williams—341
Losses in rocksalt, P. L. Bayley—355
Potential and charge of a protein surface in a liquid, H. A. Abramson, J. Daniel—773(A)
Weiss law for Rochelle salt, H. Müller—500(A)

Discharge of electricity in gases (see also **Electrons in gases**)
Initiation of discharges in high vacua, J. W. Beams—382(A)
Momentum transfer to cathode surfaces by impinging ions, E. S. Lamar—169
Momentum transfer theory, K. T. Compton, E. S. Lamar—1058(A)
Photoelectric currents in gases, L. A. Young, N. E. Bradbury—34
Photoelectric and metastable atom emission of electrons, C. Kenty—181
Plasma, Cs vapor, W walls, I. Langmuir—224
Secondary emission of electrons, S. D. Gvosdover—146(L)

Discharge of electricity in high vacua (see also **Thermionic emission; High voltage tubes and machines**)
Initiation of discharges in high vacua, J. W. Beams—382(A)

Disintegration of nucleus (see **Nucleus**)

Dissociation
Of CO by electron impact, W. W. Lozier—776(A)

Dissociation, heat of (see also **Molecular structure and constants**)
Of diatomic Bi, G. M. Almy, F. M. Spark—1043(A)

Dynamics
Figures on circular plates, R. C. Colwell—782(A)
Filtration of elastic waves in rods, R. B. Lindsay—1051(A)

Efficiency of ionization
He^{++} produced by single electron impact, W. Bleakney—378(A)
K-electron ionization of Ag by cathode rays, D. L. Webster, W. W. Hansen, F. B. Duveneck—384(A); 839

Elasticity
Analysis of compressibility data, H. M. Evjen—215(A)
Constants of rocks, W. A. Zisman—501(A)
Hooke's law and tensor analysis, C. Kaplan—137
Plasticity of rocksalt and its dependence upon water, R. B. Barnes—82(L)
Young's modulus and temperature changes, J. Zacharias—377(A)

Electrical circuits
Direct-current amplifier, L. A. Turner—1050(A)
Luminous discharge between Lecher wires, W. D. Hershberger, H. A. Zahl, M. J. E. Golay—1042(A)

Electrical conductivity and resistance (see also **Electrons in metals**)
Absolute determination of the ohm, H. L. Curtis, C. Moon, C. M. Sparks—1041(A)
Of Bi single crystals in magnetic fields, O. Stierstadt—577
Discontinuities of magnetoresistance, C. W. Heaps—763(L)
Effect of tension in Sb single crystals, M. Allen—502(A); 569
Passage of photoelectrons through mica, N. E. Bradbury, L. A. Young—84(L)
Of a rare gas irradiated by its own resonance radiation O. S. Duffendack, R. W. Smith—374(A)
Resistivity of expanded conductors, P. I. Wold—383(A); 1042(A)
Suggested explanation of superconductivity, J. Frenkel—907
Superconductivity of Sn at radiofrequencies, F. B. Silsbee, R. B. Scott, F. G. Brickwedde, J. W. Cook—1050(A)

Electrical oscillations and waves
Absorption and reradiation of short waves, C. R. Fountain—384(A)
Double refraction effects in K.-H. layers, H. R. Mimno, P. H. Wang—769(L)
Reflecting layers of the upper atmospheres, R. C. Colwell, I. O. Myers—774(A)

Electrolytes
Foundation of Debye's theory, O. Halpern—495(L)

Electromagnetic theory
Constitutive relations and tensor analysis, C. Kaplan—137
Torque on Faraday's magnet, E. H. Kennard—587(L)
Uncertainty of the field of a particle, G. Lemaitre—148(L)
Variational principles in electromagnetism, H. Bateman—481

Electromagnetic waves (see **Electrical oscillations and waves**)

Electron diffraction (see also **Scattering of electrons**)
Effect of spin interaction; polarization, J. H. Howey—499(L)
Fine structure of beams from Au crystal and from Ag on Au, H. E. Farnsworth—900
By liquids, L. R. Maxwell—1057(A)
Of low-speed electrons; surface effects, H. E. Farnsworth—778(A)
By single molecules, L. R. Maxwell, M. E. Jefferson, V. M. Mosley—777(A)
By stibnite and galena, A. G. Emslie—1056(A)
Surface effects, L. H. Germer—724; 777(A); H. E. Farnsworth—778(A)
By a W single crystal, W. T. Sproull—516
In zinc oxide and its interpretation, K. Lark-Horovitz, H. J. Yearian—376(A)

Electron lenses
For cosmic rays, W. F. G. Swann, W. E. Danforth, Jr.—778(A)
Electron beams in high vacuum, V. K. Zworykin—778(A)
Electron optics; production of beams, C. J. Davisson—777(A)

Electrons in gases
Electron capture cross sections, L. A. Young, N. E. Bradbury—1054(A)

Electrons in metals (see also **Electrical conductivity and resistance**)
Brillouin's unifield statistics, R. B. Lindsay—780(A)
Contact potential difference, A. T. Waterman—1048(A)
Flow in large direct currents, E. Weber—781(A)

Electrons, scattering of (see also **Electrons in gases**)
From Hg atom, E. B. Jordan, R. B. Brode—112; 211(A)
Perturbation theory and electron scattering, E. L. Hill—303(L)

Electrons, secondary
Ejected by Hg metastable atoms on a W surface, S. Sonkin—788
From a metal collector immersed in a discharge, S. D. Gvosdover—146(L)
By metastable atoms, S. Sonkin, H. W. Webb—776(A)
Reflection of electrons from liquid Hg, R. B. Brode, E. B. Jordan—1057(A)
From Ta, C. L. Utterback, E. A. Williams—212(A)

Electrons, thermionic (see **Thermionic emission**)

Electro-optical effects (see also **Photoelectric effect and properties**)
Kerr effect in gases, C. W. Bruce—1044(A)
Kerr effect of nonpolar molecules; theory, R. Serber—1003
Kerr effect theory, R. Serber—1011
Kerr's law at high fields, J. A. Hootman—1044(A); 749

Electrophoresis
Comparison of values, R. T. Birge—211(A)
Thermodynamic relation in the theory, O. Halpern—372(A)

Electrostatics
Semi-infinite medium of variable conductivity, L. B. Slichter—373(A)

e/m
By means of photoelectrons excited by x-rays, G. G. Kretschmar—417
By a new deflection method, F. G. Dunnington—211(A); 404

Energy states of atoms (see also **Spectra, atomic**)
Almost closed shell configuration, M. H. Johnson, Jr.—632
Interaction of configurations: $sd-p^2$, R. F. Bacher—264
Low states of the heaviest elements, Ta-You Wu, S. Goudsmit—496(L)
Vector model and the Pauli principle, M. H. Johnson, Jr.—627
Wave functions for $1s$, $2s$ and $2p$ states, P. M. Morse, L. A. Young—501(A)

Entropy
Of H_2O, W. F. Giauque, M. F. Ashley—81(L)
Of N_2, CO and H_2, W. E. Deming, L. S. Deming—1051(A)

Equations of state
Cooperative phenomena, F. Zwicky—270

Errata
Efficiency of electron release by metastable atoms, E. W. Pike—144(L)
Projective relativity and the Einstein-Mayer unified field theory, B. Hoffmann—946(L)

Evaporation
Of Mo in high electrostatic fields, G. B. Estabrook—383(A)
Possible explanations of vaporization in high fields, A. G. Worthing—383(A)

Excitation of atoms and molecules
Light excitation in Ne by Li ions, A. J. Dempster, R. E. Holzer—365(L)

Ferromagnetism (see **Magnetic properties**)

Field currents
Effect of high series resistance, R. W. Mebs—1058(A)
Effect of temperature on currents from Mo and W, A. J. Ahearn—1058(A)

Fine structure (see **Spectra, etc. and Hyperfine structure**)

Fluorescence
Of chlorophyll, H. V. Knorr, V. M. Alberts—379(A)
Effect of foreign gases in Hg, R. T. MacDonald, G. K. Rollefson—177
I fluorescence by I and A, quenching of, J. F. Koehler—1044(A)
Of substances containing vitamin A, J. W. Woodrow, A. R. Schmidt—379(A)

Galvanomagnetic effects
Gyromagnetic ratios for Ni and Co, S. J. Barnett—384(A)
Magnetization by the rotation of a magnetic field, S. J. Barnett—384(A)

Geophysics
Actinouranium and the geologic time, F. Western, A. E. Ruark—205(L); 781(A)
Flow of liquids through porous media, R. D. Wyckoff, H. G. Botset—373(A)
Isotopes of U, Th, and Pb and geologic time, C. S. Piggot—51
Seismic wave speeds and elastic constants of rocks, W. A. Zisman—501(A)
Variable conductiviy of surface layers, L. B. Slichter—373(A)

Hall effect
In large direct currents, E. Weber—781(A)

High voltage tubes and machines
Electrostatic production of high voltage, R. J. Van de Graaff, K. T. Compton, L. C. Van Atta—149; 382(A)
High velocity Hg ions, W. M. Coates, D. H. Sloan—212(A)
Method of doubling the energy of electrons, J. W. M. DuMond—1049(A)
New design for discharge tube, L. C. Van Atta, R. J. Van de Graaff, H. A. Barton—158
Portable electrostatic generator, E. H. Bramhall, R. J. Van de Graaff—382(A)
Production of 4,800,000 volt H_2 ions, M. S. Livingston, E. O. Lawrence—212(A)
Vacuum tube and Tesla coil, D. H. Sloan—213(A)
X-ray tube and constant potential transformer equipment, C. A. Poole, W. S. Werner—774(A)

Hydrodynamics
Boundary layer flow, H. L. Dryden—1046(A)
Capacity of channels having uniform inflow, K. H. Beij—1046(A)
National Hydraulic Laboratory, H. N. Eaton—1046(A)
Pressure loss for flow in curved pipes, G. H. Keulegan—1046(A)
Schroedinger's rule and stress-tensors, H. Bateman—363(L)
Theory and experiment, L. B. Tuckerman—1045(A)
Tides, P. Schureman—1046(A)

Hyperfine structure (see also **Nuclear spin**)
Absorption and emission in optically excited Hg vapor, M. L. Pool, S. J. Simmons—1045(A)
In the Al III line, 5722.6A, R. C. Williams, G. B. Sabine—362(L)
Anomalies, S. Goudsmit, R. F. Bacher—1045(A); 894
Calculation of nuclear magnetic moments, S. Goudsmit—636
Of Co, N. S. Grace—762(L)
Intermediate coupling and magnetic moments, G. Breit, L. A. Wills—1044(A)
Magnetic depolarization, A. C. G. Mitchell—780(A)
Paschen-Back effect; polarization of Cd λ2288A, N. P. Heydenburg—640
Polarization of resonance radiation, A. C. G. Mitchell—887
Separation of Na levels, N. P. Heydenburg—1045(A)
In Sr; lack of, A. N. Benson, R. A. Sawyer—766(L)

Instruments (see **Methods and instruments**)

Intensities in spectra (see also **Spectra, etc.**)
Between 1650A and 1240A with a photoelectric cell, W. M. Powell—779(A)

Ionization by neutrons
Gases at high pressures, T. W. Bonner—871

Ionization by positive ions (see also **Ionization potentials**)
In A, Ne, He N_2, CO H_2, and Hg by *K*-ions, C. A. Frische—160

Ionization potentials
Of A, Ne and He by A, Ne and He atoms, C. J. Brasefield—776(A); 785
Of N and acetylene, J. T. Tate, P. T. Smith, A. L. Vaughan—1054(A)
Of sulphur dioxide; products of ionization, H. D. Smyth, D. W. Mueller—121
Of water vapor; products of ionization, H. D. Smyth, D. W. Mueller—116

Ionization by radiation (see **Photoionization**)

Ionization, thermal
Electric-furnace ionization effect, A. S. King—214(A)

Ions in gases
Electron capture cross sections, L. A. Young, N. E. Bradbury—1054(A)
Nature of gas ions, O. Luhr—1053(A)
Types of ions in O_2 and H_2, C. D. Bock—1053(A)

Isotopes
Of Bi by magneto-optic method, F. Allison, E. R. Bishop—47
Of He; search for, J. T. Tate, P. T. Smith—672(L)
Of Kr, Se, Cd and Ge, K. T. Bainbridge—1056(A)
Lyman series lines of H, S. S. Ballard, H. E. White—941(L)
Mass of Be^9, K. T. Bainbridge—367(L)
Mass ratio of the Li isotopes, A. McKellar—215(A)
Masses of Ne^{20}, B^{11}, Ne^{22}, F^{19}, K. T. Bainbridge—424

Isotopes (Continued)
Of Ne; relative abundance, W. Bleakney—1056(A); K. T. Bainbridge—1056(A)
Of Pb by magneto-optic method, E. R. Bishop, M. Lawrenz, C. B. Dollins—43
Of Ra by magneto-optic method, E. R. Bishop, C. B. Dollins—48
And radioactive families, E. R. Bishop—38
Radioactivity of Be^9, R. M. Langer, R. W. Raitt—585 (L); 1055(A)
Search for Ne^{22}, K. T. Bainbridge—1060(A)
Separation of H_2 isotopes by fractional description, H. S. Taylor, A. J. Gould, W. Bleakney—496(L)
Spectroscopic search for H^3, G. N. Lewis, F. H. Spedding —964
Spin of H_2 isotope, G. N. Lewis, M. F. Ashley—837(L)
Of U, Th, and Pb and their geophysical significance, C. S. Piggot—51
Of U, Th and Tl by magneto-optic method, R. Goslin, F. Allison—49

Joule-Thomson effect
In He, J. R. Roebuck, H. Osterberg—60

Kerr effect (see **Electro-optical effects**)

Kinetic theory of gases
Transport phenomena in Einstein-Bose and Fermi-Dirac gases, E. A. Uehling, G. E. Uhlenbeck—552

Liquids
Air-segment in capillary, W. Schriever, J. F. Evans—372(A)
Electron diffraction, L. R. Maxwell—1057(A); L. R. Maxwell, M. E. Jefferson, V. M. Mosley—777(A)
Flow of liquids through porous media, R. D. Wyckoff, H. G. Botset—373(A)
Structure in ionic solutions, G. W. Stewart—1057(A)
X-ray diffraction of liquid films, K. Lark-Horovitz, E. P. Miller—1060(A)

Logic of physics
Many-valued truths, E. T. Bell—1033(L); F. Zwicky—1031(L)
New type of reasoning and some consequences, F. Zwicky—1031(L)

Low temperature phenomena
Temperatures below 1°K by demagnetization of $Gd_2(SO_4)_3 \cdot 8H_2O$, W. F. Giauque, D. P. MacDougall—768(L)

Luminescence
Decay and light absorption in phosphorescent materials, D. H. Kabakjian—378(A)
From solidified gases, L. Vegard—1060(A)

Magnetic properties (see also **Magnetostriction**)
Of B_2H_6, R. S. Mulliken—765(L)
Barkhausen; effect of a circular field with torsion, L. Tonks, K. J. Sixtus—70
Diamagnetic susceptibilities and polarizabilities, G. W. Brindley—1030(L)
Diamagnetism of carbon tetrachloride, benzene and toluene; temperature variation, G. F. Boeker—756
Dipole energy in crystals, L. W. McKeehan—1022
Dipole energy in hexagonal crystals, L. W. McKeehan —1025
Dipole fields in unstrained cubic crystals, L. W. McKeehan—913
Discontinuities of magnetoresistance, C. W. Heaps—763(L); 945(L)
Discontinuities and regions of reversed magnetization, L. Tonks, K. J. Sixtus—931
Dynamic aspect of ferromagnetic theory, F. Bitter—377(A)
Electromotive forces and Barkhausen discontinuities, C. W. Heaps—763(L); 945(L)
Of Fe-Ni alloys under pressure, R. L. Steinberger—502(A)
Of ferromagnetic crystals; models, F. Bitter—655
Ferromagnetism and cooperative phenomena, F. Zwicky —270
Fields in cubic crystals; effect of cold-working, L. W. McKeehan—924
Gyromagnetic ratios of Ni and Cs, S. J. Barnett—384(A)
Intrinsic magnetic field and specific heat, J. Kunz—377(A)
Magnetization as a function of temperature, P. S. Epstein—216(A)
Paramagnetic saturation in a single crystal, W. J. de Haas, J. v. d. Handel, C. J. Gorter—81(L)
Paramagnetic susceptibilities and crystalline fields, W. G. Penney—485
Permeability of Fe at high frequencies, W. Arkadiew —671(L)
Search by deflection-experiments for the Dirac isolated magnetic pole, M. A. Tuve—770(L)
Susceptibilities of common gases, G. G. Havens—992
Susceptibilities of Pb, Ag and their alloys, C. G. Montgomery, W. H. Ross—358

Magnetostriction
In Bi single crystals, A. Wolf, A. Goetz—213(A)

Mass defects
Of He, E. Wigner—252

Mass-spectrograph (see also **Methods and instruments**)
Comparison of masses of He and H^1, K. T. Bainbridge —103; 378(A)
Mass of Be^9 and the atomic weight of Be, K. T. Bainbridge—367(L)
Products of ionization in N and acetylene, J. T. Tate, P. T. Smith, A. L. Vaughan—1054(A)
Theory of the inverse first power electrostatic field, G. C. Munro—213(A)

Mean life
Effect of foreign gases on Hg in 2^3S_1 state, R. T. Macdonald, G. K. Rollefson—177
From magnetic depolarization, A. C. G. Mitchell—780 (A)
Of states of Hg atom, A. C. G. Mitchell—887

Mechanics (see **Dynamics**)

Mechanics, quantum—atomic structure and spectra
Configuration interaction in complex spectra, C. W. Ufford—1047(A)
Configuration p^5p of rare gases, G. H. Shortley—1047(A)
Dispersion and absorption of He, J. A. Wheeler—258
Electron affinity spectrum of H_2, C. K. Jen—540
Energy levels and Zeeman effect, D. R. Inglis, N. Ginsburg—194
Interaction of valence electron with closed shell, M. H. Johnson, Jr., G. Breit—1047(A)
Variable scale wave functions, J. P. Vinti, P. M. Morse—337
Wave functions for Bo, F. W. Brown, C. G. Dunn—1047(A)
Wave functions for $1s$, $2s$ and $2p$ states, P. M. Morse, L. A. Young—501(A)

Mechanics, quantum—general
Approximations by Rayleigh-Ritz variation method, J. K. L. MacDonald—830
Conservation theorems, E. L. Hill—780(A)
Continuous β-ray spectrum, A. Bramley—767(L)
Disintegration of Li by protons, J. R. Oppenheimer—380(A)
Interaction between atoms with s-electrons, N. Rosen, S. Ikehara—5; N. Rosen—671(L)
Kowalewski's top, Otto Laporte—548; 782(A)
Orthogonality of scattered wave functions, E. H. Kennard, 378(A)
Particle with a q-number charge, V. Rojansky—207(L)
Perturbation theory and electron scattering, E. L. Hill—303(L)
Pressure broadening of spectral lines, H. Margenau—129; 374(A)
Probabilities of K-electron ionization by cathode rays, D. L. Webster, W. W. Hansen, F. B. Duveneck—384(A); 839
Properties of similar particles, E. E. Witmer, J. P. Vinti—780(A)
Uncertainty of electromagnetic field of a particle, G. Lemaitre—148(L)

Mechanics, quantum—molecular structure and spectra
Of Be H; table of integrals, C. Ireland—329
C state of Li_2, W. H. Furry—361(L)
Electronic states; theory of the double bond; polyatomic molecules, R. S. Mulliken—279
Perturbations due to spin-orbit forces, J. H. Van Vleck—1047(A)
Structure of ethylene and theory of double bond, W. G. Penney—1048(A)

Mechanics, quantum—of solid bodies
Constitution of metallic Na, E. Wigner, F. Seitz—804; 1048(A)

Mechanics, statistical
Averages for perturbed systems, R. Serber—320
Brillouin's unified statistics, R. B. Lindsay—780(A)
Transport phenomena in Einstein-Bose and Fermi-Dirac gases, E. A. Uehling, G. E. Uhlenbeck—552

Metals
Constitution of metallic Na, E. Wigner, F. Seitz—804; 1048(A)
$CuAl_2$ in duralumin, G. L. Clark, H. A. Smith—305(L)
Transparent films of Mn, J. B. Nathanson—383(A)

Metastable atoms and molecules
Action of Hg atoms on W surfaces, S. Sonkin—788; S. Sonkin, H. W. Webb—776(A)
^2D-level of N, H. Stücklen, E. P. Carr—944(L)
And efficiency of ionization, E. W. Pike—144(L)
Emission of electrons from surfaces, C. Kenty—181

Meteorology (see also **Aerodynamics**)
Effect of turbulence on cup anemometers, H. L. Dryden, W. C. Mock, Jr., G. B. Schubauer—1046(A)
High speed ions of stellar origin, R. Gunn—380(A)
Receiving circuit for K.-H. layer observations, H. R. Mimno, P. H. Wang—774(A); double refraction effects—769(L)
Reflecting layers of the upper atmospheres, R. C. Colwell, I. O. Myers—774(A)
Theory of cup anemometer, G. Grimminger—1042(A)
Turbulence in the atmosphere, L. P. Harrison—1042(A)

Methods and instruments
α-particles from ionization chamber materials, J. A. Bearden—376(A)
Al-Mg mirrors, H. W. Edwards—205(L); J. Strong—498(L)
Applications of the absolute differential calculus to physics, C. Kaplan—137
Attainment of temperatures below 1°K, W. F. Giauque, D. P. MacDougall—768(L)
Ballistic galvanometer in high vacuum, W. B. Ellwood—1042(A)
Characteristics and functions of Thyratrons, A. W. Hull—379(A); see *Physics*, Vol. **4**, p. 66 (1933)
Circuit for coincidence measurements of cosmic rays, B. Sparks, W. Pickering—214(A)
Cosmic-ray hodoscope and a circuit of recording multiply coincident discharges, T. H. Johnson—379(A)
Determination of work functions, G. W. Fox, R. M. Bowie—1057(A)
Dielectric constant at high frequencies, J. L. Oncley, J. W. Williams—341
Direct-current amplifier, L. A. Turner—1050(A)
Doubling energy of electrons, J. W. M. DuMond—1049(A)
Effect of turbulence on cup anemometers, H. L. Dryden, W. C. Mock, Jr., G. B. Schubauer—1046(A)
Electrostatic production of high voltage, R. J. Van de Graaff, K. T. Compton, L. C. Van Atta—149; 382(A)
Estimates of probable error using prior information, W. E. Deming—1059(A)
Etching and rocking curve widths of x-rays, K. V. Manning—1050(A)
Evaporation technique for production of Al mirrors, J. Strong—498(L)

Methods and instruments (Continued)
Focussing method for the angular distribution of electrons; collisions with gases, A. P. Gagge—776(A)
Fusion method for determining Ra content of rocks, R. D. Evans—211(A)
High potential porcelain x-ray tube, R. Crane, C. C. Lauritsen—212(A)
High tension electrostatic voltmeter, J. E. Shrader—774(A)
High vacua in large metal chambers, M. S. Livingston—214(A)
High voltage discharge tube, L. C. Van Atta, R. J. Van de Graaff, H. A. Barton—158
Intensification of x-ray spectrograms, E. Wainer—303(L)
Intensity measurements with photoelectric cell, W. M. Powell—779(A)
Interferometer for analyzing vibrations, H. Osterberg—819
Logarithmic protractor, H. J. Yearian—779(A)
Luminous discharge between Lecher wires, W. D. Hershberger, H. A. Zahl, M. J. E. Golay—1042(A)
Magnetic velocity selector for molecular beams, I. I. Rabi, V. W. Cohen—377(A); 1044(A)
Mechanical interval recorder, W. Albertson—501(A)
Mechanical interval sorter for complex spectra, G. R. Harrison—779(A)
Micro-moving pictures of protozoa, R. Franklin, A. J. Allen, E. McDonald—1041(A)
Modulator for K.-H. layer recording, H. R. Mimno, P. H. Wang, P. B. King—501(A)
Nuclear studies using the Van de Graaff generator, M. A. Tuve, L. R. Hafstad, O. Dahl—1055(A)
Photographic determination of the half-period of Th *B*, T. R. Wilkins, S. D. Burdick—1059(A)
Porous plug experiment, J. R. Roebuck, H. Osterberg—60
Portable high tension voltmeter, H. T. Clark—382(A)
Portable roentgenmeter, C. C. Lauritsen—212(A)
Properties of metals in the region of ultra-short electromagnetic waves, G. Potapenko, R. Sanger—210(A)
Receiving circuit for K.-H. layer observations, H. R. Mimno, P. H. Wang—774(A)
Response of desensitized emulsion to α-rays, T. R. Wilkins, R. N. Wolfe—1052(A)
Reststrahlen plates, preparation of, R. B. Barnes—31
Sensitive stellar photoelectric photometer, S. Smith—211(A)
Separation of H_2 isotopes by fractional desorption, H. S. Taylor, A. J. Gould, W. Bleakney—496(L)
Specified wave forms with the photoelectric siren, H. P. Knauss—375(A)
Spectra from striated muscle, A. Sandow—1041(A)
Theory of the inverse first power electrostatic field, G. C. Munro—213(A)
Transmission band of alkali metals in ultraviolet, R. W. Wood—779(A)
Use of A in measuring cosmic rays and γ-rays, J. J. Hopfield—675
Vacuum tube for measuring small unidirectional currents and voltages, G. Failla—774(A)
Vacuum tube methods, for corpuscular radiation, J. R. Dunning—380(A)
X-ray absorption by the use of the FP-54 Pilotron, L. M. Heil, J. E. Edwards—1
X-ray tube and constant potential transformer equipment, C. A. Poole, W. S. Werner—774(A)

Molecular beams (see **Atomic and molecular beams**)

Molecular structure and constants
Of beryllium hydride; table of integrals, C. Ireland—329
Calculation of molecular vibration frequencies, D. H. Andrews, V. Deitz, S. Goldheim—373(A)
Of CO_2, A. Adel, D. M. Dennison—716
Of CO, G. H. Dieke, J. W. Mauchly—12
C state of Li_2, W. H. Furry—361(L)
Electronic configuration of B_2H_6, R. S. Mulliken—765(L)
From envelopes of absorption bands, S. L. Gerhard, D. M. Dennison—197
Of ethylene and theory of double bond, W. G. Penney—1048(A)
Ground state of H_3, H. M. James, A. S. Coolidge—588(L)
Of H_2O, W. F. Giauque, M. F. Ashley—81(L)
Inner shells and atomic interactions, H. M. James—589(L)
Interaction between atoms with *s*-electrons, N. Rosen, S. Ikehara—5; N. Rosen—671(L)
Of ozone, W. S. Benedict—580(L)
Of ozone and SO_2, R. M. Badger, L. G. Bonner—305(L)
Predissociation and energy curves, O. K. Rice—501(A)

Multiplets (see **Spectra, atomic**)

Neutrons
Collision radius, I. I. Rabi—838(L)
Disintegration by a slow neutron, W. D. Harkins, D. M. Gans, H. W. Newson—208(L)
Emission of γ-rays by nuclei excited by neutrons, W. D. Harkins—362(L)
Ionization of gases at high pressures, T. W. Bonner—871
Mass of the neutron, A. v. Grosse—143(L)
Model of; collision with protons, F. N. D. Kurie—672(L)
Neutron of high velocity; nuclear disintegration, W. D. Harkins, D. M. Gans, H. W. Newson—584(L)
Nitrogen disintegration, F. N. D. Kurie—771(L)
In the nucleus, A. Landé—I, 620; II, 624
Scattering and absorption, J. R. Dunning, G. B. Pegram—497(L); 775(A)
Search by deflection-experiments for the Dirac isolated magnetic pole, M. A. Tuve—770(L)

Nuclear spin (see also **Hyperfine structure**)
Of Be, A. E. Parker—1035(L)
Of CO, N. S. Grace—762(L)
Of H^2, G. N. Lewis, M. F. Ashley—837(L)
Of Li^7 from hyperfine structure, N. M. Gray—1044(A)
Of N, R. F. Bacher—1044(A); 1001
Of Na, I. I. Rabi, V. W. Cohen—582(L)

Nucleus
Conquest of the physical world, B. Davis—379(A)
Disintegration of B by protons, M. G. White, E. O. Lawrence—304(L)
Disintegration of elements of medium atomic number, M. A. Tuve, L. R. Hafstad, O. Dahl—942(L)
Disintegration of F^{19}, K. T. Bainbridge—424
Disintegration of Li by high speed protons, M. C. Henderson—98; 380(A); J. R. Oppenheimer—380(A)
Disintegration of N^{14} by capture of neutron, W. D. Harkins, D. M. Gans, H. W. Newson—584(L); 1055(A)
Disintegration by a slow neutron, W. D. Harkins, D. M. Gans, H. W. Newson—208(L)
Disintegration of swiftly moving protons, M. S. Livingston, E. O. Lawrence—369(L)
Equilibrium theory of the abundance of the elements, T. E. Sterne—768(L)
Hemi-alpha groups and energy of binding, W. D. Harkins—1059(A)
Mass defect, E. Wigner—252
Moment of Co, N. S. Grace—762(L)
Nuclear energy levels, W. D. Harkins—362(L)
Radioactivity of Be^{9}, R. M. Langer, R. W. Raitt—585 (L)
Relative radii of nuclei, T. W. Bonner—871
Spin of Na nucleus, J. Joffe, H. C. Urey—761(L)
Structure of, A. Landé, I—620; II—624
Studies using the Van de Graaff generator, M. A. Tuve, L. R. Hafstad, O. Dahl—1055(A)
Theory of the abundance of the elements, T. E. Sterne —585(L)

Optical constants and properties (see also **Electro-optical effects**)
Absorption and dispersion of O_2, R. Ladenburg, C. C. Van Voorhis—315
Of the alkali metals, R. W. Wood—1052(A)
Depolarization, optical anisotropy and Kerr effect, R. Serber—1003
Dispersion and absorption of He, J. A. Wheeler—258
Index of refraction of CO_2, F. L. Brown—373(A)
Index of refraction of water and its thermal history, V. K. La Mer, M. L. Miller—207(L)
Refractive indices of metals in thin films, J. B. Nathanson—383(A)

Optical instruments (see **Methods and instruments**)

Paschen-Back effect
Of hyperfine structure of Cd λ2288A, N. P. Heydenburg —640
Of resonance Na radiation, N. P. Heydenburg—1045(A)

Philosophy of physics (see **Logic**)

Photoelectric effect and properties
Of Cs vapor, J. Kunz—1052(A)
Energy distribution; electrons from Zn surfaces, N. E. Bradbury—502(A)
Energy distribution of photoelectrons, L. A. DuBridge —727
Equivalent circuit of a blocking-layer photo-cell, L. A. Wood—375(A)
Passage of photoelectrons through mica, N. E. Bradbury, L. A. Young—84(L)
Sensitive stellar photoelectric photometer, S. Smith—211(A)
Sensitivity and long wave limit of Mg, R. J. Cashman, W. S. Huxford—811
From surfaces in the rare gases, C. Kenty—181
Work function in the extreme ultraviolet, C. Kenty—776(A)
And x-ray polarization, D. G. Hare, P. Kirkpatrick—000(L)

Photography
Frequency of flash and the photographic intermittency effect, B. O'Brien, V. L. Parks—1052(A)
Herschel effect with x-rays, J. M. Blair—85(L)
Response of desensitized emulsion to α-rays, T. R. Wilkins, R. N. Wolfe—1052(A)

Photoionization
By hard γ-rays; secondary effects, E. J. Workman—859

Photometry
Intensification of spectrograms, E. Wainer—303(L)
Logarithmic protractor, H. J. Yearian—779(A)
Measurement in the Schumann region, E. G. Schneider —779(A)
Measurements between 1650A and 1240A with a photoelectric cell, W. M. Powell—779(A)

Piezoelectric effect
Analyzing vibrations of a quartz plate, H. Osterberg—819
Harmonic thickness of piezoelectric quartz plates, S. Nishikawa, Y. Sakisaka, I. Sumoto—363(L)

Plasmoidal discharges (see **Discharge of electricity in gases**)

Polarization, electrical (see also **Dielectrics and dielectric properties**)
Polarizabilities of ions from spectra, J. E. Mayer, M. G. Mayer—605

Polarization of radiation
Depolarization and optical anisotropy, R. Serber—1003
And hyperfine structure, A. C. G. Mitchell—887
Of resonance Na radiation, N. P. Heydenburg—1045(A)

Polymerization
Of water, V. K. La Mer, M. L. Miller—207(L)

Positive electron
Observed in Wilson chamber cosmic-ray tracks, C. D. Anderson—491; 1056(A)
Production by γ-rays, C. D. Anderson, S. A. Neddermeyer—1034(L)

Potentials, critical (see also **Ionization potentials**)
Light excitation in Ne by Li ions, A. J. Dempster, R. E. Holzer—365(L)

Predissociation
Crossing of molecular potential energy curves, O. K. Rice—501(A)

Probability of ionization (see **Efficiency of ionization**)

Quenching of radiation
I fluorescence by I and A, J. F. Koehler—1044(A)

Radio (see **Electric oscillations and waves**)

Radioactivity
Actinouranium and the geologic time, F. Western, A. E. Ruark—205(L); 781(A)
α-particles from ionization chamber materials, J. A. Bearden—376(A)
Of Be^9, R. M. Langer, R. W. Raitt—585(L); 1055(A)
Continuous β-ray spectrum, A. Bramley—767(L)
Fusion method for determining Ra content of rocks, R. D. Evans—211(A)
Half-period of Th *B*; photographic method, T. R. Wilkins, S. D. Burdick—1059(A)
Ionization by hard γ-rays, E. J. Workman—859
Radioactive families, E. R. Bishop—38
Range of α-particles from Th; Geiger-Nuttall law, F. N. D. Kurie, G. D. Knopf—311; 781(A)

Raman spectra
Of CO_2, A. Adel, D. M. Dennison—716
Of five higher alcohols, P. L. Bayley—373(A)
Scattering by crystalline H halides, E. O. Salant, D. Callihan—590(L)

Ramsauer scattering
Photoelectric currents in gases, L. A. Young, N. E. Bradbury—34

Recombination
Of atomic H_2, I. Amdur—208(L)
Spectra in Cs discharge, F. L. Mohler—374(A)

Relativity
Absolute motion and the ether-drift experiment, D. C. Miller—1054(A)
Interpretation of heat in relativistic thermodynamics, R. C. Tolman, H. P. Robertson—564
Projective relativity and the unified field theory, B. Hoffman—615; Erratum—946(L)

Resistance, electrical (see **Electrical conductivity and resistance**)

Resonance radiation
Of Br vapor, H. Plumley—495(L)
Paschen-Back effect of Cd λ2288A, N. P. Heydenburg—640

Reststrahlen
Preparation of plates; reflecting power, R. B. Barnes—31

Scattering of electrons and ions (see also **Electrons in gases; Electron diffraction**)
Elastic scattering by Hg, E. B. Jordan, R. B. Brode—112; 211(A)
Elastic scattering in Ne, A. L. Hughes, J. H. McMillen—875
K-ions in gases, C. A. Frische—160
Magnetic focussing method, A. P. Gagge—776(A)
Small-angle inelastic scattering in He, Hg and H_2, S. N. Van Voorhis—777(A)

Scattering of light (see **Raman spectra**)

Secondary electrons (see **Electrons, secondary**)

Solid state (see **Mechanics, quantum—of solid bodies**)

Sound (see **Acoustics**)

Spectra, absorption (see also **Absorption of light**)
Of ammonia in ultraviolet, J. K. Dixon—711
Of benzene in the liquid, solid and vapor states, C. F. Leberknight—967
In crystals, F. H. Spedding—143(L)
Of diatomic Bi, G. M. Almy, F. M. Sparks—1043(A)
Envelopes of infrared bands, S. L. Gerhard, D. M. Dennison—197
Fine structure of infrared bands of methane, W. V. Norris, H. J. Unger—467
Infrared bands of ammonia, H. J. Unger—123
Infrared bands of acetylene; new arrangement, G. B. B. M. Sutherland—883
Rotation spectra of NH_3 and PH_3 gases under high resolving power, H. M. Randall, N. Wright—1043(A)

Spectra, atomic
Almost closed shell configuration, M. H. Johnson, Jr.—632
Of Ca I-like ions, W. M. Cady—214(A); 322
Of Cb V and Mo VI, M. W. Trawick—1047(A)
By collisions of the second kind, O. S. Duffendack, K. Thomson—106
Configuration interaction; for Ti II and Zr II, C. W. Ufford—1047(A)
Configuration f^5p of rare gases, G. H. Shortley—1047(A)
Energy levels and Zeeman effect, D. R. Inglis, N. Ginsburg—194
Of H^2; fine structure, G. N. Lewis, F. H. Spedding—964
Interaction of valence electron with closed shell, M. H. Johnson, Jr., G. Breit—1047(A)
Lyman series lines of H^2, S. S. Ballard, H. E. White—941(L)
Metastable ^2D-level of N, H. Stücklen, E. P. Carr—944(L)
Multiplets of Cb III and Mo IV, A. Y. Eliason—745
Of Ni, Zeeman effect, H. H. Marvin, A. E. Barager—973
Pt I-like isoelectronic sequence, G. K. Schoepfle—374(A)
Of Sb VI and Te VII; Pd I isoelectronic sequence, G. K. Schoepfle—742
Of S I and S II, J. E. Ruedy—1045(A)
Transformations in complex spectra, G. H. Shortley—451
Vector model and the Pauli principle, M. H. Johnson, Jr.—627
Wave functions for Bo, F. W. Brown, C. G. Dunn—1047(A)

Spectra, continuous
Electron affinity spectrum, O. Oldenberg—374(A); 534
Electron affinity spectrum of H_2, C. K. Jen—540

Spectra, general
Auroral spectrum, J. Kaplan—214(A)
Electron affinity spectrum of I, O. Oldenberg—374(A); 534
Pressure broadening; calculated, H. Margenau—129; 374(A)
Pressure effects of foreign gases on Na, H. Margenau, W. W. Watson—1053(A)
Recombination spectra of a Cs discharge, F. L. Mohler—374(A); 1052(A)

Spectra, molecular (see also **Molecular structure and constants**)
Analysis of GeO band spectrum, R. W. Shaw—1050(A)
Analysis of infrared of CO_2, A. Adel, D. M. Dennison—1043(A)
From atomic I vapor, O. Oldenberg—501(A)
Of BaH, W. W. Watson—9; 375(A)
Of CS, F. H. Crawford, W. A. Shurcliff—766(L)
Electronic states; double bond; polyatomic molecules, R. S. Mulliken—279
Extreme ultraviolet of H, C. R. Jeppesen—214(A)
Of H^1 H^2, M. F. Ashley—770(L)
Infrared of CO_2, A. Adel, D. M. Dennison—716
Internal resonance in CO, G. H. Dieke—780(A)
Magnetic rotation spectrum of Na, W. R. Frederickson, C. R. Stannard—1043(A)
Of MgF, F. A. Jenkins, R. Grinfeld—943(L)
New band of the OH molecule, H. L. Johnston, D. H. Dawson, M. K. Walker—374(A); 473; 1057
New lines in the electronic spectrum of OH, K. Chamberlain, H. B. Cutter—771(L)
OH band (2, 2), H. L. Johnston, D. H. Dawson—580(L)
Resonance spectrum of Br, H. Plumley—495(L)
Rotation vibration spectrum of H_2S, A. D. Sprague, H. H. Nielsen—375(A)
Spin-orbit forces, J. H. Van Vleck—1047(A)
Third positive group of CO bands, G. H. Dieke, J. W. Mauchly—12
Vibrational isotope effects, E. O. Salant, J. E. Rosenthal—581(L)

Spectroscopy, technique
Al mirrors, H. W. Edwards—205(L); J. Strong—498(L)
Application of the mechanical interval recorder, W. Albertson—501(A)
Intensification of x-ray spectrograms, E. Wainer—303(L)
Mechanical interval sorter for complex spectra, G. R. Harrison—779(A)

Sputtering
Of Pt by Hg ions, J. E. Henderson, E. Gideon—601
Transparent films of Mn, J. B. Nathanson—383(A)

Standards (see **Constants, physical**)

Stark effect
Theory of; applications to Ni, Li, C and A, E. U. Condon—648

Statistical mechanics (see **Mechanics, statistical**)

Superconductivity (see **Electrical conductivity and resistance**)

Surface phenomena
Electrical potential of amorphous and crystalline amphoteric surfaces in liquids, O. Winterstein, H. A. Abramson—372(A)

Symposia
On cosmic rays, G. L. Locher—381(A); R. A. Millikan, H. V. Neher—381(A); A. H. Compton—382(A); See under Cosmic radiation

Thermal conductivity
Convective heat flux from parallel plates, R. B. Kennard—372(A)
Of metals in the range 0 to 600°C, M. S. Van Dusen, S. M. Shelton—1052(A)

Thermal diffusivity
Precise determination for Zn, R. H. Frazier—135; 762(L)

Thermal expansion
Of Bi single crystals, T. L. Ho, A. Goetz—213

Thermionic emission of electrons, emitting surfaces
Films of Cs on W; space charge sheath and image force, I. Langmuir—224
New method for determining work functions, G. W. Fox, R. M. Bowie—1057(A)
Thermionic and adsorption characteristics of Pt on W, R. W. Sears, J. A. Becker—1058(A)
Thermionic and adsorption characteristics of Th on W, W. H. Brattain, J. A. Becker—428; 1058(A)
Work function of composite surfaces, W. H. Brattain, J. A. Becker—1058(A)

Thermodynamics
Calculations from spectroscopic data, L. S. Kassel—364(L)
Cooperative phenomena, F. Zwicky—270
Free energy of formation and entropy of water, W. F. Giauque, M. F. Ashley—81(L)
Interpretation of heat in relativistic thermodynamics, R. C. Tolman, H. P. Robertson—564

Vacuum tubes
For measuring small unidirectional currents and voltages, G. Failla—774(A)

van der Waals forces
From pressure broadening of spectral lines, H. Margenau—374(A)

Vaporization (see **Evaporation**)

Viscosity
Of mixtures of liquids at high pressures, R. B. Dow—502(A)

X-rays, absorption
Anomaly in the Cb *K* limit, C. J. Burbank—83(L)

X-rays, absorption (Continued)
Secondary absorption and fine structure, C. D. Cooksey, S. T. Stephenson—670(L)
By the use of the FP-54 Pliotron, L. M. Heil, J. E. Edwards—1

X-rays, diffraction scattering, reflection, refraction and polarization
Absolute reflectivities of single crystals, P. Kirkpatrick, P. A. Ross—596
Broadening of lines of cold-worked Al, L. Thomassen, J. E. Wilson—763(L)
Diffraction in ionic solutions, G. W. Stewart—1057(A)
Diffraction of liquid films, K. Lark-Horovitz, E. P. Miller—1060(A)
Effect of temperature on the intensity of x-rays, J. J. Shonka—947
Effect of temperature on the reflection of x-rays, A. Goetz, R. B. Jacobs—213(A)
Etching and the relative intensities of double Laue spots, M. Y. Colby, S. Harris—562
Intensity of Compton radiation, E. O. Wollan—955
Laue photographs from oscillating quartz, S. Nishikawa, Y. Sakisaka, I. Sumoto—363(L)
Measurement of polarization, D. G. Hare, P. Kirkpatrick—1037(L)
Scattering by gaseous dichlorbenzenes, W. C. Pierce—145(L)
Scattering from powdered crystals, G. E. M. Jauncey, F. Pennell—505
Scattering from sylvine, G. G. Harvey—591; 707

X-rays, emission
Of Cb and Sb, P. A. Ross and P. Kirkpatrick—1036(L)
Effects of chemical combination on S $K\alpha$ and $K\beta$, J. Valasek—612
By fast Hg ions, W. M. Coates—837(L)
Fluorescence yield from U, R. J. Stephenson—527
Intensity of the Ag K-lines as a function of voltage and emergence angle, D. L. Webster, W. W. Hansen, F. B. Duveneck—385(A)
Interpretation of satellites, H. C. Wolfe—376; 221
K-series of W, J. C. Hudson, H. G. Vogt—1049(A)
Relation between line and continuous spectra, W. W. Hansen, K. B. Stoddard—701
Relative intensities of K-series lines of elements 24 to 52, J. H. Williams—1049(A)
Relative intensity of satellites, F. R. Hirsch, Jr., F. K. Richtmyer—1049(A)
Wave-lengths of Ag Mo, Cu Fe and Cr K-series lines, J. A. Bearden, C. H. Shaw—921; 1050(A)
Widths of lines of K-series of W, F. K. Richtmyer, S. W. Barnes—1049(A)

X-rays, spectra and spectroscopy, wave-length measurements
Etching and rocking curve widths, K. V. Manning—1050(A)
Intensification of spectrograms, E. Wainer—303(L)
Measurement of polarization, D. G. Hare, P. Kirkpatrick—1037(L)
Rocking curve; widths of lines, S. W. Barnes, L. D. Palmer—1050(A)
Wave-lengths of $K\alpha_1$ lines of Ag, Mo, Cu, Fe and Cr, J. A. Bearden, C. H. Shaw—1050(A); 92; 1059(A)

X-rays, tubes, apparatus
High voltage tube and constant potential transformer equipment, C. A. Poole, W. S. Werner—774(A)
High voltage tube and Tesla coil, D. H. Sloan—213(A)
Method of doubling the energy of electrons, J. W. M. DuMond—1049(A)
Porcelain x-ray tube, R. Crane, C. C. Lauritsen—212(A)
Portable roentgenmeter, C. C. Lauritsen—212(A)

Zeeman effect
Arc sprectrum of Ni, H. H. Marvin, A. E. Baragar—973
Atomic energy levels in complex spectra, D. R. Inglis, N. Ginsburg—194
In crystals, F. H. Spedding—214(A)
Of Pb II, III and IV, J. B. Green, R. A. Loring—459
For perturbed N_2^+ terms, A. E. Parker—780(A)

THE AMERICAN INSTITUTE OF PHYSICS

INCORPORATED

GOVERNING BOARD

For The American Physical Society:

KARL T. COMPTON, *Chairman*
Massachusetts Institute of Technology

GEORGE B. PEGRAM, *Secretary*
Columbia University

JOHN T. TATE
University of Minnesota

For The Optical Society of America:

PAUL D. FOOTE
The Gulf Companies

LOYD A. JONES
Eastman Kodak Company

F. K. RICHTMYER
Cornell University

For The Acoustical Society of America:

H. D. ARNOLD
Bell Telephone Laboratories

HARVEY FLETCHER
Bell Telephone Laboratories

DAYTON C. MILLER
Case School of Applied Science

For The Society of Rheology:

E. C. BINGHAM
Lafayette College

WHEELER P. DAVEY
Pennsylvania State College

A. STUART HUNTER
du Pont Rayon Company

For The American Association of Physics Teachers:

HOMER L. DODGE
University of Oklahoma

PAUL E. KLOPSTEG
Central Scientific Company

FREDERIC PALMER
Haverford College

ADMINISTRATIVE STAFF

HENRY A. BARTON, *Director*
JOHN T. TATE, *Adviser on Publications*
MADELINE M. MITCHELL, *Editorial Secretary*
Office: 11 East 38th Street, New York, New York

JOURNALS

THE PHYSICAL REVIEW
REVIEWS OF MODERN PHYSICS
PHYSICS
JOHN T. TATE, *Editor*
J. W. BUCHTA, *Assistant Editor*
E. C. BINGHAM, *Rheology Editor*

THE JOURNAL OF THE OPTICAL SOCIETY OF AMERICA
THE REVIEW OF SCIENTIFIC INSTRUMENTS WITH PHYSICS NEWS AND VIEWS
F. K. RICHTMYER, *Editor*

THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA
F. R. WATSON, *Editor*

THE JOURNAL OF CHEMICAL PHYSICS
HAROLD C. UREY, *Editor*

THE AMERICAN PHYSICS TEACHER
DUANE ROLLER, *Editor*

The American Institute of Physics is a membership corporation of which five societies are the founding members. These five societies are the principal national societies devoted to pure and applied physics in America. Their names, and those of their representatives who form the Governing Board of the Institute, are given above. The principal work of the Institute is the publication of journals sponsored by the societies or by itself. The names and the editors of the journals are also listed above.

The broad purpose of the Institute is to represent, in all matters of wide or common interest, the five thousand or more members and subscribers associated with the Founder Societies and the journals. It aims to advance the science and profession of physics, and to promote cooperation between pure research, the applied sciences and the industries. To achieve these aims, it is greatly in need of an endowment and is empowered by law to administer grants and funds of any kind.

CONTENTS

PHYS. REV. 43 (1933) JUNE 15

Effect of Temperature on the Intensity of X-Rays Scattered by Powdered Sodium Fluoride . . . . . J. J. SHONKA 947

Measurements of Intensity of Compton Modified Radiation by Means of Filters . . . . . E. O. WOLLAN 955

Spectroscopic Search for H^3 in Concentrated H^2 . . . . . GILBERT N. LEWIS AND FRANK H. SPEDDING 964

Infrared Absorption of Benzene in the Liquid, Solid and Vapor States . . . . . C. E. LEBERKNIGHT 967

Zeeman Effect in the Arc Spectrum of Nickel . . . . . H. H. MARVIN AND A. E. BARAGAR 973

Further Studies in the Spectrum of the OH Molecule: A New (2,2) Band; Satellite Series in λ3122; Λ-Type Doubling and Electronic Spin Doubling . . . . . DAVID H. DAWSON AND HERRICK L. JOHNSTON 980

Magnetic Susceptibilities of Some Common Gases . . GLENN G. HAVENS 992

Note on the Magnetic Moment of the Nitrogen Nucleus . . R. F. BACHER 1001

Theory of Depolarization, Optical Anisotropy, and the Kerr Effect . . . . . ROBERT SERBER 1003

Calculation of Statistical Averages for Perturbed Systems . ROBERT SERBER 1011

Magnetic Dipole Energy in Homogeneously Strained Cubic Crystals . . . . . L. W. MCKEEHAN 1022

Magnetic Dipole Energy in Hexagonal Crystals . . . L. W. MCKEEHAN 1025

Letters to the Editor:

Diamagnetic Susceptibilities and Polarizabilities of Ions . . G. W. BRINDLEY 1030

On a New Type of Reasoning and Some of Its Possible Consequences . F. ZWICKY 1031

Remarks on the Preceding Note on Many-Valued Truths . . . . E. T. BELL 1033

Positrons from Gamma-Rays . CARL D. ANDERSON AND SETH H. NEDDERMEYER 1034

Partial Resolution of the Be I Line λ4572.69 and Probable Spin of the Be^9 Nucleus . . . . . ALLAN E. PARKER 1035

Weak X-Ray Lines of Columbium and Antimony . . . . . P. A. ROSS AND PAUL KIRKPATRICK 1036

Lateral Distribution of Photoelectrons and the Measurement of X-Ray Polarization . . . . . DONALD G. HARE AND PAUL KIRKPATRICK 1037

Angular Distribution of Cosmic-Ray Particles . . . . . J. C. STEARNS AND R. D. BENNETT 1038

Hyperfine Structure and Isotopic Constitution of Tungsten . . . . . NORMAN S. GRACE AND H. E. WHITE 1039

Proceedings of the American Physical Society:

WASHINGTON MEETING, April 27–29, 1933; Minutes and Abstracts 1–102, Author Index 1040

Author Index to Volume 43 1063

Analytic Subject Index to Volume 43 1073

LANCASTER PRESS, INC., LANCASTER, PA.